

# Commande Par Modes de Glissement en Robotique

**PROGRAMME  
UNIT-GDR ROBOTIQUE**



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# Application of Sliding Mode Control to Robotic Systems

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## 0.1. Introduction

Most research and applications in robot control deal with electrically actuated robots (this is generally due to ease of their use and low cost) but use of pneumatic and hydraulic systems is increasing. The latter are used in robotic systems when large forces and direct drive possibilities with high capabilities are required. This justifies their extensive use in industrial applications. This chapter deals with the application of sliding mode control and passive feedback systems for mechanical systems encountered in robotics involving pneumatic and hydraulic actuators. We show that sliding mode control is a viable approach for such systems. Simple examples are studied first in order to introduce a methodology for this kind of control techniques.

<sup>1</sup>Référence: Chapitre 13 du livre édité par W. Perruquetti et J. P. Barbot: Sliding Mode Control in Engineering, Editeur Marcel Dekker, Inc., 2001. N. K. M'Sirdi, N.Nadjar-Gauthier. Sliding mode control application for robotic systems.

The mathematical model of a hydraulically or pneumatically actuated system is highly nonlinear and time-varying. Several energy conversions are present (electro-mechanical to hydraulic or pneumatic pressure and then to mechanical motion). Generally the control of such systems has been first based on classical or PID feedback approaches [1, 2]. Next the intent was to enhance the control by use of state space design and adaptive control [3, 4, 5, 6]. Standard or linearization-based control design methods have some drawbacks for pneumatic and hydraulic systems; this is due to the lack of knowledge of the model and parameters. The approximation by locally linear models is not applicable [7, 8]. Consequently the well known control methods like the computed torque or classic controllers are not directly applicable.

Recent applications in robotics involve complex systems with regard to nonlinearities, time variations, and performance requirements [9, 10]. Linearization-based methods or computed torque have been suggested in robotics as an effective way of using the nonlinear model of the system in the control law [11]. However, the dynamic parameters used in the control law must match the real ones [12]. In practical cases, neglected dynamics remain after modeling (nonlinear frictions, thermodynamic or hydraulic effects and parameter variations). The inability to consider the total dynamic model is "penalizing" for decoupling and compensation. These problems are caused by the fact that thermodynamics parameters depend on initial conditions, on temperature, pressure, added to offsets and cable effects. Anyway, this kind of control is rather sophisticated and remains complex to be implemented in real time (for fast motion) [13]. Robotic applications revealed the need for further investigation in order to enhance control robustness and reduce the implementation complexity [14]. In those cases we found that it was more efficient to use passivity-based controllers and sliding mode approach which enhanced the robustness of control by exploiting the system's robotic properties [15, 16]. They provided good performances whatever the robot configuration and desired speed.

For such system the control structure requires robustness of the feedback controller to parameter changes

and disturbances. These performances can be obtained by sliding mode control [17]. Many applications of variable structure control in robotics have been reported [18, 19, 20]. Exact modeling is not necessary, since the control is based only on knowledge of uncertainties or variation bounds of the system model[9].

The main objective of this chapter is the design of a robust control law by use of a sliding mode approach. The considered problem for sliding mode control design can be stated as follows: given a desired sliding manifold function of the system's states ( $s(x) = 0$ ), which can be nonlinear or time varying, determine a control (or input  $u$ ) such that sliding mode occurs on this sliding surface. Then the desired performance can be achieved by an involved reduced-order dynamics in the sliding regime. We show that it gives a viable alternative for high performance tasks in industrial applications. The control stability can be studied by use of Lyapunov theory and the method can be shown to obey passivity property.

The organization of this chapter is as follows. First we present some basic features on modeling of mechanical systems and the involved properties of frictions and inertia effects and actuator limits. Then we recall some key points on passive systems and hyperstability theory, in order to make comprehensive the approach of sliding mode control design based on system hyperstability. The passivity property is illustrated for different mechanical systems. Then the sliding mode control design by this approach is applied for some chosen examples and stability analysis is presented to emphasize the robustness and control parameters effects.

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## 0.2. Modeling and properties of robotic systems

### 0.2.1. Dynamics of mechanical systems

#### 0.2.1.1. Euler-Lagrange systems

For mechanical systems, the dynamic equations or the model can be formulated by means of energy quantities [21]. The model of a rigid mechanical system, with  $n$  degrees of freedom ( $n$  DOF), can be obtained by use of the Lagrange method [22, 23]:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \Upsilon \quad (0.2.1)$$

where  $L = E_c - E_p$  is the Lagrangian function,  $E_c = \frac{1}{2} \dot{q}^T M(q) \dot{q}$  is the kinetic energy, and  $E_p$  potential energy and  $\Upsilon$  are the applied external forces / torques.

Recall that  $q$ ,  $\dot{q}$ ,  $\ddot{q}$ ,  $\tau$  denote respectively the  $(n \times 1)$  vectors of joint positions, speeds, accelerations, and torques.  $M(q)$  is the  $(n \times n)$  generalized inertia matrix,  $G(q)$  is the  $(n \times 1)$  vector of gravitational forces. The matrix  $C(q, \dot{q})$  (Centripetal and Coriolis effects) is commonly obtained by use of Christoffel symbols and the matrix  $\frac{1}{2} \dot{M}(q) - C(q, \dot{q})$  is skew symmetric [24]. This leads to the general equation form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_v(\dot{q}) = \tau \quad (0.2.2)$$

where the term  $F_v(\dot{q})$  has as components  $F_{v_i}(\dot{q}_i)$  the friction and disturbance torques (all the friction effects, angular, linear and nonlinear terms and disturbances) for each joint.

### 0.2.1.2. Physical properties

The following physical properties of the rigid robots (with revolute joints) can be used for control [25, 26].

- 1)  $\exists \alpha_0, \alpha_1 \in \mathbb{R}$  such that  $\alpha_0 I_n < M(q) < \alpha_1 I_n, \forall q$
- 2)  $\exists \alpha_2 \in \mathbb{R}$  such that  $\|C(q, z)\| < \alpha_2 \|z\|, \forall q, \forall z$
- 3)  $\exists \alpha_3 \in \mathbb{R}$  such that  $\|G(q)\| < \alpha_3, \forall q$
- 4) Frictions and load disturbance torques are bounded by [27]:  $\exists \alpha_4, \alpha_5, \alpha_6 \in \mathbb{R}$  such that  $\|F_v(\dot{q})\| < \alpha_4 + \alpha_5 \|\dot{q}\|, \forall \dot{q}$ .
- 5) Real systems have limited velocities and accelerations, then we have  $\|\dot{q}\| < \dot{q}_{\max}$  and  $\|\ddot{q}\| < \ddot{q}_{\max}$ .

This limitation is introduced to take into account real physical limits of actuators and of power systems and perturbations.

The model equation (0.2.2) can be written in state space form with state components:  $x_1 = q$ ,  $x_2 = \dot{q}$  ( $x = (x_1^T, x_2^T)^T$ ) and measurable output  $y = q = x_1$ :

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1) [C(x_1, x_2)x_2 + G(x_1) - \tau] \\ y = x_1 \end{cases} \quad (0.2.3)$$

### 0.2.2. Control design approach

Variable structure control systems design can be realized in several ways. For our point of view, passivity-based design or the hyperstability approach is the most straightforward and practical method. It allows us to exploit the physical properties of the system for the control design. Then the obtained control law is more suited to the system and easier to tune. In what follows, we recall some features of passive systems theory.

### 0.2.2.1. Passive systems and hyperstability

It is not our objective, in this section, to give a detailed presentation of passivity, but only to introduce the needed notions for control design (see details in [28, 29, 21, 30, 31, 32, 33]).

**Proposition 1.** A passive system verifies the following property [34]:

$$E(t_1) = E(0) + E_s(0, t_1) - E_L(0, t_1)$$

where  $E(t_1)$  and  $E(0)$  represent, respectively, the system energy at time  $t_1$  and its initial value at  $t = 0$ ;  $E_s(0, t_1)$  is the supplied energy during  $[0, t_1]$ , and  $E_L(0, t_1)$  represents the lost energy dissipated in frictions during  $[0, t_1]$ .

For passive systems (with input  $u$  and output  $y$ ), the following property is always verified (the Popov inequality) [34]:

$$\exists \gamma : \gamma_0^2 < \infty, \quad \int_{t_0}^{t_1} y^T u dt \geq -\gamma_0^2 \quad (0.2.4)$$

For system (0.2.2) we can write the expression of time derivative of its kinetic energy:

$$\frac{1}{2} \frac{d}{dt} (\dot{q}^T M(q) \dot{q}) = \dot{q}^T [\tau - G(q)] \quad (0.2.5)$$

we obtain by integration [ $q_o = q(0)$ ]:

$$\begin{aligned} & \int_0^t \dot{q}^T (\tau - G(q)) dt = \\ & \frac{1}{2} (\dot{q}^T(t_1) M(q(t_1)) \dot{q}(t_1)) - \frac{1}{2} (\dot{q}_o^T M(q_o) \dot{q}_o) \end{aligned} \quad (0.2.6)$$

$\tau - G(q) \rightarrow y = \dot{q}$  is passive.

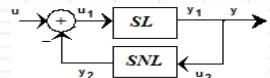
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Figure 0.2.1: Linear system with nonlinear feedback

In order to introduce the control approach let us consider the dynamic system of Figure (0.2.2.1), where the transfer function of the linear system (block SL) is  $H(p)$  and we assume that  $(A, B)$  is completely controllable and  $(A, C)$  is completely observable for a minimal realization  $(A, B, C, D)$ . The input  $u$  is assumed equal to zero.

$$H(p) = \frac{y_1(p)}{u_1(p)} = D + C[pI - A]^{-1}B \quad (0.2.7)$$

$$\dot{x} = Ax + Bu_1 = Ax - B(y_2 - u) \quad (0.2.8)$$

$$y_1 = Cx + Du_1 = Cx - D(y_2 - u) \quad (0.2.9)$$

The feedback block (SNL), may be generally a nonlinear time variant subsystem:  $y_2 = f(u_2, t, \tau)$  with  $\tau \leq t$ . It is assumed to verify the sector condition (see Figure 0.2.2.1)

$$\phi(0) = 0 \text{ and } k_1 u_2^2 \leq \phi(u_2).u_2 \leq k_2 u_2^2 \quad \forall u \in R \quad (0.2.10)$$

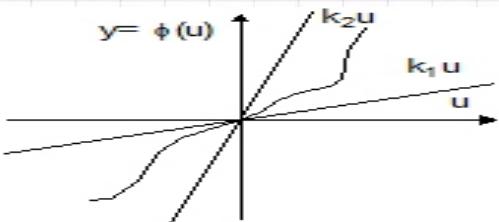


Figure 0.2.2: Conicity property for a nonlinear block ( $k_1 u^2 \leq \phi(u_2).u_2 \leq k_2 u^2 \quad \forall u \in R$ ).

**Définition 1.** The nonlinear block (SNL) is passive if it verifies the Popov inequality (for all  $t_1 > 0$ ).

$$\int_{t_o}^{t_1} y_2(\tau)^T.u_2(\tau)d\tau \geq -\gamma_o^2, \text{ with } \gamma_o^2 < \infty \quad \forall t \geq 0 \quad (0.2.11)$$

**Définition 2.** A transfer function  $H(p)$  with  $p$  complex  $p = \sigma + j\omega$  is strictly positive real (SPR) if:

- 1) The poles of  $H(p)$  are in the half plane  $\text{Re}(p) < 0$ ,
- 2)  $H(j\omega) + H^T(-j\omega)$  is positive definite Hermitian for all real  $\omega$ .

The linear system (block SL), can be characterized also by use of its minimal state space representation  $(A, B, C, D)$ , by means of the following lemma (**positive real lemma**).

**Lemme 1. positive real lemma:** Let  $H(p)$  be an  $(m \times m)$  matrix of real rational function of the complex variable  $p$ , with  $H(\infty) < \infty$ , and  $(A, B, C, D)$  the minimal state space realization of  $H(p)$  (assumed controllable and observable). Then  $H(p)$  is positive real if and only if there exist real matrices  $P, L$ , and

$W$  with  $P$  symmetric positive definite such that:

$$A^T P + PA = -LL^T, \quad PB = C^T - LW, \quad (0.2.12)$$

$$\text{and } W^T W = D + D^T \quad (0.2.13)$$

For the stability analysis of the class of nonlinear systems that can be represented in the form of Figure (0.2.2.1), the following theorems are very useful.

