



# Systemes de locomotion hybrides roues-pattes

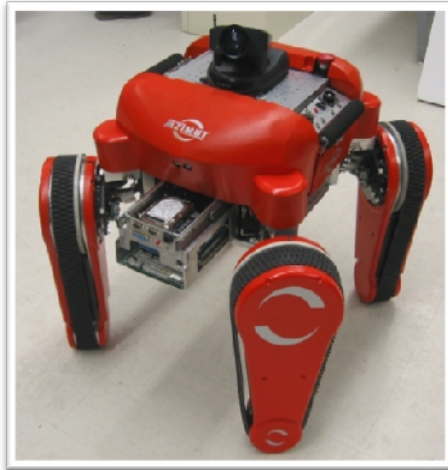
ISIR – UPMC

Christophe Grand et Faïz Ben Amar

# Plan

1. Introduction
2. Architecture mécatronique des robots Hylos
3. Modélisation cinémato-statique
4. Commande pour le contrôle de posture
5. Optimisation de la distribution des forces

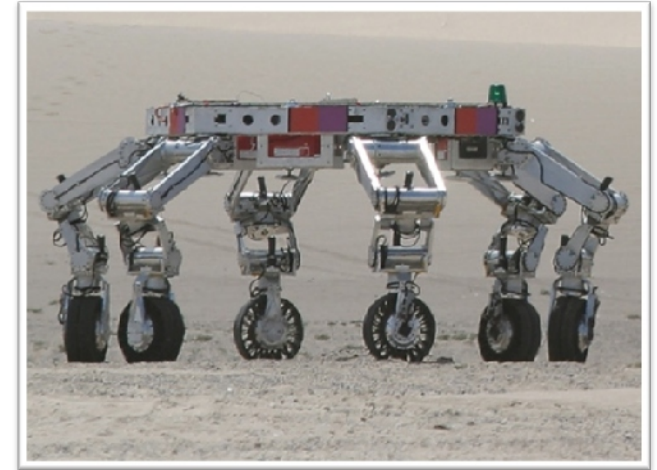
# Introduction



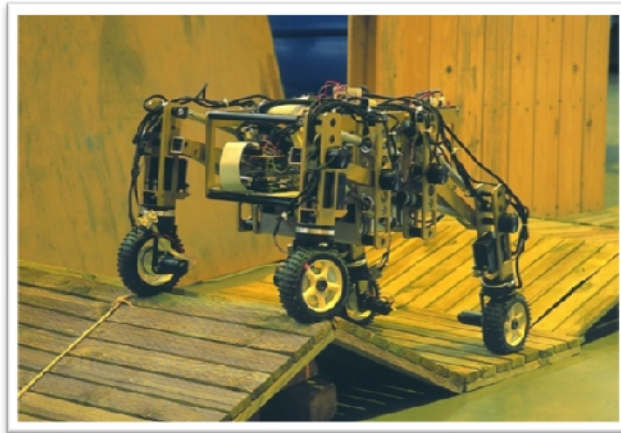
*Azimut* [Laborius/2004]



*Hybtor* [HUT / 2001]



*Athlete* [JPL / 2004]



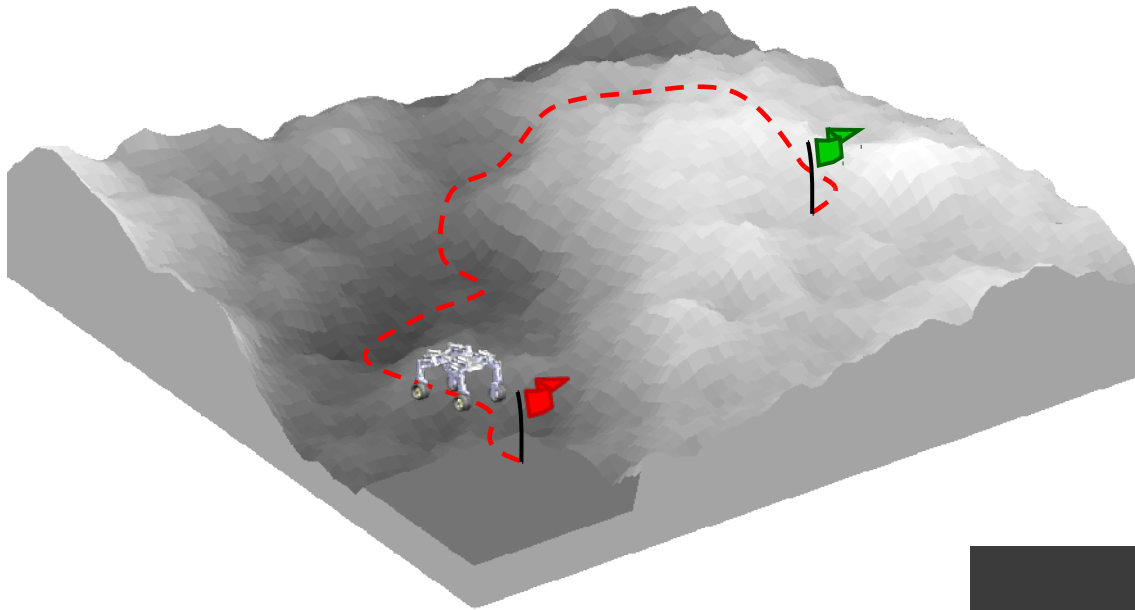
*HyLoS I* [UPMC / 2001]



*HyLoS II* [UPMC/2005]