**Théorème 1. Hyperstability:** *The system of Figure (0.2.2.1) is hyper-stable if and only if the transfer function of the linear block  $H(p)$  is positive real (PR) and the nonlinear time-varying feedback block is passive; Every solution  $x(x(0), t)$  of the system satisfies the following property:*

$$\|x(t)\| < \delta (\|x(0)\| + \gamma_o), \quad \delta > 0, \quad \gamma_o > 0, \quad \forall t > 0 \quad (0.2.14)$$

**Théorème 2. Asymptotic hyperstability:** *The system of Figure (0.2.2.1) is asymptotically hyperstable if and only if the transfer function of the linear block  $H(p)$  is strictly positive real (SPR) and the nonlinear time-varying feedback block is passive. Every solution  $x(x(0), t)$  of the system satisfies the property (0.2.14) with  $\lim_{t \rightarrow \infty} x(t) = 0$ , for any bounded input  $u_1(t)$ .*

## 0.2.2.2. System decomposition and design objective

For control design, the problem is to find an equivalent feedback system which obviates passive parts of the system dynamics and allows us to find linear and nonlinear control terms in order to ensure asymptotic hyperstability [35]. In what follows we emphasize some interesting properties for connections of passive systems. We can note explicitly that by connecting hyperstable systems in parallel or in feedback, as

shown by Figure (0.2.2.2), we obtain a hyper-stable system, i.e. hyperstability is preserved by connections in feedback or in parallel. This property is not valid for serial or cascade connections.

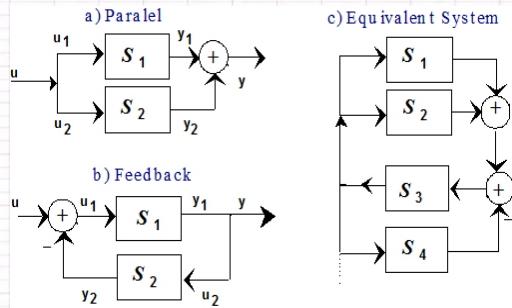


Figure 0.2.3: Passive systems associations.

Connecting the passive blocks  $S_1$  and  $S_2$  in parallel as in Figure (0.2.2.2a), leads to:  $u = u_1 = u_2$  and  $y = y_1 + y_2$  and then we have:  $\int_0^{t_1} y^T u dt = \int_0^{t_1} y_1^T u_1 dt + \int_0^{t_1} y_2^T u_2 dt$ .

The same considerations can be made for the feedback connection of two passive blocks  $S_1$  and  $S_2$ , as in Figure (0.2.2.2b). We have  $u = u_1 + y_2$  and  $u_2 = y_1 = y$ , and then  $\int_0^{t_1} y^T u dt = \int_0^{t_1} y^T (u_1 + y_2) dt = \int_0^{t_1} y_1^T u_1 dt + \int_0^{t_1} y_2^T u_2 dt$ . We can then state the following lemma.

**Lemme 2.** *Combination of two passive blocks in parallel gives a passive system.*

**Lemme 3.** *Feedback connection of two passive blocks in parallel gives a passive system.*

Then for complex passive systems, often by choosing a new system state vector, we can find a passive equivalent feedback system as a combination of parallel and feedback connections of  $n$  passive subsystems.

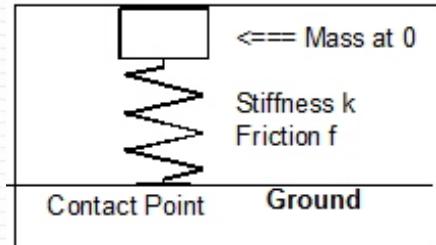


Figure 0.2.4: Mass and spring system.

These properties are very important for stability analysis and control design of complex systems (Figure 0.2.2.2c).

For mechanical system as robot manipulators or legged robots, knowledge of physical properties allows us to point out passive subsystems in the system modeling. These passive blocks can be used to find an equivalent system and then complete this scheme by appropriate control terms in order to ensure asymptotic hyperstability.

### 0.2.3. Examples

#### 0.2.3.1. Mass and spring systems

**0.2.3.1.1. A second-order example** Let us consider as an example a mass  $m$  attached to the ground through a spring with stiffness  $k$  (Figure 0.2.3.1.1).

The damping coefficient (friction) is denoted  $f$ . The system equation can be written  $m\ddot{x} + f(\dot{x}) + k(x) = u$  (the input is either the gravity force  $u = -mg$  or zero if we assume its compensation by a spring initial

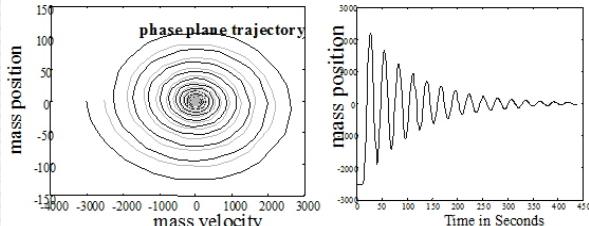


Figure 0.2.5: Trajectories in the phase plane and versus time.

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compression) for a linear case we have  $f(\dot{x}) = f\cdot\dot{x}$  and  $k(x) = k\cdot x$ .

This system, in free motion, starting from an initial position with some initial velocity has a behavior like the trajectory represented in the phase space and versus time by figure (0.2.3.1.1). This behavior is function of the system parameters.

Let us now assume that as input  $u$ , we can modify the spring length or position of its attachment to the mass (or simply the spring force by an additional term  $u$ ):  $\ddot{x} + \frac{1}{m}f\cdot\dot{x} + \frac{1}{m}k\cdot x = \frac{1}{m}u$ . Note that for this system, if we consider as input  $u$  and as output the position  $x$  ( $H(p) = \frac{1}{mp^2+fp+k}$ ), the system is not passive because it has a relative degree greater than unity.

**Remarque 1.** We can consider as output  $y$  a function of the velocity  $\dot{x}$  and the position  $x$ , such as the transfer from  $u$  to  $y$  to be SPR. For example, we can take  $y = \dot{x} + \lambda x$ , so we have as transfer function  $H(p) = \frac{p+\lambda}{mp^2+fp+k}$  and then this system is SPR if some conditions on the transfer function parameters are respected (all coefficients positive and  $\lambda < \frac{f}{m}$ ). This key point shows that use of an auxilliary signal  $y$  is important for two main features: 1) its use allows us to involve, for the control performance, the passivity characteristic of the system (we will see later that the system dynamics appears as a feedback block in the equivalent feedback system); 2) this choice must respect the dynamic of the system:  $\lambda < \frac{f}{m}$  means that the N.K. M'Sirdi [nkms@free.fr](mailto:nkms@free.fr)

introduced zero, in order to render the transfer function SPR, must be compatible with the system damping ratio.

For stabilization (at  $x = 0$ ), PD control can be applied to this system. Let us consider the following control law  $u = -k_0(\dot{x} + \lambda_0 x)$ . In closed-loop, we obtain  $\ddot{x} + \frac{1}{m}(f.\dot{x} + k_0\dot{x}) + \frac{1}{m}(k.x + k_0\lambda_0 x) = 0$ .

Stability is ensured if  $\phi_1(\dot{x})\dot{x} = (f.\dot{x} + k_0\dot{x})\dot{x} > 0$  and  $\phi_2(x)x = (k.x + k_0\lambda_0 x) > 0 \forall x, \dot{x} \in \mathbb{R} \times \mathbb{R}$ . This can be easily proved by use of the Lyapunov method with as Lyapunov candidate function  $V = \frac{1}{2}m.\dot{x}^2 + \frac{1}{2} \int_0^x (k.y + k_0\lambda y)dy$ . We obtain  $\dot{V} = -\dot{x}(f.\dot{x} + k_0\dot{x})$ . We can see clearly that  $V$  and  $-\dot{V}$  are positive if and only if stiffness and damping function verify the positivity property  $v.\phi_i(v) > 0$ .

The behavior of the system in closed-loop remains sensitive to the parameters of the system and the control  $(\ddot{x} + \frac{1}{m}(f.\dot{x} + k_0\dot{x}) + \frac{1}{m}(k.x + k_0\lambda x) = 0)$ .

For stabilization by use of sliding mode control, we can also define a commutation surface by  $s = \dot{x} + \lambda x$  [see the previous remark for SPR linear transfer function  $H(p)$ ] and apply as control law  $u = -k_0 \text{sign}(s)$  or  $u = -k_0 \text{sat}(s)$  (passive element) with the saturation function defined (for some small constant  $\varepsilon$ ) by:

$$\text{sat}(s_i) = \begin{cases} 1 & s_i > \varepsilon \\ s_i/\varepsilon & |s_i| \leq \varepsilon \\ -1 & s_i < -\varepsilon \end{cases} \quad (0.2.15)$$

The  $\text{sat}$  function verifies the passivity condition represented by Figure (0.2.3.1.1). It can be easily verified that this leads us to the scheme of Figure (0.2.2.1), with, as linear SPR transfer block  $H(p) = \frac{S(p)}{U(p)} = \frac{p+\lambda}{mp^2+fp+k}$  and nonlinear feedback block  $y_2 = f(u_2, t, \tau) = k_0 \text{sat}(s)$  which is passive. Time response and phase space trajectories obtained by this control are represented in Figure (0.2.3.1.1).

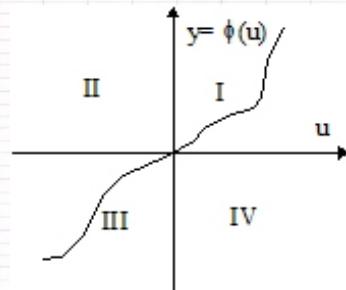


Figure 0.2.6: Memoryless passive element characteristic.

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**Remarque 2.** *Sliding mode control allows high speed responses independently from the system damping and parameters. This performance is easy to obtain with respect of the key points of the previous remark.*

*If  $f$  is too small or negative (unstable system), a preliminary feedback can be used to reinforce the system dissipation and simplify the choice of commutation surface  $s$ . With PD precompensation we have  $f' = f + k_0$  and  $k' = k + k_0\lambda_0$  instead of  $f$  and  $k$ , respectively, and then  $\frac{S(p)}{U(p)} = \frac{p+\lambda}{mp^2+f'p+k'}$ . The new constraint becomes  $\lambda < \frac{f'}{m} = \frac{f+k_0}{m}$ .*

**0.2.3.1.2. Mechanical impedance** In robotic applications involving manipulations, very often contacts between the robot and its environment appear. An example is represented by Figure (0.2.3.1.2). It can be modeled by:

$$M\ddot{x} + B\dot{x} + K.x = F \quad (0.2.16)$$

where  $M$  is the moving mass,  $B$  the damping coefficient or friction, and  $K$  the stiffness of contact (robot + environment). The applied forces are noted  $F$  and  $x$  is the cartesian position. The mechanical impedance

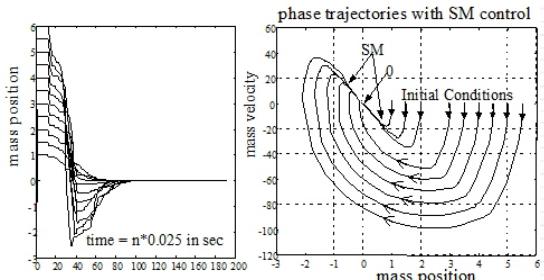


Figure 0.2.7: Behavior of the mass with passive SM-control.

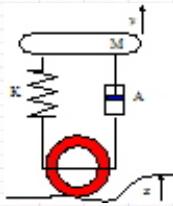


Figure 0.2.8: Suspension system.

[a transfer between force and velocity  $F = Z(\dot{x})$ ] is defined, in linear cases ( $M$ ,  $B$ , and  $K$  are three constants), by a symmetric positive definite transfer matrix:

$$Z(p) = \frac{Mp^2 + Bp + K}{p}, \text{ with } F(p) = Z(p)\dot{x}(p) \quad (0.2.17)$$

It corresponds to the energy function defined by

$$W(t) = \frac{1}{2} \{ \dot{x}^T M \dot{x} + x^T(t) K x(t) \} \quad (0.2.18)$$

Variation of energy is equal to the one supplied by  $F$  minus the friction loss:  $\dot{W}(t) = \dot{x}^T M \ddot{x} + x^T(t) K \dot{x}(t)$ ; using equation (0.2.16) we obtain:  $\dot{W}(t) = \dot{x}^T(F - B\dot{x} - Kx + Kx) = \dot{x}^T F - \dot{x}^T B\dot{x}$ . Then it can be shown that the transfer matrix  $Z(p)$  is positive real and satisfies the passivity property:

$$\int_0^t \dot{x}^T F d\tau = W(t) - W(0) + \int_0^t V(\tau) d\tau \geq -\gamma_0^2 \quad (0.2.19)$$

$$\text{with } \gamma_0^2 = W(0) \text{ and } V = \dot{x}^T B \dot{x} \quad (0.2.20)$$

This feature can be exploited for control of the behavior in case of contact between the system and its environment; this is the case for vehicles and legged robots [36]. Note that this system has the same properties as the previous simple example and then the sliding mode control is able to give the same type of results and the previously pointed out particularities can be physically interpreted. For example, preliminary PD feedback is not necessary if the system is damped enough. This is the case of the vehicle suspensions.

### 0.2.3.2. Pneumatic actuated systems

Figure (0.2.3.2) represents a pneumatic robot leg with two rigid links and two rotational joints. This robot leg is an experimental platform at the LRP. Each joint is actuated by a pneumatic cylinder (double effects linear jacks) driven by an electro-pneumatic servo-valve. The obtention of the dynamic actuator's model [1, 2, 4, 8, 11] is based on the study of the flow stage supplied with fixed pressure  $P_a$  and energy N.K. M'Sirdi nkms@free.fr

conversions. This system has the following dynamic model (see appendix):

$$\begin{cases} (ml + M(q))\ddot{q} + C'(q, \dot{q})\dot{q} + G'(q) = \tau \\ \dot{\tau} = Ji - B\tau - E\dot{q} \text{ with } \tau = K(\Delta P_p - \Delta P_n) \end{cases} \quad (0.2.21)$$

where  $m$  is the mass of the cable and of the piston that is negligible compared with the inertia of the segment,  $l$  is the radius of the pulley, and  $i$  is the servo-valve input current.

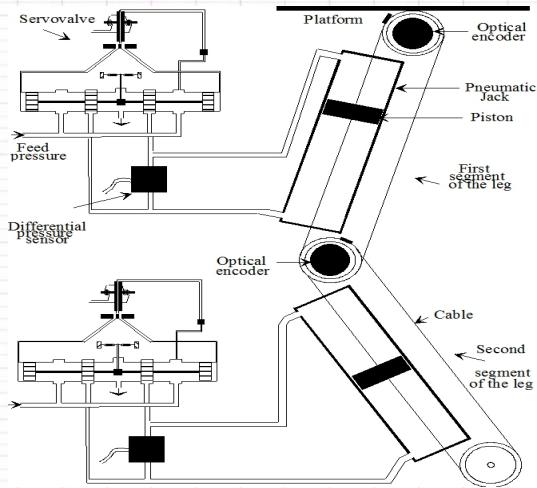


Figure 0.2.9: Pneumatic robot leg.

The output  $\tau$  can be obtained from the measure of the differential pressure applied to the piston.  $J$ ,  $B$  and  $E$  are  $(2 \times 2)$  diagonal matrices called the thermodynamics parameters depending on the temperature gas characteristics and initial conditions of pressures and chambers volume.  
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Note that the first equation of the system (0.2.21) verifies the Popov inequality (0.2.4) and then corresponds to a passive transfer between  $u = \tau - G(q)$  and  $\dot{q}$ . The second equation also involves an SPR transfer function ( $H(p) = \frac{E}{B+p}$ ). And then the system can be represented as in Figure (0.2.2.1) by a linear block ( $H(p) = \frac{E}{B+p}$ , SPR) in feedback connection with a nonlinear passive block such as:  $M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau - G(q)$  and  $\dot{\tau} + B\tau = -E(\dot{q} - \frac{J}{E}i)$ . Note that parameters  $J$ ,  $B$ , and  $E$  are assumed constant for this representation.

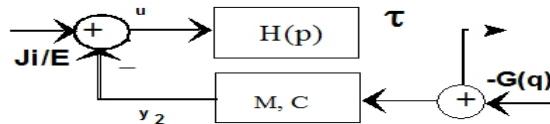


Figure 0.2.10: Passive equivalent feedback system of the pneumatic robot.

It is then obvious that the sliding mode control will be designed in order to give a passive equivalent feedback system and impose the desired dynamics.

### 0.2.3.3. Hydraulic robot manipulator

In this section, we show that a hydraulic robot manipulator designed for underwater applications (Figure 0.2.3.3 shows how a joint is actuated), has several similarities in its features with a pneumatic leg. Thus the same control approach can be applied in order to obtain good performances and robustness. From the robot dynamic equations (see for details [5][6] [37]) and introducing a term  $F_v$  comprising all the friction effects (angular, linear and nonlinear terms and disturbances), we can obtain the complete dynamic model

of the hydraulic actuated manipulator.

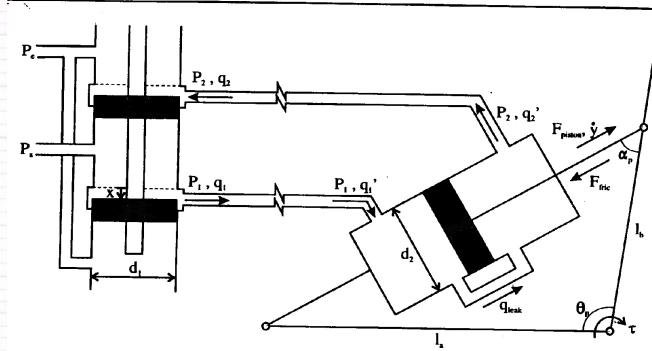


Figure 0.2.11: Scheme of one joint hydraulic actuation.

Note that  $J$ ,  $k_o$ ,  $B$ , and  $E$  are  $(n \times n)$  nonlinear diagonal matrices and depend on the actuator variables [6]. We note that the model is expressed in the same form as the pneumatic robot, so the same kind of equivalent feedback representation can be found.

Let us consider in what follows another way of representation, assuming that pressure or torque is not available for measurement. For control, we can consider as state variables the positions, velocities, and accelerations. Then the robot model can be rewritten in one stage to eliminate torque from model equations. The torque derivative is obtained analytically by derivation of its expression (arguments are

dropped for ease of notation):

$$\dot{\tau} = M\ddot{q} + \dot{M}\ddot{q} + C\ddot{q} + \dot{C}\dot{q} + \dot{G} + \dot{F}_v \quad (0.2.24)$$

We then obtain, for the global system, an equation independent of the applied torque or differential pressure measurements as follows:

$$\begin{aligned} J.k_o.i &= M\ddot{q} + (\dot{M} + C + BM)\ddot{q} + \dot{C} + \dot{F}_v + \\ &+ (\dot{C} + BC + E)\dot{q} + BG + BF_v \end{aligned} \quad (0.2.25)$$

This equation is rewritten as follows for simplicity:

$$J.k_o.i = M\ddot{q} + C\ddot{q} + \beta\ddot{q} + \gamma\dot{q} + \delta \quad (0.2.26)$$

with  $\beta = \dot{M} + BM$ ;  $\gamma = \dot{C} + E + BC$  and  $\delta = \dot{G} + \dot{F}_v + BG + BF_v$ . The latest equation will be used for control design and stability analysis. In this way, we can remark the presence of the passive transfer  $u = M\ddot{q} + C\ddot{q}$  with  $u = J.k_o.i - (\beta\ddot{q} + \gamma\dot{q} + \delta)$ .

### 0.3. Sliding mode for robot control

Sliding mode control is one of the most suitable methods to deal with systems having large uncertainties, nonlinearities, and bounded external disturbances. This approach has attracted intense research interest in the past decade for robot manipulators [38, 39, 19]. The sliding mode control is designed by means of the passive systems approach [34]. This simplifies the design and allows us to exploit the physical system

properties in a direct way. For boundedness of the derivative of the inertia matrix and the Coriolis and centrifugal terms we need the following lemma [40].

**Lemme 4.** (*Bernstein Lemma*). *If  $x(t)$  is a bounded signal ( $|x(t)| \leq M$ ) and has its frequency spectrum bounded by pulsation  $\omega_m$ , then all its derivatives are bounded such as  $\left| \frac{d^n}{dt^n} x(t) \right| \leq (\omega_m)^n M$ .*

### 0.3.1. Sliding mode control for a pneumatic system

#### 0.3.1.1. Sliding mode control design

Let us consider the dynamic model equation (0.2.21) of the robot leg and rewrite it as follows (arguments are dropped for ease of notation):

$$\begin{aligned} M\ddot{q} + C\ddot{q} + H(q, \dot{q}, \ddot{q}) &= J_i & (0.3.1) \\ H(q, \dot{q}, \ddot{q}) &= (\dot{M} + BM)\ddot{q} + (\dot{C} + E + BC)\dot{q} + (\dot{G} + BG) \end{aligned}$$

Usually the components of the matrices  $M, C, G, J, B$ , and  $E$  are not well-known but only the estimated terms  $\widehat{M}, \widehat{C}, \widehat{G}, \widehat{J}, \widehat{B}$ , and  $\widehat{E}$  and bounds of the errors  $\widetilde{M}, \widetilde{C}, \widetilde{G}, \widetilde{J}, \widetilde{B}$ , and  $\widetilde{E}$  can be available:

$$\widetilde{M} = \widehat{M} - M, \widetilde{C} = \widehat{C} - C, \widetilde{G} = \widehat{G} - G, \widetilde{J} = \widehat{J} - J, \widetilde{B} = \widehat{B} - B, \widetilde{E} = \widehat{E} - E$$

For ease of presentation we will assume  $J$  is well-known. If this is not the case this approach can be easily extended. Note that, for sliding mode control, the estimates can be chosen constant ( $\widehat{M} = M_0 = cst$ ,  $\widehat{C} = C_0 = cst, \dots$ ). Robustness toward structure errors is guaranteed by the sliding mode control [17]. Let

us define the tracking error vector as

$$\begin{bmatrix} e \\ \dot{e} \\ \ddot{e} \end{bmatrix} = \begin{bmatrix} q - q^d \\ \dot{q} - \dot{q}^d \\ \ddot{q} - \ddot{q}^d \end{bmatrix} \quad (0.3.2)$$

Let  $\ddot{q}_r$  be called the reference acceleration and defined as a function of the desired trajectories  $q^d, \dot{q}^d, \ddot{q}^d$  and errors:

$$\ddot{q}_r = \ddot{q}^d - \Lambda_1 (\dot{q} - \dot{q}^d) - \Lambda_2 (q - q^d) \quad (0.3.3)$$

Then the chosen switching surface  $s$  (function of the output trajectory error) is given by:

$$s = \ddot{e} + \Lambda_1 \dot{e} + \Lambda_2 e = \ddot{q} - \ddot{q}_r \quad (0.3.4)$$

$\Lambda_1 = \text{diag}(\lambda_1^1, \dots, \lambda_n^1)$   $\Lambda_2 = \text{diag}(\lambda_1^2, \dots, \lambda_n^2)$  are diagonal matrices with strictly positive components.

We can apply a partial feedback using the estimated nominal model  $Ji = Ji_{eq} + v$ ,  $v = K_{sat}(s)$  (the *sign* function can be also used but chattering may occur due to measurement noise). Choosing  $Ji_{eq}$  as follows gives us in closed-loop the passive equivalent system of Figure (0.3.1.1).

$$Ji_{eq} = \widehat{M}q_r^{(3)} + \widehat{C}\ddot{q}_r + \widehat{H}(q, \dot{q}, \ddot{q}) - f_v s \quad (0.3.5)$$

$$\widehat{H}(q, \dot{q}, \ddot{q}) = \left( \widehat{M} + \widehat{B}\widehat{M} \right) \ddot{q} + \left( \widehat{C} + \widehat{E} + \widehat{B}\widehat{C} \right) \dot{q} + \left( \widehat{G} + \widehat{B}\widehat{G} \right)$$

The closed-loop system can be expressed:

$$M\dot{s} + Cs = -f_v s + v - \delta \quad (0.3.6)$$

$$\delta = \left( \widetilde{M}q_r^{(3)} + \widetilde{C}\ddot{q}_r + H - \widehat{H} \right) \quad (0.3.7)$$

where  $f_v$  is a positive diagonal matrix.

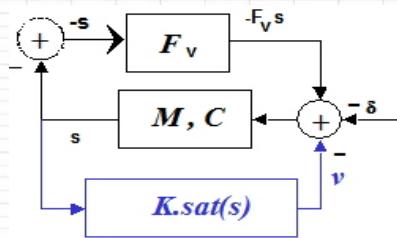


Figure 0.3.1: Equivalent feedback system for the SM-controlled robot.