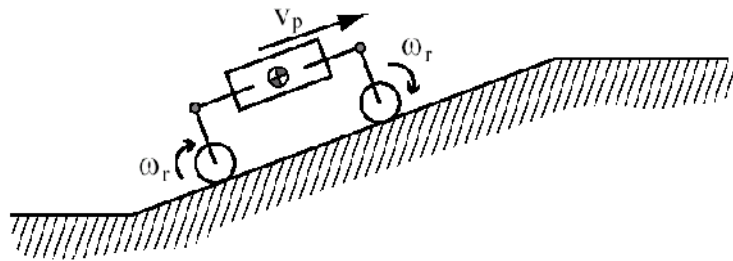
# Introduction

Intérêt de la locomotion hybride roues-pattes

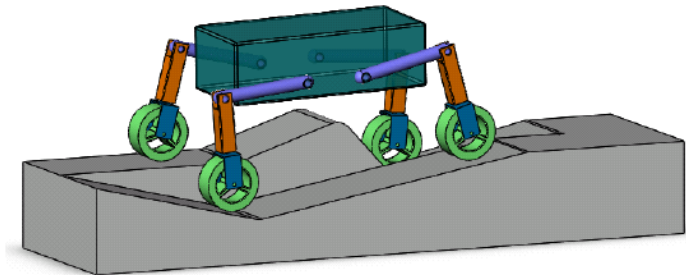


# Introduction

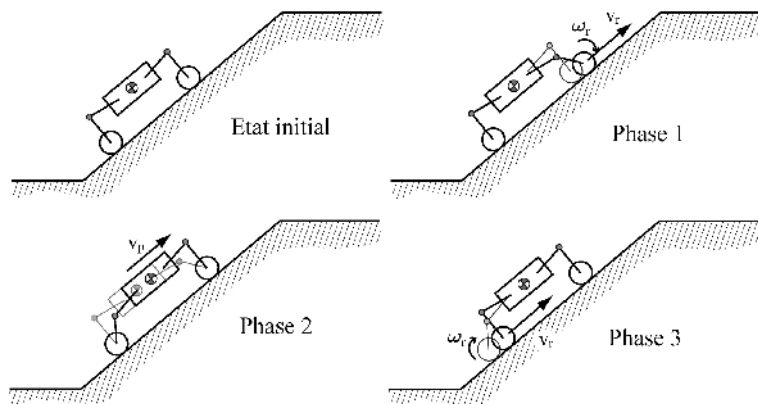
## Les modes de locomotion



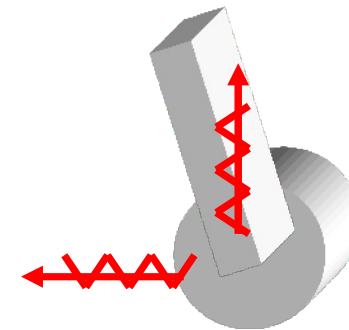
Mode 1 : Roulement simple



Mode 2 : Reconfiguration

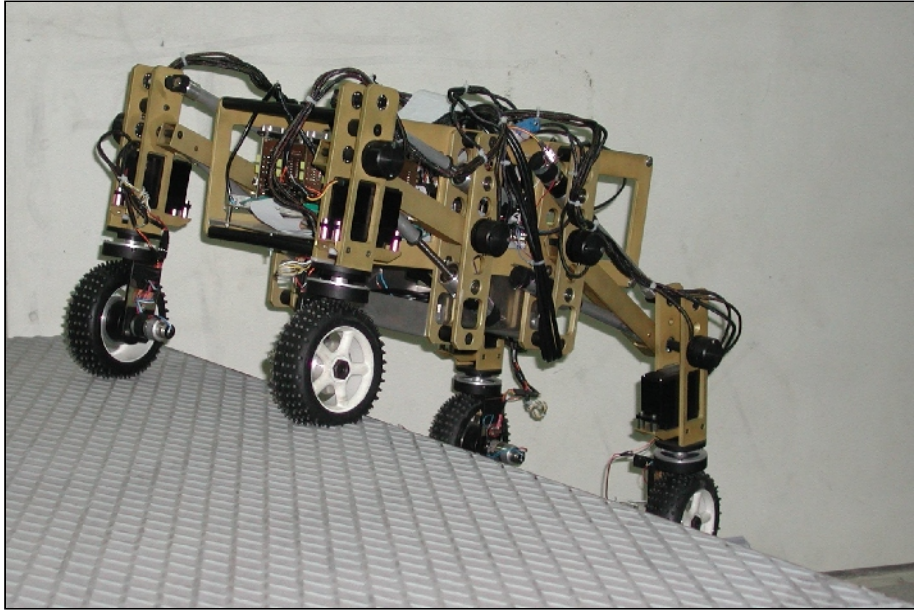


Mode 3 : Périltisme



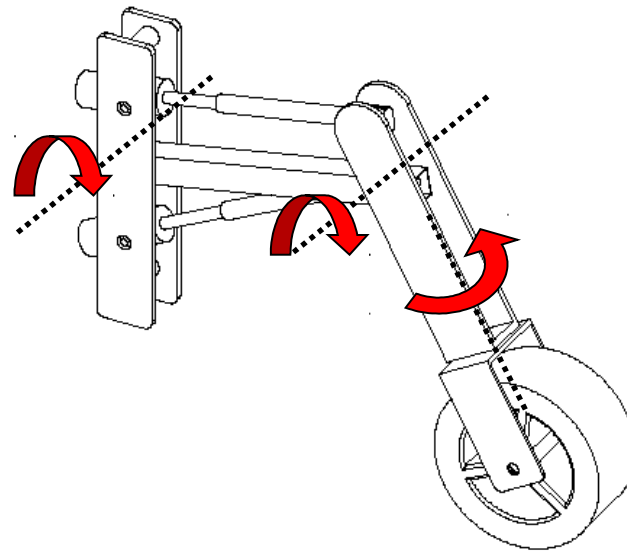
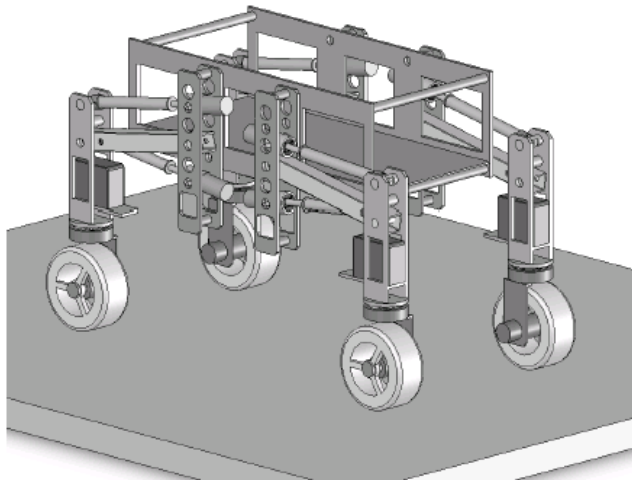
Mode 4 : Franchissement

# Architecture Hylos 1

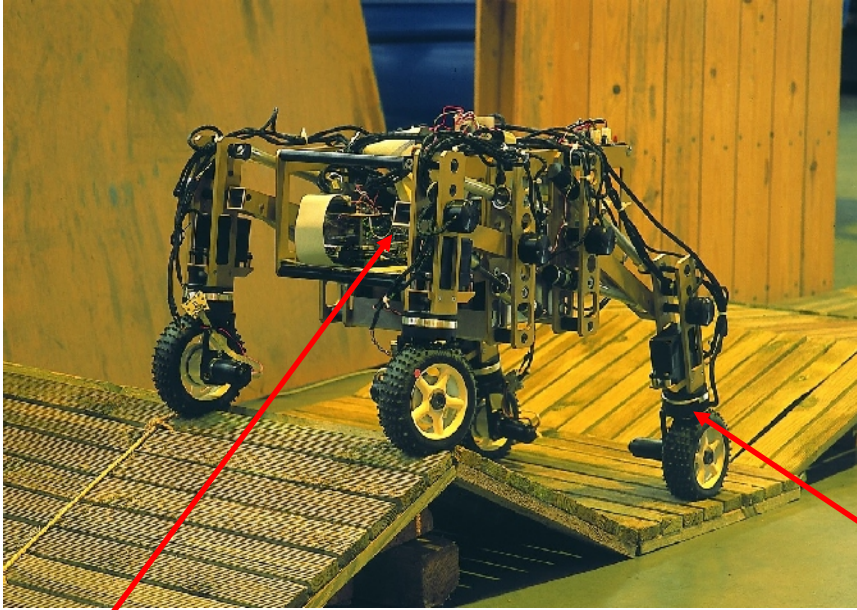


## Caractéristiques du robot

Masse	15 kg
Dimensions	70x30x40 cm
Garde au sol	10-30 cm
Vélocité	0.6 m/s
ddl	16
Énergie	Batterie (NiCd)

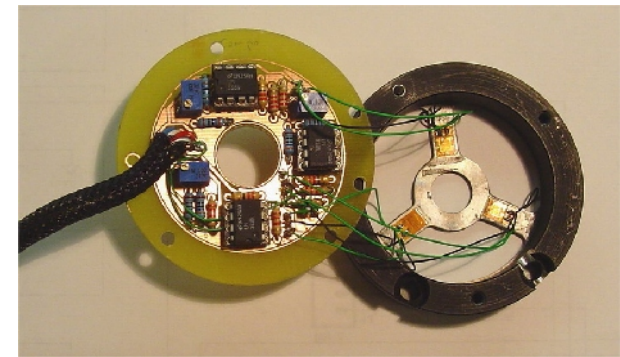


# Architecture Hylos 1



## Perception

- 1 Inclinomètre 2 axes roulis/tangage
- Potentiomètres articulations
- Codeurs optiques
- 4 capteurs d'effort 3 axes