The equivalent feedback system is composed of a linear block with gain  $f_v$ , a nonlinear feedback block composed by the mechanical dynamic parts and then, in feedback with this one, a nonlinear block for control commutation. The input  $\delta$  represent a perturbation due to the modeling error. We can see clearly that this perturbation can be tackled (matched uncertainty) by use of the sliding mode control term  $v = K_{sat}(s)$  to enhance the robustness of the controlled system.

### 0.3.1.2. Stability analysis

In order to prove the stability of the system in closed loop and see how to adjust the control parameters, we can consider as a Lyapunov function candidate

$$V(s, t) = \frac{1}{2} s^T M s \quad (0.3.8)$$

Differentiating (0.3.8) with respect to time yields

$$\begin{aligned} \dot{V}(s, t) &= \frac{1}{2} s^T M \dot{s} + s^T [J_i - M q_r^{(3)} - C \ddot{q} - (\dot{M} + B M) \ddot{q} \\ &\quad - (\dot{C} + E + B C) \dot{q} - (\dot{G} + B G)] \end{aligned}$$

Owing to the preceding equations, the derivative  $\dot{V}(s, t)$  becomes

$$\dot{V}(s, t) = -s^T f_v s - s^T K \text{sat}(s) - s^T (\widetilde{M} q_r^{(3)} + \widetilde{C} \ddot{q}_r + \widetilde{H})$$

with

$$K \geq \left| \widetilde{M} q_r^{(3)} + \widetilde{C} \ddot{q}_r + \widetilde{H} \right|$$

To ensure  $\dot{V} < 0$ , we choose  $K = \Delta M |q_r^{(3)}| + \Delta C |\ddot{q}_r| + \Delta H$ , where these bounds are

$$\Delta M \geq |M - \widetilde{M}| ; \Delta C \geq |C - \widetilde{C}| ; \Delta H \geq |H - \widetilde{H}|$$

Using the Bernstein lemma [40], we can prove the boundedness of the estimation errors appearing above in the Lyapunov function.

**Lemme 5.** Using the rigid robot properties and Equations (0.3.1) and (0.3.5), it can be proved that

$$i) \left\| \tilde{H} \right\| \leq \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\|; \text{ ans}$$

$$ii) \left\| \tilde{M} \ddot{\tilde{q}_r} + \tilde{C} \ddot{\tilde{q}_r} + \tilde{H} \right\| \leq \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\| + \alpha_2 \|\ddot{\tilde{q}_r}\| + \alpha_1 \|\ddot{\tilde{q}_r}\| = \eta(\ddot{\tilde{q}_r}, \ddot{\tilde{q}_r}, \ddot{q}, \dot{q})$$

This, when applied to the Lyapunov derivative, leads to  $\dot{V}(s) \leq -s^T(f_v s - K \text{sat}(s)) + \eta \|s\|$ , with a certain bound  $\eta$  positive valued function or constant depending of the maximum velocity, acceleration and desired jerk trajectory. In order to ensure  $\dot{V}(s) < 0$ , in the presence of model uncertainties, we see from the previous equation that we can choose  $K \geq \eta_o \geq \eta(\ddot{\tilde{q}_r}, \ddot{\tilde{q}_r}, \ddot{q}, \dot{q})$ , which give us

$$\dot{V}(s) \leq -s^T f_v s - (K - \eta_o) \|s\| < 0 \quad (0.3.9)$$

Then we can conclude to the stability of the system:  $s$  converges to a neighborhood of zero. Once the system state trajectory reaches a neighborhood of the switching surface, subsequent motion of the state trajectories involves the sliding of the trajectories on the surface.

### 0.3.2. Sliding mode control of a hydraulic robot

Let us rewrite the dynamic model of the hydraulic underwater manipulator as follows<sup>2</sup>:

$$J k_o i = M \ddot{q} + C \ddot{q} + H(q, \dot{q}, \ddot{q}) \quad (0.3.10)$$

$$H(q, \dot{q}, \ddot{q}) = \beta \ddot{q} + \gamma \dot{q} + \delta \quad (0.3.11)$$

The matrices  $M, C, G, J, B$ , and  $E$ , defined above, are not known but only estimates can be available

<sup>2</sup>This work was done in collaboration with the LIRMM.

for  $M_o, C_o, G_o, J_o, B_o$ , and  $E_o$ . The estimated terms are chosen constant ( $M_o = cst, \dots$ ). We assume that bounds on the estimation errors are known. The tracking error vector is defined by Equation (0.3.2) and  $\ddot{q}_r$  the acceleration reference is defined in Equation (0.3.3) with  $\Lambda_i = \text{diag}(\lambda_1^i, \dots, \lambda_n^i)$  diagonal matrices with strictly positive components. We choose as switching surface  $s$  function of the state error given by

$$s = \ddot{q} - \ddot{q}_r = \ddot{e} + \Lambda_1 \dot{e} + \Lambda_2 e \quad (0.3.12)$$

Assume  $k_o$  to be known, for simplicity, and choose the control  $i$  (based on the nominal model of the robot) as follows [37]:

$$J_o k_o i = M_o \ddot{q}_r + C_o \ddot{q}_r + H_o(q, \dot{q}, \ddot{q}) - f_v s - K_{\text{sat}}(s) \quad (0.3.13)$$

where  $f_v$  is a positive diagonal matrix chosen for transient duration adjustment and  $H_o(q, \dot{q}, \ddot{q}) = (\dot{M}_o + B_o M_o) \ddot{q} + (\dot{C}_o + E_o + B_o C_o) \dot{q} + \dot{G}_o + \dot{F}_{vo} + B_o G_o + B_o F_{vo}$ .

Let us denote the parameter estimation errors (for control) as follows:  $\widetilde{M} = J_o J_o^{-1} M_o - M$ ,  $\widetilde{C} = J_o J_o^{-1} C_o - C$ ,  $\widetilde{H} = J_o J_o^{-1} H_o - H$ .

This leads us to the same equivalent feedback scheme as for the pneumatic robot leg (see Figure 0.3.1.1), which emphasizes how the passive dynamics of the robot are involved in the control design. In this way the control involves a term [ $v = K_{\text{sat}}(s)$ ] used to tackle uncertainties on the model and the involved perturbation (which, in this manner, verify the matching condition).

Stability analysis can also be considered in the same line as for the pneumatic case by use of the Lyapunov function candidate:  $V(s) = \frac{1}{2} s^T M s$ .

Differentiating  $V(s)$  with respect to time, using the model equations and the system's passivity property

$(\frac{1}{2}\dot{M} - C)$  is skew symmetric), we obtain

$$\dot{V} = -s^T(f_v s + K \text{sat}(s) - \widetilde{M} \ddot{\tilde{q}_r} - \widetilde{C} \ddot{\tilde{q}_r} - \widetilde{H}) \quad (0.3.14)$$

**Lemme 6.** Using the rigid robot properties and Equations (0.3.10) and (0.3.13), it can be proved that

- i)  $\|\widetilde{H}\| \leq \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\|$ ; and
- ii)  $\|\widetilde{M} \ddot{\tilde{q}_r} + \widetilde{C} \ddot{\tilde{q}_r} + \widetilde{H}\| \leq \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\| + \alpha_2 \|\ddot{\tilde{q}_r}\| + \alpha_1 \|\ddot{\tilde{q}_r}\| = \eta(\ddot{\tilde{q}_r}, \ddot{\tilde{q}}, \ddot{q}, \dot{q})$

This, when applied to the Lyapunov derivative, leads to  $\dot{V}(s) \leq -s^T f_v s - s^T K \text{sat}(s) + \eta \|s\|$ , with certain bound  $\eta$  positive valued function or constant depending of the maximum velocity, acceleration, and desired jerk trajectory. Recall that accelerations and velocities are limited for real systems ( $\|\dot{q}\| < \dot{q}_{\max}$  and  $\|\ddot{q}\| < \ddot{q}_{\max}$  then  $\|\dot{q}\|^2 < \dot{q}_{\max} \|\dot{q}\|$ ). In order to ensure  $\dot{V}(s) < 0$ , in the presence of model uncertainties and system limitations, we see from the previous equation and (0.3.14) that we can choose  $K \geq \eta_o \geq \eta(\ddot{\tilde{q}_r}, \ddot{\tilde{q}}, \ddot{q}, \dot{q})$ . It leads us to

$$\dot{V}(s) \leq -s^T f_v s - (K - \eta_o) \|s\| \leq 0 \quad (0.3.15)$$

Then we can conclude to the stability of the system and  $s$  converges to a neighborhood of zero.

### 0.3.3. Simulation results

#### 0.3.3.1. Results for the hydraulic underwater manipulator

Hereafter, we will illustrate the behavior of the proposed control for 2 DOF of an underwater manipulator (Slingsby TA9)<sup>3</sup> with hydraulic actuators [5, 6, 37]. Sliding mode control has been applied in joint space. The robot is simulated using Matlab - Simulink software packages. In simulations, we must take into account the physical positions and pressure limits and choose appropriate a priori estimates for the control. We take as control  $J_o k_o i = M_o \ddot{q}_r + C_o \ddot{\dot{q}}_r - f_v s - K \text{sign}(s)$  with  $H_o = 0$  and  $M_o, C_o$  equal to their median values. Control is implemented in discrete time with  $T_s = 1ms$  as sampling time. The results presented in Figures (0.3.3.1) and (0.3.3.1), have been obtained with  $f_v = K = 2500$ .

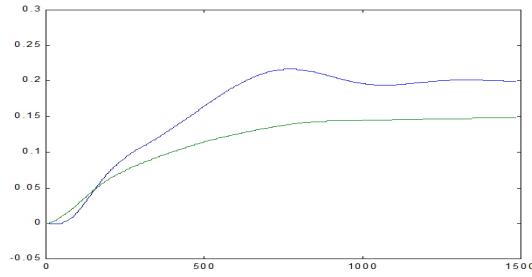


Figure 0.3.2: Positions in radians, versus time.

With these values the input current remains less than 1 volt, in the admissible zone, without saturation. The desired positions are 0.15 and 0.2 rad. The components  $M_o \ddot{q}_r$  and  $C_o \ddot{\dot{q}}_r$  anticipate the main dynamics effects. The feedback adjusts compensation by use of weights on position, velocity and acceleration errors.

<sup>3</sup>This work has been done in collaboration with the LIRMM.

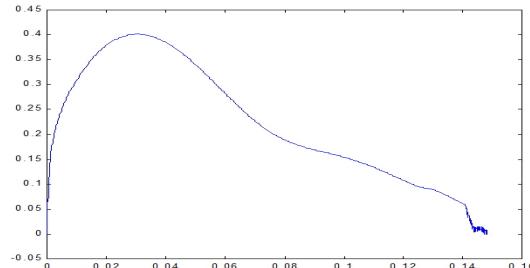


Figure 0.3.3: Phase plane for joint 1 (velocity versus position).

These two components contribute for perturbation damping and rejection. The increase of  $f_v$  allows us to enhance the time response and damp the acceleration, velocity, and position errors, during the transient, depending on weights by means of  $\Lambda_1$  and  $\Lambda_2$  (recall that  $s = \ddot{e} + \Lambda_1\dot{e} + \Lambda_2e$ ). The commutation in control enhances the robustness versus modeling errors and perturbations like friction variation  $F_v$  and torque disturbance. With  $K = 0$ , we can obtain, in the regulation case, a stable behavior with small oscillations of positions. The chattering may be eliminated by use of the commutation function (0.2.15). These features emphasize the effectiveness of the sliding control scheme for underwater manipulators.

### 0.3.3.2. Simulation results for the pneumatic robot

The following results show the obtained position velocity and acceleration errors. We can also see that no chattering appears on the control. Simulation results emphasize the robustness of the sliding mode control and the ease of its adjustment.

The sliding mode control design by use of passivity approach leads to efficient and robust controllers for pneumatic systems and manipulators with hydraulic actuators. The control law involves an acceleration

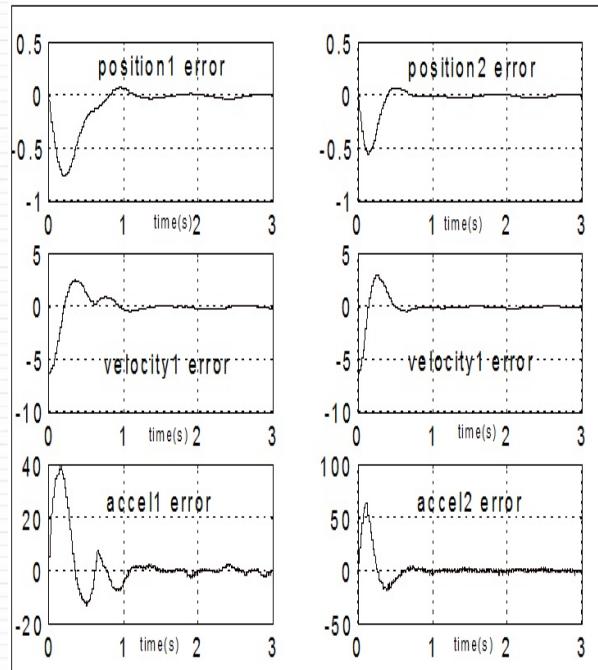
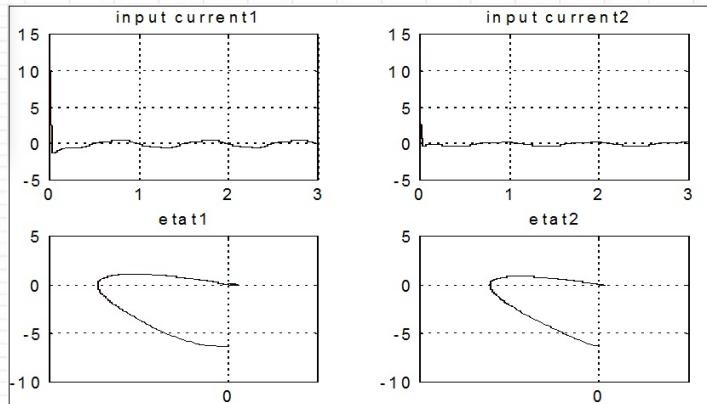


Figure 0.3.4: Position, velocity and acceleration errors.

Figure 0.3.5: Input currents and phase plane errors  $\dot{e}$  versus  $e$ .

## 0.4. SM observers based control

Several structures are possible [46, 20] for joint observation and control. We present, briefly, one of them in this section, which is an extension of the sliding observer described in [39, 49], for the pneumatic robot leg. We suppose the parameters of the model unknown and design a sliding mode observer and sliding mode controller, and then prove the closed-loop stability.

### 0.4.1. Observer design

Introducing the state vector  $x = (x_1^T, x_2^T, x_3^T)^T$  ( $x_1 = q, x_2 = \dot{q}, x_3 = \ddot{q}$ ), the model (0.3.1) can be rewritten in the following state-space representation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = f_4(x, i) \\ y = x_1 = q \end{cases} \quad (0.4.1)$$

with

$$f_4(x, i) = -M^{-1}[\left(\dot{M} + C + BM\right)x_3 + \left(\dot{C} + BC + E\right)x_2 + \left(BG + \dot{G}\right) - Ji]$$

This state-space representation is observable if only joint positions are measured [47]. In order to estimate the complete state  $\hat{x} = (\hat{x}_1^T, \hat{x}_2^T, \hat{x}_3^T)^T$  (joint positions, velocities, and accelerations) used in the control law, a nonlinear sliding structure for the state observer is considered [46, 47, 48, 19, 20].

$$\begin{cases} \dot{\hat{x}}_1 = -\Gamma_1 \tilde{x}_1 + \hat{x}_2 - \Lambda_1 sgn(\tilde{x}_1) \\ \dot{\hat{x}}_2 = -\Gamma_2 \tilde{x}_1 + \hat{x}_3 - \Lambda_2 sgn(\tilde{x}_1) \\ \dot{\hat{x}}_3 = -\Gamma_3 \tilde{x}_1 + f_4(\hat{x}, i) - \Lambda_3 sgn(\tilde{x}_1) + v \end{cases} \quad (0.4.2)$$

The term  $v$  is added in order to guarantee the stability of the observer error dynamics against the parameters uncertainties. Actually, this term is needed to account for the interaction between controller and observer.  $\Gamma_1, \Gamma_2, \Gamma_3$  are positive diagonal matrices.  $\Gamma_1 = diag(\gamma_{11}, \gamma_{21})$ ,  $\Gamma_2 = diag(\gamma_{12}, \gamma_{22})$ , and  $\Gamma_3 = diag(\gamma_{13}, \gamma_{23})$ ; they are chosen such that the linear part of the system is asymptotically stable. The

matrices  $\Lambda_1, \Lambda_2$  are chosen positive diagonal.  $\Lambda_1 = diag(\lambda_{11}, \lambda_{21})$ ,  $\Lambda_2 = diag(\lambda_{12}, \lambda_{22})$ , and the nonlinear matrix  $\Lambda_3(.)$  will be defined later.

The dynamics of the observation error  $\tilde{x} = \hat{x} - x$  is then

$$\begin{cases} \dot{\tilde{x}}_1 = -\Gamma_1 \tilde{x}_1 + \tilde{x}_2 - \Lambda_1 sgn(\tilde{x}_1) \\ \dot{\tilde{x}}_2 = -\Gamma_2 \tilde{x}_1 + \tilde{x}_3 - \Lambda_2 sgn(\tilde{x}_1) \\ \dot{\tilde{x}}_3 = -\Gamma_3 \tilde{x}_1 + f_4(\hat{x}, i) - f_4(x, i) \\ \quad -\Lambda_3 sgn(\tilde{x}_1) + v \end{cases} \quad (0.4.3)$$

In order to obtain the system's dynamics behavior inside the attractive region [49], we proceed step by step. First, we show that  $\tilde{x}_1 = 0$  has an attractive region under some conditions on velocities with the Lyapunov function  $V_1 = \frac{1}{2} \tilde{x}_1^T \tilde{x}_1$  [39]. It is obvious that  $\dot{V}_1 < 0$  under the conditions

$$|\tilde{x}_2^i| < +\lambda_{i1} + \gamma_{i1} |\tilde{x}_1^i| \quad i \in \{1, 2\} \quad (0.4.4)$$

Thus, the domains defined above and the hyperplane  $\tilde{x}_1 = 0$  are attractive. On the intersection of the region (0.4.4) and  $\tilde{x}_1 = 0$ , we consider the obtained reduced dynamics of the observation error. That is, in the mean average [50], the behavior is described by<sup>4</sup>

$$\dot{\tilde{x}}_1 = \tilde{x}_2 - \Lambda_1 \overline{sgn}(\tilde{x}_1) = 0 \quad (0.4.5)$$

$$\dot{\tilde{x}}_2 = \tilde{x}_3 - \Lambda_2 \Lambda_1^{-1} \tilde{x}_2 \quad (0.4.6)$$

In a second step, to show that  $\tilde{x}_2$  tends toward zero (existence of an attraction region), let us consider a

<sup>4</sup> $\overline{sgn}$  denotes the function equivalent, in the mean, to the  $sgn$  effect.

second Lyapunov function defined by:  $V_2 = \frac{1}{2}\tilde{x}_2^T\tilde{x}_2$ . Then,

$$\dot{V}_2 = \tilde{x}_2^T \dot{\tilde{x}}_2 = \tilde{x}_2^T (\tilde{x}_3 - \Lambda_2 \Lambda_1^{-1} \tilde{x}_2)$$

$$\dot{V}_2 = \tilde{x}_2^T \tilde{x}_3 - \tilde{x}_2^T \Lambda_2 \Lambda_1^{-1} \tilde{x}_2$$

So  $\dot{V}_2 < 0$  under the conditions [39]

$$\|\tilde{x}_3\| < \lambda_{\min} \{ \Lambda_2 \Lambda_1^{-1} \} \|\tilde{x}_2\| \quad (0.4.7)$$

The domains defined above and the hyperplane  $\tilde{x}_2 = 0$  are then attractive.

If we consider only one link, the intersection of the regions (0.4.7), and the hyperplanes  $\tilde{x}_1 = 0$ ,  $\tilde{x}_2 = 0$  is reduced to a segment. On this segment, the reduced dynamics of the observation error can be written as

$$\begin{cases} \dot{\tilde{x}}_2 = \tilde{x}_3 - \Lambda_2 \overline{sgn}(\tilde{x}_1) = 0 \\ \overline{sgn}(\tilde{x}_1) = \Lambda_2^{-1} \tilde{x}_3 \end{cases} \quad (0.4.8)$$

That is

$$\dot{\tilde{x}}_3 = f_4(\hat{x}, i) - f_4(x, i) - \Lambda_3 \Lambda_2^{-1} \tilde{x}_3 + v \quad (0.4.9)$$

with

$$\begin{aligned} f_4(\hat{x}, i) = & -\widehat{M}^{-1} \left( \dot{\widehat{M}} + \widehat{C} + \widehat{B}\widehat{M} \right) \hat{x}_3 \\ & + \left( \widehat{C} + \widehat{B} + \widehat{C}\widehat{E} \right) x_2 + \left( \widehat{B}\widehat{G} + \dot{\widehat{G}} \right) - \widehat{J}i \end{aligned}$$

We can write

$$\begin{aligned}\dot{\tilde{x}}_3 &= -M^{-1} \left( \dot{M} + C + BM + M\Lambda_3\Lambda_2^{-1} \right) \tilde{x}_3 \\ &\quad - \overline{M^{-1}C} \hat{x}_3 + \overline{M^{-1}J} i - H_1 + v\end{aligned}\tag{0.4.10}$$

with

$$\begin{aligned}H_1 &= \left( \overline{M^{-1}\dot{M}} + \overline{M^{-1}BM} \right) \hat{x}_3 + \left( \overline{M^{-1}\dot{C}} + \overline{M^{-1}BC} + \overline{M^{-1}E} \right) x_2 \\ &\quad + \left( \overline{M^{-1}BG} + \overline{M^{-1}\dot{G}} \right)\end{aligned}$$

where the notation corresponds to:

$$\begin{cases} \overline{AB} = \widehat{A}\widehat{B} - AB \\ \overline{ABC} = \widehat{A}\widehat{B}\widehat{C} - ABC \end{cases}$$

$(A, B, C)$  are three matrices, members of the set

$$\left\{ M^{-1}, \dot{M}, C, \dot{C}, G, \dot{G}, B, E, J \right\}$$

In this section we have finally obtained the reduced dynamics of the observation error. Next, the convergence condition on  $\tilde{x}_3$  will be studied in closed-loop, with the equation of tracking error.

## 0.4.2. Tracking error equation: observer and control

The control objective is to track the desired position, velocity, and acceleration  $\{x_1^d, x_2^d, x_3^d\}$ , time-varying trajectories. The sliding mode controller as developed previously [48] has the following structure:

$$i = \widehat{J}^{-1} \left( \widehat{M}\dot{x}_3^r + \widehat{C}x_3^r + \widehat{H} - F_v s - k\text{sign}(s) \right)$$

where

$$s = \ddot{e} + \delta_1 \dot{e} + \delta_2 e = (\widehat{x}_3 - x_3^d) + \delta_1 \dot{e} + \delta_2 e = \widehat{x}_3 - x_3^r$$

$$\dot{s} = e^{(3)} + \delta_1 \ddot{e} + \delta_2 \dot{e} = (\widehat{\dot{x}}_3 - \dot{x}_3^d) + \delta_1 \ddot{e} + \delta_2 \dot{e} = \widehat{\dot{x}}_3 - \dot{x}_3^r$$

$x_3^r$  is the acceleration reference signal and

$$\widehat{H} = \left( \widehat{M} + \widehat{B}\widehat{M} \right) \widehat{x}_3 + \left( \widehat{C} + \widehat{B}\widehat{C} + \widehat{E} \right) x_2 + \widehat{G} + \widehat{B}\widehat{G}$$

If we apply this control law to the system, we obtain

$$\begin{aligned} \dot{s} &= JM^{-1} \left[ - (F + J^{-1}C) s - K\text{sgn}(s) + J^{-1}M\dot{\widehat{x}}_3 \right. \\ &\quad \left. + J^{-1} \left( \dot{M} + C + BM \right) \widetilde{x}_3 + \overline{J^{-1}M} \dot{x}_3^r + \overline{J^{-1}C} x_3^r + H_2 \right] \end{aligned} \quad (0.4.11)$$

where  $F = \widehat{J}^{-1}F_v$ ,  $K = \widehat{J}^{-1}k$ ,  $F_v$  and  $k$  are positive diagonal matrices, and

$$H_2 = \left( \overline{J^{-1}\dot{M}} + \overline{J^{-1}BM} \right) \widehat{x}_3 + \left( \overline{J^{-1}\dot{C}} + \overline{J^{-1}BC} + \overline{J^{-1}E} \right) x_2 \\ + \overline{J^{-1}\dot{G}} + \overline{J^{-1}BG}$$