## Commande



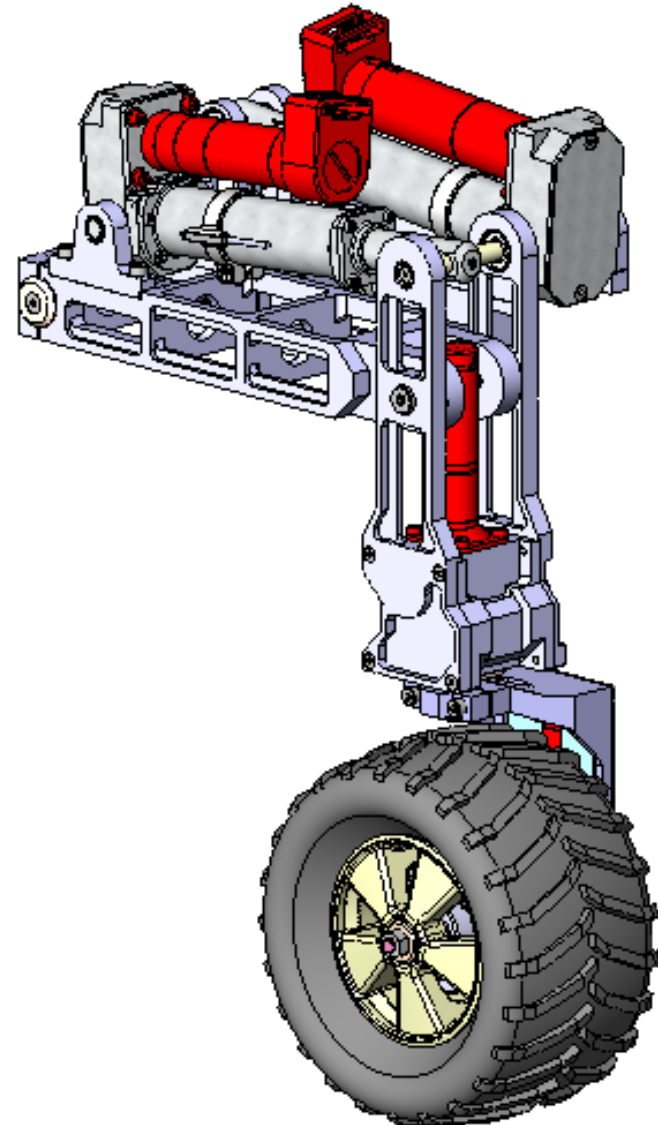
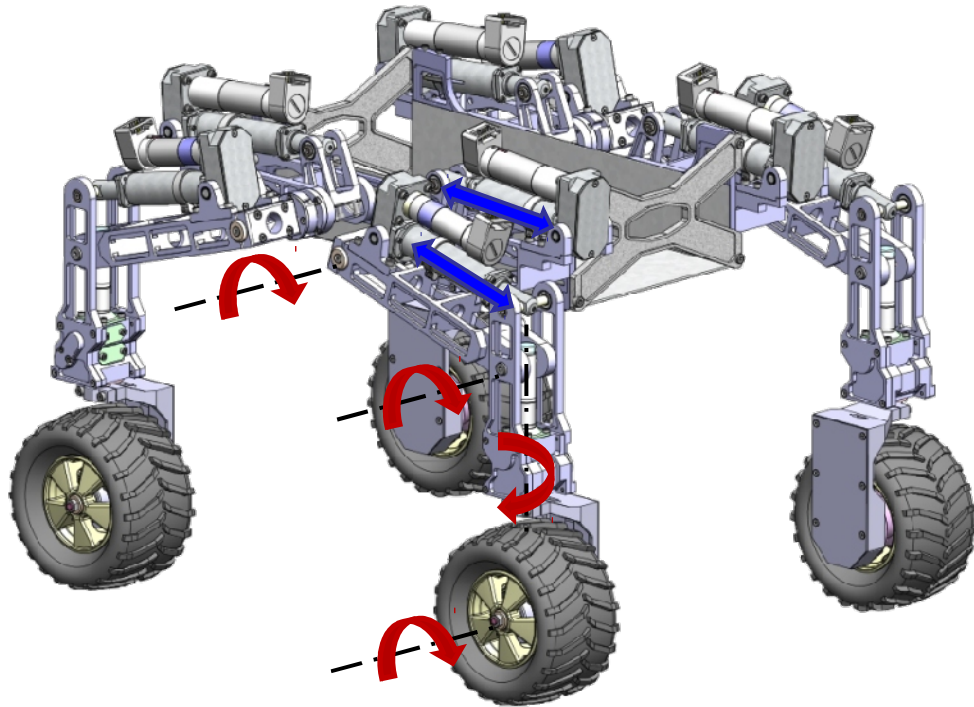
PC104



MPC555

Bus CAN  
(1 Mbits/S)

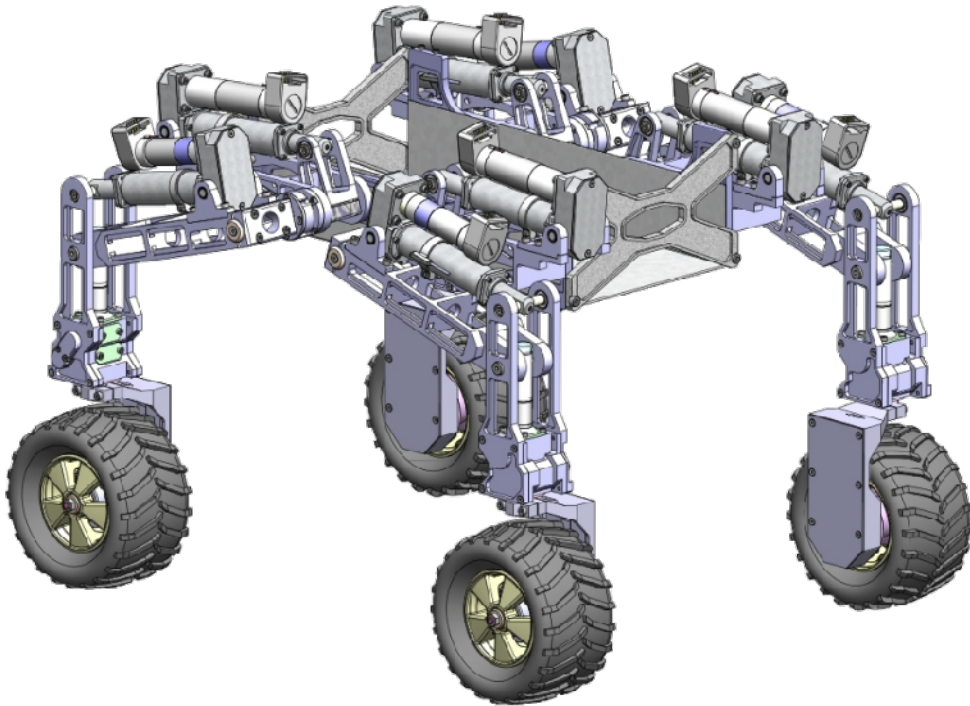
# Architecture mécatronique



<b>Masse</b>	25 kg
<b>Dimensions</b>	70 x 50 x 40 cm (L x l x h) 14 cm ( $\varnothing_{\text{roue}}$ )
<b>Variation de la garde au sol</b>	10 cm à 40 cm
<b>Vitesse</b>	2 m/s
<b>ddl</b>	16 (4/patte)

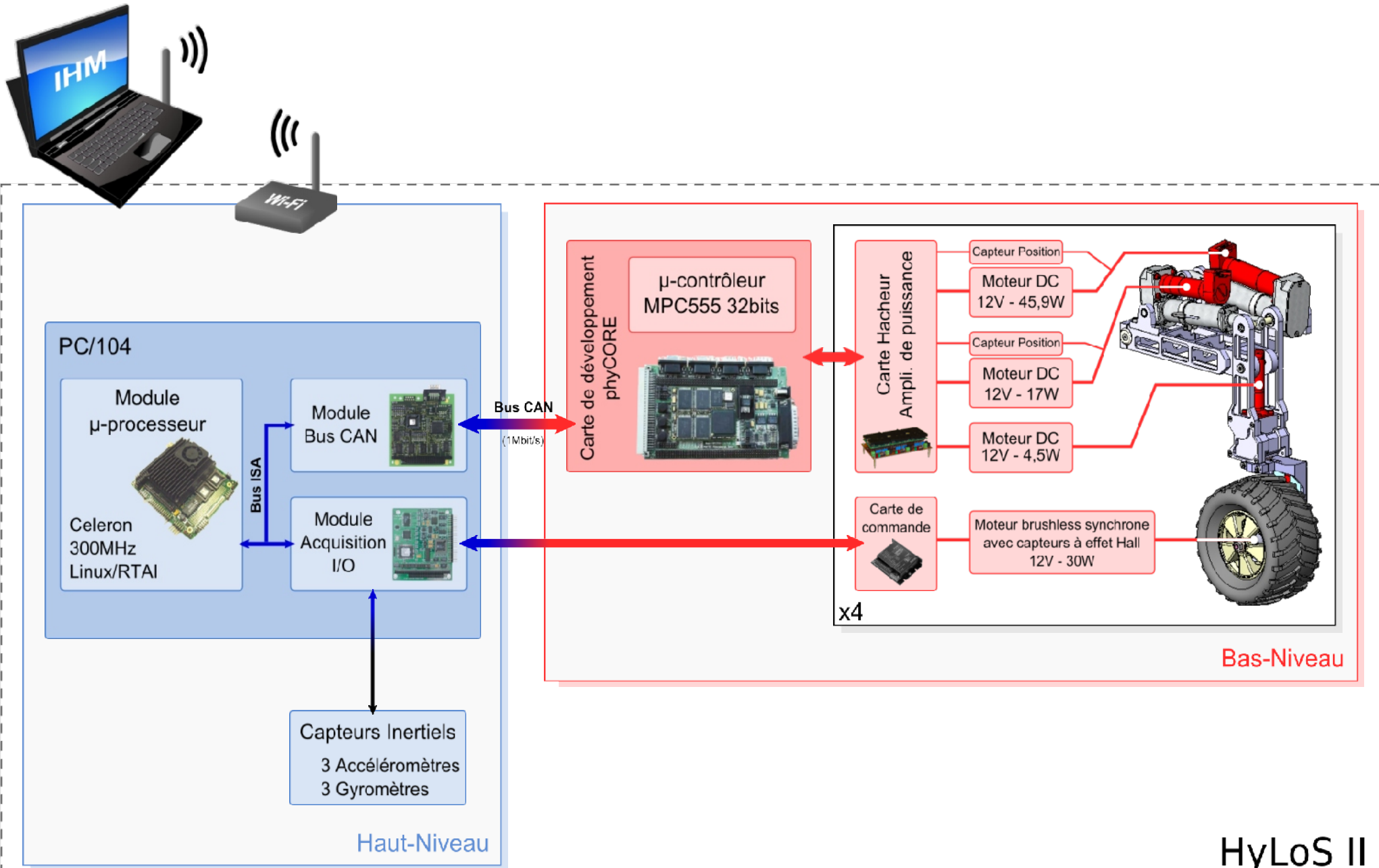


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# Architecture mécatronique



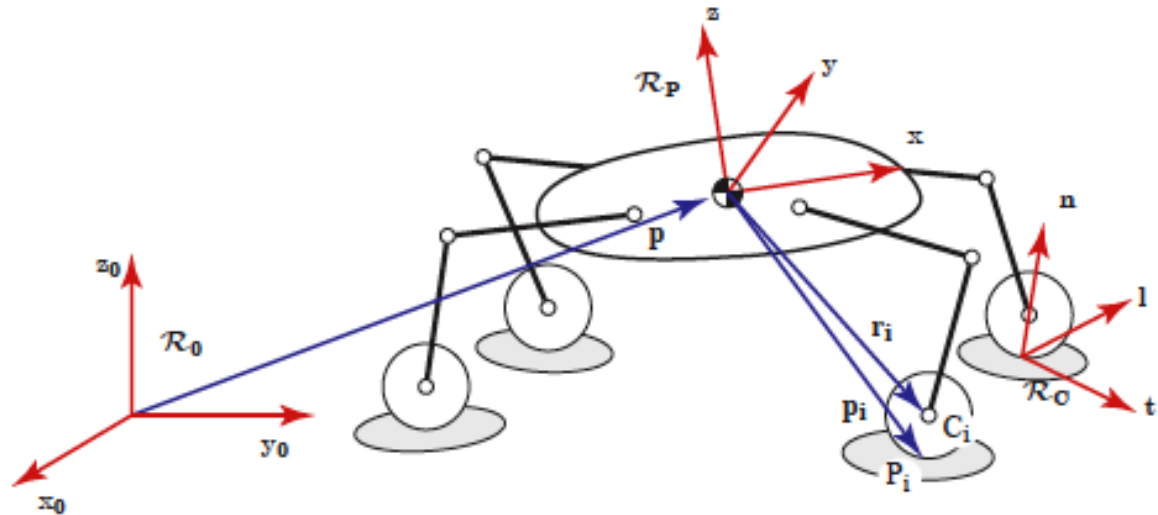
# Modélisation cinémato-statique

Inertial frame:  $\mathcal{R}_0$

Main body frame:  $\mathcal{R}_p$

Transformation:  $\mathbf{x}^t = (\mathbf{p}^t, \phi^t)$

$$\mathbf{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \phi = \begin{pmatrix} \varphi \\ \psi \\ \theta \end{pmatrix}$$

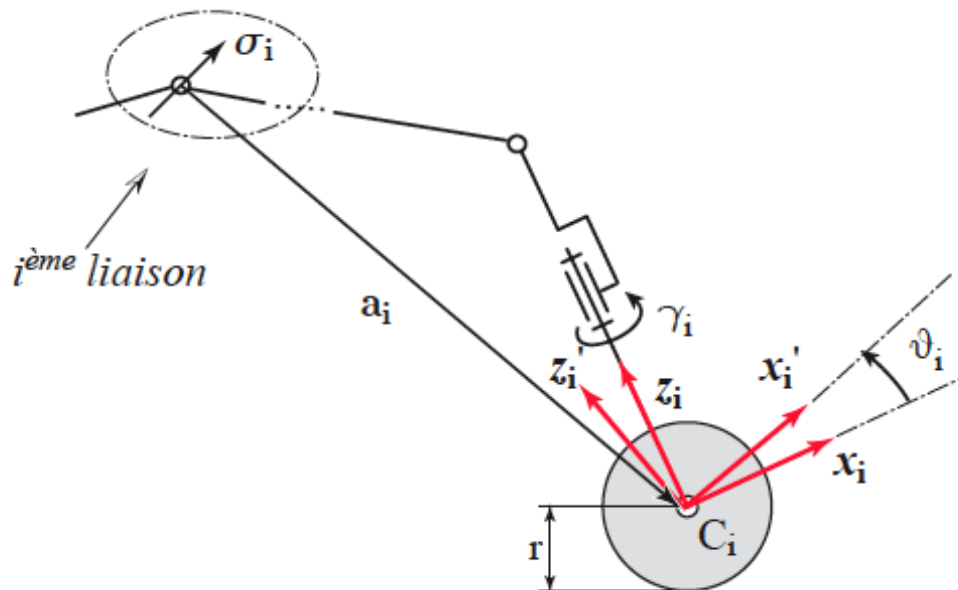


Leg parameters:  $\theta_i = \begin{pmatrix} \alpha_i \\ \beta_i \\ \dots \end{pmatrix}$

Wheel configuration:  $\chi_i = \begin{pmatrix} \gamma_i \\ \vartheta_i \end{pmatrix}$

Global parameters:

$$\mathbf{q}^t = (\mathbf{x}^t, \theta_1^t, \chi_1^t, \theta_2^t, \chi_2^t, \dots)$$



# Modélisation cinémato-statique

Wheel-soil contact give a velocity constraint at contact point  $P_i$  for each contact (pseudo closed loop) :