### 0.4.3. Stability of observer based control

The closed-loop analysis is performed on the basis of the reduced order manifold dynamics (0.4.10) and the tracking error dynamics (0.4.11). Then we define the augmented state vector  $z(t) = (s^T; \widetilde{x}_3^T)^T$  with as equation:

$$\begin{cases} \dot{s} = JM^{-1} [ - (F + J^{-1}C)s - K sgn(s) \\ \quad + J^{-1}M\dot{\widetilde{x}}_3 + J^{-1}(\dot{M} + C + BM)\widetilde{x}_3 \\ \quad + \overline{J^{-1}M\dot{x}_3^r} + \overline{J^{-1}C\dot{x}_3^r} + H_2 ] \\ \dot{\widetilde{x}}_3 = -M^{-1} (\dot{M} + C + BM + M\Lambda_3\Lambda_2^{-1})\widetilde{x}_3 \\ \quad - M^{-1}C\widehat{x}_3 + \overline{M^{-1}Ji} - H_1 + v \end{cases}$$

The following Lyapunov function is used.

$$V(s, \widetilde{x}_3) = \frac{1}{2}s^T J^{-1}Ms + \frac{1}{2}\widetilde{x}_3^T \widetilde{x}_3$$

The time derivative of  $V$  is given by

$$\begin{aligned}\dot{V} &= s^T J^{-1} M \dot{s} + \frac{1}{2} s^T J^{-1} \dot{M} s + \tilde{x}_3^T \tilde{x}_3 \\ &= -s^T F s - \tilde{x}_3^T Q \tilde{x}_3 + s^T (\beta_1 - K sgn(s)) - \tilde{x}_3^T \beta_2 + \tilde{x}_3^T v\end{aligned}$$

where

$$\begin{aligned}Q &= \widehat{M}^{-1} \dot{\widehat{M}} + \widehat{B} + \widehat{C} + \Lambda_3 \Lambda_2^{-1} \\ \beta_1 &= J^{-1} M \tilde{x}_3 + J^{-1} (\dot{M} + C + B M) \tilde{x}_3 + \overline{J^{-1} M \dot{x}_3^r} + \overline{J^{-1} C x_3^r} + H_2 \\ \beta_2 &= \left( \overline{M^{-1} \dot{M}} + \overline{M^{-1} B M} + \widetilde{C} \right) \tilde{x}_3 + \overline{M^{-1} C} \widehat{x}_3 + H_1 - \overline{M^{-1} J i}\end{aligned}$$

The stability of the closed-loop system is studied under the physical assumption of bounded system states ( $\|x(t)\| < \infty, \forall t \geq 0$ ) and the following assumptions:

$$\begin{array}{ll} \|M(x_1)\| \leq \alpha_1; & \|\widehat{M}(x_1)\| \leq \alpha'_1; \\ \|\dot{M}(x_1)\| \leq \alpha_1 w_m; & \|\dot{\widehat{M}}(x_1)\| \leq \alpha'_1 w_m; \\ \|\dot{M}(x_1)^{-1}\| \leq \alpha''_1; & \|\dot{\widehat{M}}(x_1)^{-1}\| \leq \overline{\alpha''}_1; \\ \|C(x_1, x_2)\| \leq \alpha_2 \|x_2\|; & \|\widehat{C}(x_1)\| \leq \alpha'_2 \|x_2\|; \\ \|\dot{C}(x_1, x_2)\| \leq \alpha_2 w_m \|x_2\|; & \\ \|G(x_1)\| \leq g_0; & \|\dot{G}(x_1)\| \leq g_0 w_m; \\ \|J\| \leq J_m; & \|\widehat{J}\| \leq J'_m; \end{array}$$

$$\|J^{-1}\| \leq J''_m; \quad \|B\| \leq b_m;$$

These assumptions are compatible with the real system, and come from mechanical properties of the system and limited bandwidth of the real signals. The following constants  $f_0$ ,  $q_0$ ,  $\beta_{01}$ , and  $\beta_{02}$  are defined by  $f_0 = \lambda_{\min}\{F\}$ ,  $q_0 = \lambda_{\min}\{Q\}$ , and

$$\begin{aligned} \beta_{01} &= \sup_{0 \leq \tau \leq t} \|\beta_1(\tau)\| \\ &= \sup_{0 \leq \tau \leq t} [\eta_1 \|\tilde{x}_3\| + (\eta_2 + \eta_3 \|x_2\|) \|\tilde{x}_3\| \\ &\quad + \eta_4 \|\dot{x}_3^r\| + \eta_5 \|x_2\| + k_0 + k_1 \|x_2\| \\ &\quad + k_2 \|x_2\|^2 + k_3 \|\hat{x}_3\|] \end{aligned}$$

$$\begin{aligned} \beta_{02} &= \sup_{0 \leq \tau \leq t} \|\beta_2(\tau)\| \\ &= \sup_{0 \leq \tau \leq t} [(\eta'_1 + \eta'_2 \|x_2\|) \|\tilde{x}_3\| + \eta'_3 \|x_2\| \|\hat{x}_3\| \\ &\quad + \eta'_4 \|i\| + k'_0 + k'_1 \|x_2\| + k'_2 \|x_2\|^2 \\ &\quad + k'_3 \|\hat{x}_3\|] \end{aligned}$$

where  $\eta_i, \eta'_i, k_i, k'_i$  are positive constants. The signal  $v$  can then be defined as [49]

$$v = \begin{cases} -\beta_{02} \frac{\tilde{x}_3}{\|\tilde{x}_3\|} & \text{if } \|\tilde{x}_3\| \neq 0 \\ 0 & \text{if } \|\tilde{x}_3\| = 0 \end{cases}$$

Note that on the sliding surface,  $x_3 = \hat{x}_3 - \Lambda_2 \operatorname{sgn}(\tilde{x}_1)$ , then  $v$  can be rewritten for implementation  $v = N.K. M'Sirdi nkms@free.fr$

$\frac{-\beta_{02}}{\|\Lambda_2\|} \Lambda_2 sgn(\tilde{x}_1)$ . The time derivative of  $V(s, \tilde{x}_3)$  can then be bounded as follows.

$$\dot{V} \leq -f_0 \|s\|^2 - q_0 \|\tilde{x}_3\| + \|s\| (\beta_{01} - K) + \|\tilde{x}_3\| \beta_{02} - \frac{\tilde{x}_3^T \tilde{x}_3}{\|\tilde{x}_3\|} \beta_{02}$$

$$\dot{V} \leq -f_0 \|s\|^2 - q_0 \|\tilde{x}_3\| + \|s\| (\beta_{01} - K)$$

where  $Q$  and  $F$  are diagonal positive definite matrices, and  $K$  and  $\Lambda_3$  are chosen such as:

$$K \geq \beta_{01}$$

$$\Lambda_3 = \Lambda_2 \left( Q - \widehat{M}^{-1} \dot{\widehat{M}} - \widehat{M}^{-1} \widehat{B} \widehat{M} - \widehat{C} \right)$$

So,  $\dot{V}$  is strictly negative and thus  $\tilde{x}_3$  and  $e$  tend asymptotically to zero. The system is thus asymptotically stable.

#### 0.4.4. Simulation results

In our simulations, we considered the sliding mode control law developed previously. Errors on the structure of system was taken into account. We took the matrix  $\widehat{C}(x_1, x_2) = 0$  and the inertia matrix  $\widehat{M}(x_1)$  constant diagonal, and we considered an error of 30% on the remaining parameters (J,B,E).

The results obtained show that the complete system, controller and observer, remained stable and showed a good estimation of the velocities and accelerations. For simulation, the following gains were used:  $\Lambda_1 = diag(40, 90)$ ,  $\Lambda_2 = 10\Lambda_1$ .

Note that we have also taken into account the limitations (saturation) of the servo-valve current of

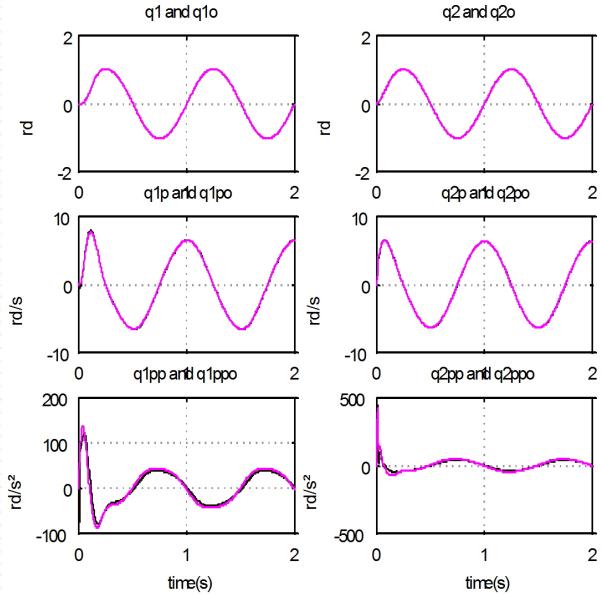


Figure 0.4.1: Actual and observed state variables

$\pm 20mA$ . For this simulation, we showed the convergence of the observed state toward the real one [Figure(0.4.4)]. We note that the convergence needs less than 0.25s and that the observer error is about 0.1% to 1% [Figure (0.4.4)]. The tracking position error is about 1% to 2%, as shown by Figure (0.4.4).

## 0.4.5. Conclusion

In this chapter we showed how sliding mode controllers can be designed using passive systems approach and coupled with a sliding mode observer for state estimation. The given sliding mode control design approach was systematic and physically well-suited with simple controllers. The passivity property of

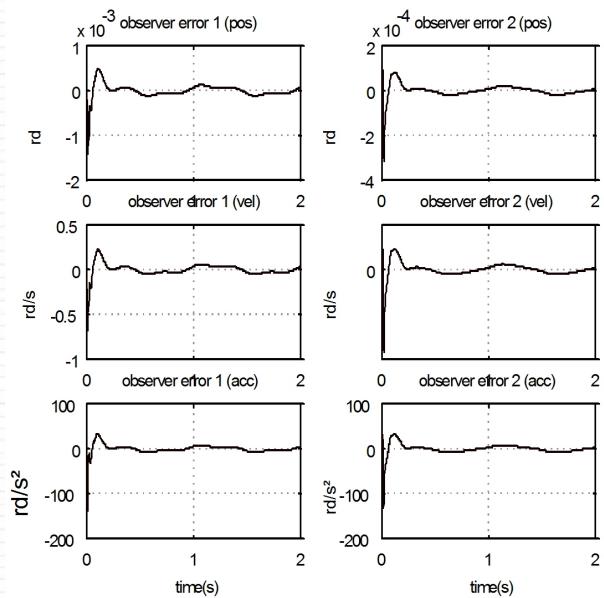


Figure 0.4.2: Observation error.

the nonlinear robot dynamics was involved to simplify the design and allowed high performance. Its use allowed us to directly choose a well-suited commutation surface and when this one was reached, in the sliding regime, the system behaved like a linear-time invariant system (with a reduced order) and was robust against modeling error, parameters variation and environment perturbations. Simulation results were presented to emphasize these features for a pneumatic robot leg and a hydraulic underwater manipulator. An observer can be used to produce estimation of velocities and accelerations for a pneumatic leg of a robot when there is a lack of knowledge of the system parameters model and structure and only angular positions are measured. The stability analysis, convergence, and simulation results emphasized robustness

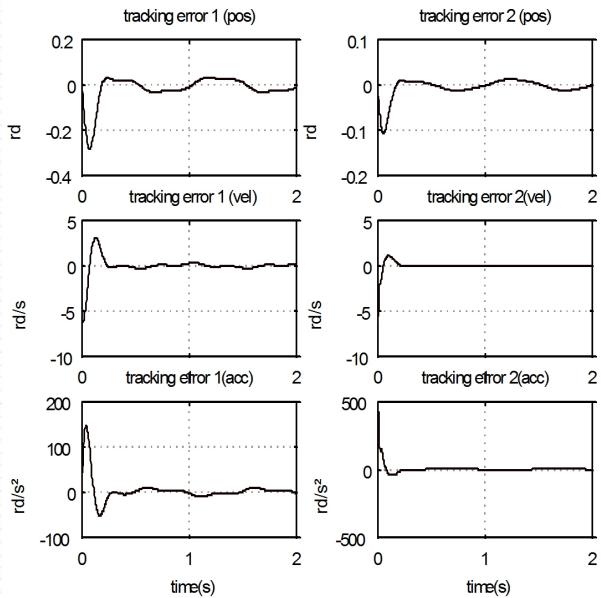


Figure 0.4.3: Tracking error.

versus uncertainties on the model parameters and efficiency of the sliding mode observers and controllers.