$$\mathbf{v}_s \triangleq \mathbf{v}_x + \mathbf{v}_p - \mathbf{v}_c$$

where

${}^p\mathbf{v}_x \triangleq \mathbf{R}\dot{\mathbf{p}} + \boldsymbol{\omega} \times \mathbf{p}_i$  is the velocity of  $P_i$  due to platform motion

${}^p\mathbf{v}_p \triangleq \dot{\mathbf{r}}_i = \mathbf{J}_{p_i}\dot{\boldsymbol{\theta}}_i$  is the velocity of  $P_i$  due to legs motion

${}^i\mathbf{v}_c \triangleq r\dot{\vartheta}_i\mathbf{t}_i$  is the circumferential velocity of the wheel

The platform angular velocity:  $\boldsymbol{\omega} = \mathbf{T}_\phi\dot{\boldsymbol{\phi}}$

$${}^p\mathbf{v}_x = \mathbf{L}_i\dot{\mathbf{x}} \quad \mathbf{L}_i = \begin{bmatrix} \mathbf{R} & -\tilde{\mathbf{p}}_i\mathbf{T}_\phi \end{bmatrix}$$

$$\mathbf{T}_\phi(\boldsymbol{\phi}) = \begin{pmatrix} 1 & 0 & -S_\psi \\ 0 & C_\varphi & C_\psi S_\varphi \\ 0 & -S_\varphi & C_\psi C_\varphi \end{pmatrix}$$