## 0.5. Appendix

### 0.5.1. Pneumatic actuators model

Each joint is actuated by an electropneumatic servo-valve with double effects linear jacks. It is made up of a current motor and of two pneumatic stages. It is composed of a set slide-sleeve and of four variable

restrictions modulated by the slide position. The variations of the flow rate of air drives the jacks. Figure 0.5.1 illustrates the flow stage of a servo-valve. The determination of the dynamic actuator's model [1]-[11] is based on the study of the flow stage supplied with fixed pressure  $P_a$ . The valve controls the air flow which is converted in pressure supply for the two chambers of the cylinder via four restrictions. The application of thermodynamic relationships has the following assumptions.

**Assumptions A:** The fluid is an ideal gas. Potential and kinetic energy within the fluid are neglected. No leakage exists between the two piston chambers.

**Assumption B:** The piston's displacement is due to small slide's variations around its central position. The pneumatic system is symmetric. The pressure equations can be linearized around the initial position.

The mass flow rate depends on the current  $i$ , the jack motion, and the chamber's pressure. Pneumatic actuator equations are derived from the thermodynamic study of the system.

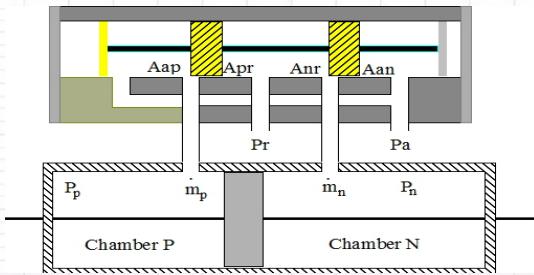


Figure 0.5.1: Flow stage of a servo-valve.

Let us define the following parameters:  $m_{ij}$  denotes the mass flow rate in restriction  $ij$

$P_p, P_n$ : pressures in chambers P and N, respectively

$P_a, P_r$ : supply and output pressures, respectively

$V_p, V_n$ : volumes in chambers P and N, respectively

$\gamma = 1.405$ : ratio of specific heat

$r = 286 J kg^{-1} K^{-1}$ : ideal gas constant

$T$  : temperature in Kelvin

$l$  : the radius of the pulley

$i$ : the motor input current

$i_0$ : initial offset current (servovalve) and  $\beta = \text{constant} \geq 0$

$S$ : denotes the cross-section area of the piston

$y$ : piston's displacement

$x$  the jack displacement

Table 1 – Thermodynamic Equations

Current action on the jack:	$i = f(x) + i_0 = \beta.x + i_0.$
Chambers fluid flow:	$\frac{dm_{ij}}{dt} = f_{ij}(x, P_{ij})$
Actuator fluid flow:	$\begin{cases} \frac{dP_p}{dt} = -\frac{\gamma.P_p}{V_p} \frac{dV_p}{dt} + \frac{rT\gamma}{V_p} \cdot \Delta\dot{m}_p \\ \frac{dP_n}{dt} = -\frac{\gamma.P_n}{V_n} \frac{dV_n}{dt} + \frac{rT\gamma}{V_n} \cdot \Delta\dot{m}_n \end{cases}$
Piston relations:	$\begin{cases} V_p = V - V_n = S.y \tau = l.F_r \\ \frac{dV_p}{dt} = -\frac{dV_n}{dt} = S\dot{y} \quad y = l.q \end{cases}$
Piston dynamic:	$\begin{cases} V(y, \dot{y}) = b_v(y, \dot{y})\dot{y} + b_c(y, \dot{y})\text{sign}(\dot{y}) \\ m\frac{dy}{dt} = S(P_p - P_n) - F_v(y, \dot{y}) - F_r \end{cases}$

The system parameters are:

$$M(q) = \begin{bmatrix} A + 4m_2l_1l_2c_2 & I_2 + 2m_2l_1l_2c_2 \\ I_2 + 2m_2l_1l_2c_2 & I_2 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} -2m_2l_1l_2\dot{q}_2s_2 & -2m_2l_1l_2(\dot{q}_1, \dot{q}_2) \\ 2m_2l_1l_2\dot{q}_1s_2 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} m_2gl_2s_{12} + m_1gl_1s_1 \\ m_2gl_2s_{12} \end{bmatrix}$$

where  $m_i$ ,  $l_i$ ,  $I_i$  are respectively the mass, length, and the segment inertia, with  $g$  the gravitation.  $A = I_1 + I_2 + 4m_2l_1^2$ ,  $c_i = \cos(q_i)$ ,  $s_i = \sin(q_i)$ ,  $c_{ij} = \cos(q_{ij})$ ,  $s_{ij} = \sin(q_{ij})$

## 0.5.2. Hydraulic manipulator model

The robot considered here is the Slingsby TA9. A detailed description and modeling of this manipulator can be found in [5]. The robot model is composed of two stages: a dynamic one for the mechanical part and a second one which will be called the hydrodynamic stage according to the hydraulic part.

The pressure, for each robot link, is converted into a force ( $F_p$  is the force applied by the piston and  $F_v$ , the friction and disturbance torques) and then into a torque by use of actuator geometric transmission and its Jacobian  $J_{pi}(q_i)$  where  $q_i$ , angular displacement of the link i.

$$J_{pi}(q_i) = \frac{l_a l_b \sin(q_i)}{\sqrt{l_a^2 + l_b^2 - 2l_a l_b \cos(q_i)}} \quad (0.5.1)$$

Lengths  $l_a$  and  $l_b$  are the geometric characteristics of the attachments of links and the actuator's cylinder. For each joint we can write, if  $y$  is the piston's displacement and  $A_2$  the cross-section area of the piston,

$$J_{p_i}(q_i)F_p = J_{p_i}(q_i)A_2P = \tau + m\frac{dy}{dt} + J_{p_i}(q_i)F_{v_i} \quad (0.5.2)$$

Joint torques and their derivatives can be expressed as a function of force as follows.

$$\tau = J_{p_i}(q_i)F_p = J_{p_i}(q_i)A_2P \quad (0.5.3)$$

The current action on the position of the valve's jack  $x$  is defined by  $x = f(i - i_o) = K_i.(i - i_o)$  with  $K_i$  the valve displacement gain and  $i_o$  the initial offset current of the servo-valve. This offset will now be neglected assuming that it is experimentally compensated ( $x = K_i i$ ).

Let us define the following parameters:

$P_s = 175 \times 10^5 N.m^{-2}$ : supply pressure,

$\rho = 870 kg.m^{-3}$  : oil density coefficient,

$\beta_o = 7 \times 10^8 N.m^{-2}$  : Bulk modulus,

$d_1$ : Spool diameter,

$k_f$  : leakage coefficient,

$V_t$  : total oil volume in chambers and connection tubes, and

$C_d$  : constant factor taking into account turbulent flow across the orifice.

Let  $P$  denote the differential pressures in the chambers and  $x$  the valve spool displacement. Let us take the flow out of the valve to be positive for output ports and by use of the square root law for turbulent flow across an orifice, assuming energy conservation and if heat exchange is neglected, we can write the following expression where  $Q_s$  is the bidirectional flow across orifices,

$$Q_s = K_v x (P_s - \text{sign}(x)P)^{\frac{1}{2}} \text{ with } K_v = \frac{C_d \pi d_1}{\sqrt{\rho}}$$

$$\text{and } Q_s = A_2 \dot{y} + k_f P + \frac{V_t}{4\beta} \dot{P}.$$

This leads to the differential pressure evolution equation

$$\dot{P} + \frac{4\beta_o k_f}{V_t} P + \frac{4\beta_o A_2}{V_t} \dot{y} = \frac{4\beta_o K_v \sqrt{P_s - \text{sign}(x)P}}{V_t} x \quad (0.5.4)$$

The force applied by the hydraulic actuator is a function of the cross-section area of the piston  $A_2$ , ( $F_p = A_2 P$ ) and obeys the following differential equation

$$\dot{F}_p + \frac{4\beta_o k_f}{V_t} F_p + \frac{4\beta_o A_2^2}{V_t} \dot{y} = \frac{4\beta_o K_v A_2 \sqrt{P_s - \text{sign}(x)P}}{V_t} K_i i$$

The pressure differential equation can be written as

$$\dot{F}_p + B_1 F_p + E_1 \dot{q} = J_1 k_o i \quad (0.5.5)$$

Expressions of the parameters  $B_1$  ,  $E_1$  ,  $J_1$  and  $k_o$  can be obtained from the two preceding equations. We are rather interested in modeling and control, by expression of torques. Using relations (0.5.2) and (0.5.3), we can obtain, from the above equation, a differential equation for torque dynamics

$$\begin{aligned} \dot{\tau} &+ \left( \frac{4\beta_o k_f}{V_t} - J_{p_i}(q_i) J_{p_i}(q_i)^{-1} \right) \tau + \frac{4\beta_o A_2^2}{V_t} J_{p_i}(q_i) \dot{q} \\ &= \frac{4\beta_o K_v A_2}{V_t} J_{p_i}(q_i) \sqrt{P_s - \text{sign}(x)P} K_i i \end{aligned}$$

$$\dot{\tau} + B \cdot \tau + E \dot{q} = J \cdot k_o \cdot i \quad (0.5.6)$$

$$\text{with } B = B(q) = \frac{4\beta_o k_f}{V_t} - J_{p_i}(q_i) J_{p_i}(q_i)^{-1},$$

$$E = \frac{4\beta_o A_2^2}{V_t} J_{p_i}(q_i), k_o = \frac{4\beta_o K_v A_2}{V_t} K_i \text{ and } J = J(x, P) = J_{p_i}(q_i) \sqrt{P_s - \text{sign}(x)P}.$$

Defining  $J_p(q) = \text{diag}(J_{p_1}(q_1), \dots, J_{p_n}(q_n))$  (positive diagonal matrix) for gain transfer between force  
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### 0.5.3. Proof of lemma

We can prove the boundedness of derivative of the inertia matrix  $\dot{M}(q)$  and the term  $\dot{C}(q, \dot{q})$ ,  $\dot{F}_v(q, \dot{q})$ .

Recall that from the robot physical properties we have

$$\|M(q)\| \leq \alpha_1, \|C(q, \dot{q})\| \leq \alpha_2 \|\dot{q}\|, \text{ and } \|F_v(\dot{q})\| \leq \alpha_4 + \alpha_5 \|\dot{q}\|$$

and suppose that  $\omega_m$  is the highest frequency of  $(q, \dot{q})$  the trajectory components (positions and velocities), then from  $M \leq \alpha_1 I_n$  it can be shown that  $\|\dot{M}(q)\| \leq \alpha_1 \omega_m$ . The same conclusion can be obtained for  $\|\dot{C}(q, \dot{q})\| \leq \alpha_2 \omega_m \|\dot{q}\|$ , and for frictions  $\|\dot{F}_v(q, \dot{q})\| \leq \alpha_4 \omega_m + \alpha_5 \omega_m \|\dot{q}\|$ .

Proof of lemma 2: Let us start by boundedness of  $J_o J_o^{-1}$  and consider  $\kappa \in R$  such as  $J = J_{p_i}(q_i) \sqrt{P_s - \text{sign}} \max(J_{p_i}(q_i)) \sqrt{P_s} = \kappa$  and take  $J_o = \kappa \implies J_o J_o^{-1} \leq 1$ . Then we can conclude that  $\widetilde{M} = J_o J_o^{-1} M_o - M \leq \alpha_1 I_n$ , and  $\|\widetilde{C}\| = \|J_o J_o^{-1} C_o - C\| \leq \alpha_2 \|\dot{q}\|$ .