# Modélisation cinémato-statique

Equations projected in each contact frame  $\mathcal{R}_i = (P_i, \mathbf{t}_i, \mathbf{l}_i, \mathbf{n}_i)$

$$\mathbf{v}_s \triangleq \mathbf{v}_x + \mathbf{v}_p - \mathbf{v}_c \quad \longrightarrow \quad \mathbf{v}_s = \mathbf{R}_i^t \mathbf{L}_i \dot{\mathbf{x}} + \mathbf{R}_i^t \mathbf{J}_{p_i} \dot{\boldsymbol{\theta}}_i - r \dot{\vartheta}_i \mathbf{t}_i$$

$$\mathbf{L} \dot{\mathbf{x}} + \mathbf{J} \dot{\boldsymbol{\Theta}} = \mathbf{v}_s$$

For  $n$  contact points  
 $3n$  scalar velocity constraints  
 $m$  joint velocities

$$\mathbf{v}_{s_i} = \begin{pmatrix} s_{t_i} \\ s_{l_i} \\ s_{n_i} \end{pmatrix}$$

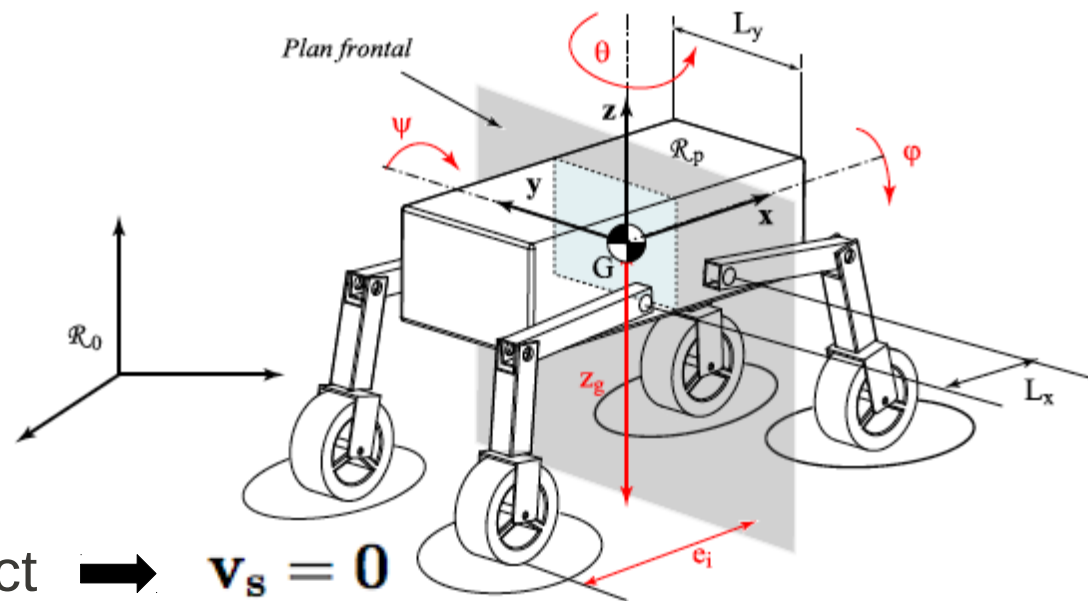
$$\boldsymbol{\Theta} = \begin{bmatrix} \theta_1 \\ \chi_1 \\ \theta_2 \\ \chi_2 \\ \vdots \\ \theta_n \\ \chi_n \end{bmatrix}_{nm \times 1}$$

$$\mathbf{L}(\mathbf{x}, \boldsymbol{\Theta}, \mathbf{n}) = \begin{bmatrix} \mathbf{R}_1^t \mathbf{L}_1 \\ \mathbf{R}_2^t \mathbf{L}_2 \\ \vdots \\ \mathbf{R}_n^t \mathbf{L}_n \end{bmatrix}_{3n \times 6}$$

$$\mathbf{J}(\boldsymbol{\Theta}, \mathbf{n}) = \begin{bmatrix} \mathbf{J}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 & & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & \mathbf{0} & & \mathbf{J}_n \end{bmatrix}_{3n \times nm}$$

$$\mathbf{J}_i = \left[ \mathbf{R}_i^t \mathbf{J}_{p_i} \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \right]_{3 \times m}$$

# Analyse de la mobilité



**Hypothesis:** no sliding in the contact  $\Rightarrow \mathbf{v}_s = \mathbf{0}$

Hylos is a 4 wheel-legged robots with 4 dof for each one:

$$n = 4 \quad \text{and} \quad m = 16$$

$\mathbf{L}\dot{\mathbf{x}} + \mathbf{J}\dot{\boldsymbol{\theta}} = \mathbf{0}$   $\Rightarrow$  correspond to  $4 \times 3 = 12$  scalar equations

6 operational and 16 joint dof(s)  $\Rightarrow$  22 velocity parameters

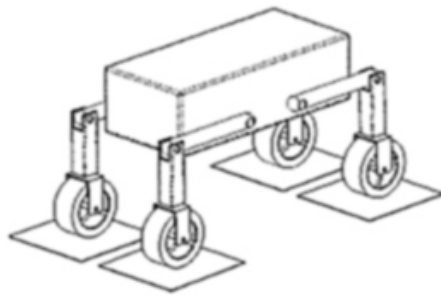
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Mobility index = 10