Friction disturbances and gravitation effects are bounded:

$$\|F_v(q, \dot{q})\| \leq \alpha_4 + \alpha_5 \|\dot{q}\|, \text{ and } \|G(q)\| < \alpha_3.$$

Let us consider bounds for the hydraulic parameters:

$$\|k_o\| = \left\| \frac{4\beta K_v A_2}{V_t} K_i \right\| \leq k_m \text{ and } \|E\| = \left\| \frac{4\beta A_2^2}{V_t} \right\| \leq e_m,$$

and for the position dependant parameter, we can write:

$$\|B\| = \|B(q)\| = \left\| \frac{4\beta k_f}{V_t} - J_{p_i}(q_i) J_{p_i}(q_i)^{-1} \right\| \leq b_m.$$

Proof of (i): By considering the expression  $H(q, \dot{q}, \ddot{q}) = (\dot{M} + BM)\ddot{q} + (\dot{C} + E + BC)\dot{q} + \dot{G} + F_v + BG + BF_v$  and  $\tilde{H} = J.J_o^{-1}H_o - H$  we can write  $\|\tilde{H}\| \leq (\alpha_1 \omega_m + \alpha_1) \|\ddot{q}\| + (\alpha_2 \omega_m \|\dot{q}\| + e_m + b_m \alpha_2 \|\dot{q}\|) \|\dot{q}\| + \alpha_3 \omega_m + \alpha_4 \omega_m + \alpha_5 \omega_m \|\dot{q}\| + b_m \alpha_3 + b_m (\alpha_4 \omega_m + \alpha_5 \omega_m \|\dot{q}\|)$

This leads to  $\|\tilde{H}\| \leq (\alpha_1 \omega_m + \alpha_1) \|\ddot{q}\| + (e_m + \alpha_5 \omega_m + \alpha_5 b_m \omega_m) \|\dot{q}\| + (\alpha_2 \omega_m + b_m \alpha_2) \|\dot{q}\|^2 + \alpha_3 \omega_m + \alpha_4 \omega_m + b_m \alpha_3 + b_m \alpha_4 \omega_m$

Then we obtain:  $\|\tilde{H}\| \leq \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\|$

Proof of (ii): By considering the inequalities:  $\|\widetilde{M}\| \leq \alpha_1$ ,  $\|\widetilde{C}\| \leq \alpha_2 \|\dot{q}\|$  and the previous result we have:  $\|\widetilde{M}\ddot{q}_r + \widetilde{C}\dot{q}_r + \widetilde{H}\| \leq \alpha_1 \|\ddot{q}_r\| + \alpha_2 \|\dot{q}_r\| + \kappa_0 + \kappa_1 \|\dot{q}\| + \kappa_2 \|\dot{q}\|^2 + \kappa_3 \|\ddot{q}\|$ .

## 0.6. Passivité des systèmes dynamiques

Afin de pouvoir utiliser la propriété de passivité des systèmes mécatroniques pour la synthèse de lois de commande, nous en rappelons ici les notions et propriétés de base. Un système passif vérifie la propriété:  $E(t_1) = E(0) + E_S(0, t_1) - E_L(0, t_1)$ ; avec  $E(0)$ , l'énergie à l'instant initial,  $E_S(0, t_1)$  l'énergie fournie au système et  $E_L(0, t_1)$  l'énergie perdue ou dissipée entre les instant 0 et  $t_1$ .

## 0.6.1. Systèmes passifs et dissipatifs, généralités et définition

Soit un système dont l'entrée est le vecteur  $u$  et la sortie un vecteur de même dimension  $y$ . Pour les systèmes mécatroniques, si les variables d'entrée sortie sont choisies convenablement pour décrire les transfert d'énergie ou de puissance, ils peuvent se mettre sous une forme vérifiant l'équation:

$$\dot{V}(t) = y^T u - g(t) \quad (0.6.1)$$

Si  $g(t) = 0, \forall u \in L_2, \forall y \in L_2, \forall t > 0$ , alors le système est conservatif, c'est le cas par exemple d'un oscillateur.

**Définition 3.** Un système vérifiant l'équation [0.6.1] est dit **passif** si  $g(t) \geq 0$  et il vérifie l'inégalité de Popov:

$$\int_0^t y^T(\tau)u(\tau)d\tau \geq -\gamma^2 \quad (0.6.2)$$

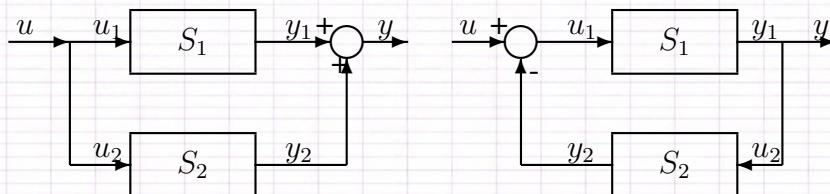
**Définition 4.** Un système passif et dit **dissipatif** si on a la propriété suivante:

$$\int_0^t y^T(\tau)u(\tau)d\tau \neq 0 \implies \int_0^\infty g(\tau)d\tau \geq 0 \quad (0.6.3)$$

Nous considérons, dans ce qui suit les systèmes vérifiant l'équation [0.6.1] et décrits par des équations en fonction des variables d'énergie et de puissance du système. Pour l'analyse de stabilité, le problème se ramène à trouver une représentation par un Système Équivalent (SE) adéquat qui permettra la mise en évidence des propriétés que nous étudions dans ce qui suit.

## 0.6.2. Association et propriétés de systèmes passifs.

Afin d'établir les propriétés intéressantes pour l'association de systèmes passifs, nous étudions les configurations en contre réaction et en parallèle représentées par la figure suivante.



Considérons la connexion parallèle de deux blocs  $S_1$  et  $S_2$  donnée par la figure (a) ci dessus, où chaque bloc est supposé passif. Ceci nous permet d'écrire:  $u = u_1 = u_2$  et  $y = y_1 + y_2$ , par conséquent, nous avons:

$$\int_{t_0}^{t_1} y^T(\tau)u(\tau)d\tau = \int_{t_0}^{t_1} y_1^T(\tau)u_1(\tau)d\tau + \int_{t_0}^{t_1} y_2^T(\tau)u_2(\tau)d\tau \quad (0.6.4)$$

On peut conclure sur la passivité du système et donc les lemmes ci dessous.

**Lemme 7.** *Un système obtenu en combinant en parallèle deux systèmes passifs est passif.*

**Lemme 8.** *Tout système obtenu par combinaison en contre réaction de deux systèmes passifs est passif.*

Considérons la connexion en contre réaction de deux blocs  $S_1$  et  $S_2$  donnée par la figure ci-dessus, où chaque bloc est passif. Ceci nous permet d'écrire:  $u = u_1 + y_2$  et  $y = y_1 = u_2$ , par conséquent, nous avons:

$$\int_{t_0}^{t_1} y^T(\tau)u(\tau)d\tau = \int_{t_0}^{t_1} y_1^T(\tau)u_1(\tau)d\tau + \int_{t_0}^{t_1} y_2^T(\tau)u_2(\tau)d\tau$$

On en conclue que, pour un choix approprié d'un nouvel état du système, la combinaison en parallèle et en contre réaction de n systèmes passifs donne un système passif. Ces propriétés sont intéressantes pour l'analyse de stabilité des systèmes complexes et surtout pour la synthèse de lois de commande pour une classe de systèmes non linéaires multivariables.

Les 2 définitions qui suivent donnent l'équivalent de ces propriétés dans le cas de fonction de transfert de systèmes linéaires.

**Définition 5.** [Positivité Réelle] Une fonction rationnelle  $h(p)$  de la variable complexe  $p=s+j\omega$  est positive réelle si:

1.  $h(p)$  est réelle pour  $p$  réelle;
2.  $\operatorname{Re}[h(p)] \geq 0$  pour tout  $\operatorname{Re}[p] > 0$ .

Cette définition nous permet d'assurer qu'une fonction rationnelle  $h(p)$  de la variable complexe  $p = s + j\omega$  est Positive Réelle si:

1.  $h(p)$  est réelle pour  $p$  réelle ;
2.  $h(p)$  n'a pas de pôles dans le demi-plan droit de Laplace,  $\operatorname{Re}[p] > 0$ ;
3. Les pôles éventuels de  $h(p)$  sur l'axe  $\operatorname{Re}[p] = 0$  ( $p = j\omega$ ) sont distincts, et les résidus qui leurs sont associés sont réels et positifs ou nuls;
4. Pour tout  $\omega$  pour lequel  $p = j\omega$  n'est pas un pôle de  $h(p)$ ,  $\operatorname{Re}[h(p)] \geq 0$ .

**Définition 6.** [Positivité Réelle Stricte (SPR)] Une fonction rationnelle  $h(p)$  de la variable complexe  $p = s + j\omega$  est Strictement Positive Réelle si:

1.  $h(p)$  est réelle pour  $p$  réelle ;
2.  $h(p)$  n'a pas de pôles dans le demi-plan droit de Laplace,  $\text{Re}[p] \geq 0$ ;
3.  $\text{Re}[h(j\omega)] > 0$ ,  $-\infty < \omega < +\infty$ .

### 0.6.3. Hyperstabilité de systèmes bouclés

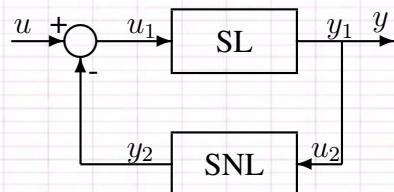
Nous considérons le système en boucle fermée de la figure ci-dessous. Le système de la chaîne directe est linéaire et invariant dans le temps (SL). Ce dernier est décrit par son équation d'état suivante:

$$\dot{x} = Ax + Bu_1 = Ax - By_2$$

$$y_1 = Cx + Du_1 = Cx - Dy_2$$

avec  $(A, B)$  commandable et  $(A, C)$  observable. Ce système est caractérisé par sa fonction de transfert donnée par:

$$H(p) = D + C[pI - A]^{-1}B$$



Le bloc non-linéaire (SNL) traduisant le retour est défini par:  $y_2 = f(u_2, t, \tau)$  avec  $\tau \leq t$ , ce dernier

vérifie l'inégalité de Popov donnée par:

$$\int_{t_0}^{t_1} y^T(\tau)u(\tau)d\tau \geq -\gamma^2 \text{ avec } \gamma^2 < \infty \forall t \geq 0 \quad (0.6.5)$$

Nous avons alors les théorèmes qui suivent [?]:

**Théorème 3** (Hyperstabilité). *Toutes solutions  $x(x(0), t)$ , du système décrit par les équations ci dessus en boucle fermée sur le bloc non-linéaire qui vérifie l'inégalité de Popov, satisfait la propriété:*

$$\|x(t)\| < \delta(\|x(0)\| + \gamma_0) \text{ avec } \delta > 0, \gamma_0 > 0 \quad \forall t \geq 0$$

*si et seulement si la matrice de fonctions de transfert  $H(p)$  est Positive Réelle.*

**Théorème 4** (Hyperstabilité Asymptotique). *Toute solution  $x(x(0), t)$ , du système décrit ci dessus en boucle fermée sur un bloc non-linéaire qui vérifie l'inégalité de Popov, satisfait l'inégalité [0.6.5] de plus nous avons (quand  $t$  tend vers 0)  $\lim(x(t)) = 0$ , pour toute entrée  $u_1(t)$  bornée, si et seulement si la matrice de fonctions de transfert  $H(p)$  est Strictement Positive Réelle.*

**Remarque:** Les théorèmes d'Hyperstabilité et d'Hyperstabilité asymptotique, donnent des conditions suffisantes pour montrer respectivement la stabilité et la stabilité asymptotique dans le cas où l'inégalité de Popov est vérifiée par le bloc de contre réaction.

## 0.7. Exemple: Passivité des robots manipulateurs rigides

Pour la modélisation d'un manipulateur, nous pouvons utiliser le formalisme de Lagrange, avec  $E(q) = \frac{1}{2}\dot{q}^T A(q)\dot{q}$  l'énergie cinétique totale du système,  $P(q)$  l'énergie potentielle totale du système. L'équation

du robot rigide est donnée par [?]:

$$A(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$$

$\tau$  est le vecteur de dimension (nx1) relatant le couple appliqué à chaque articulation et  $dE/dt$  est la puissance fournie par les forces externes généralisées.

$$\frac{dE}{dt} = \frac{1}{2} \frac{d(\dot{q}^T A(q)\dot{q})}{dt} = \dot{q}^T (\tau - G(q))$$

en intégrant on trouve (avec  $q(0) = q_0$ ):

$$\int_0^{t_1} \dot{q}^T (\tau - G(q)) dt = (\dot{q}^T(t_1) A(q(t_1)) \dot{q}(t_1))/2 - (\dot{q}_0^T A(q_0) \dot{q}_0)/2$$

ce qui permet de conclure à la relation suivante:

$$\int_0^{t_1} \dot{q}^T (\tau - G(q)) dt \geq -(\dot{q}_0^T A(q_0) \dot{q}_0)/2 = -\gamma^2$$

Pour le robot manipulateur on a pour entrée  $u = (\tau - G(q))$  et sortie  $y = \dot{q}$ , donc le transfert de  $u$  à  $y$  est passif car il vérifie l'inégalité de Popov.

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# Commande Par Modes de Glissement en Robotique

**PROGRAMME  
UNIT-GDR ROBOTIQUE**



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