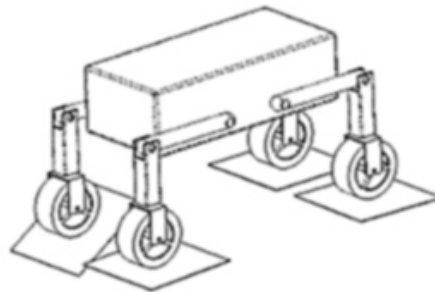
# Analyse de la mobilité

$$\mathbf{L}\dot{\mathbf{x}} + \mathbf{J}\dot{\boldsymbol{\Theta}} = \mathbf{0} \iff \mathbf{A}\dot{\mathbf{q}} = \mathbf{0} \quad \text{with} \quad \mathbf{A} = [\mathbf{L} \mid \mathbf{J}] \quad \text{and} \quad \dot{\mathbf{q}} = [\dot{\mathbf{x}} \ \dot{\boldsymbol{\Theta}}]^t$$

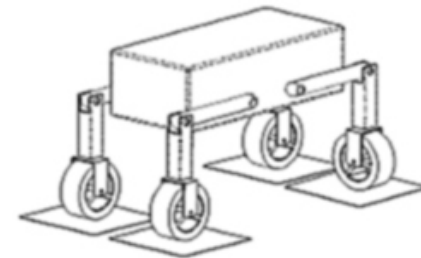
$$m_r = \dim(\mathbf{q}) - \text{rank}(\mathbf{A})$$



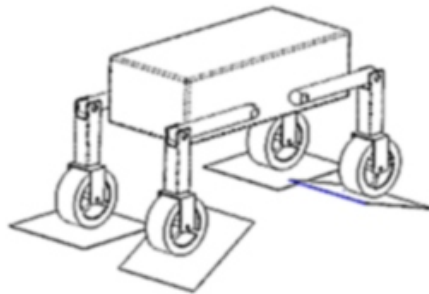
$$m_r = 12$$



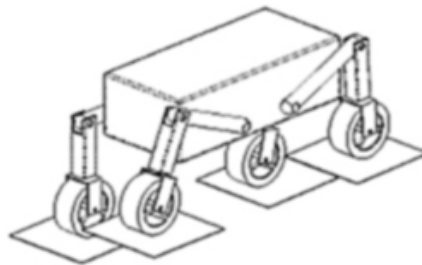
$$m_r = 11$$



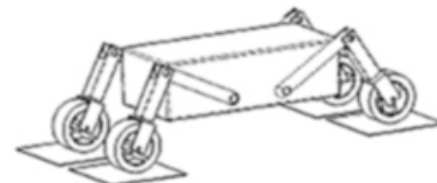
$$m_r = 11$$



$$m_r = 10$$



$$m_r = 10$$

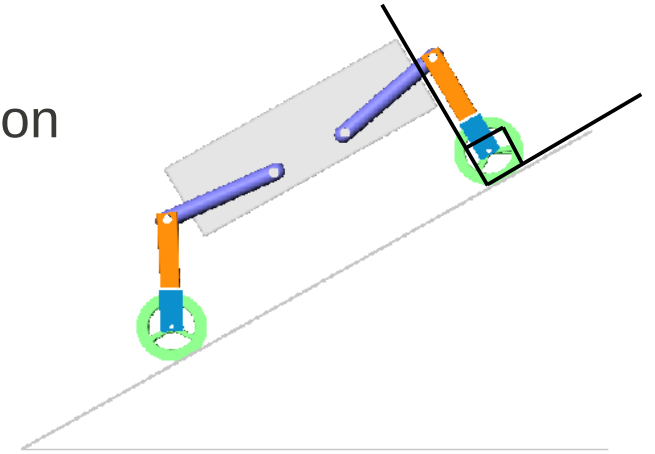


$$m_r = 10$$

# Contrôle de posture

**Hypothesis:** the steering angle velocity as limited effect on the instantaneous robot velocity

⇒ Remove  $\dot{\gamma}_i$  in velocity equations ( $\mathbf{u} = \mathbf{B}_j \dot{\Theta}$ ) and use non-holonomic conditions to solve  $\gamma_i$



$$\left\{ \begin{array}{l} \mathbf{t}_i^t \mathbf{v}_{s_1} = 0 \\ \mathbf{n}_i^t \mathbf{v}_{s_1} = 0 \\ \mathbf{B}_x \mathbf{L} \dot{\mathbf{x}} + (\mathbf{B}_x \mathbf{J} \mathbf{B}_j^t) (\mathbf{B}_j \dot{\Theta}) = 0 \end{array} \right.$$

⇒ 8 scalar equations

$$\mathbf{B}_x = \left[ \begin{array}{ccc|ccc} \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right] & & & & & 0 \\ & \ddots & & & & \\ & & & & & \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array} \right]_{2n \times 3n}$$

$$\left\{ \begin{array}{l} \mathbf{l}_i^t \mathbf{v}_{s_1} = 0 \\ \mathbf{B}_\gamma \mathbf{L} \dot{\mathbf{x}} + (\mathbf{B}_\gamma \mathbf{J} \mathbf{B}_j^t) (\mathbf{B}_j \dot{\Theta}) = 0 \end{array} \right.$$

⇒ 4 scalar equations

$$\mathbf{B}_j = \left[ \begin{array}{ccc|ccc} \left[ \begin{array}{ccc} \mathbf{I} & 0 & 0 \\ 0 & 0 & 1 \end{array} \right] & & & & & 0 \\ & \ddots & & & & \\ & & & & & \left[ \begin{array}{ccc} \mathbf{I} & 0 & 0 \\ 0 & 0 & 1 \end{array} \right] \end{array} \right]_{(l-n) \times m}$$



# Contrôle de posture

## System redundancy:

$$\dot{\mathbf{q}}^t = (\dot{\mathbf{x}}^t, \mathbf{u}^t)$$

6 + 12 dof(s)

$$\mathbf{B}_x \mathbf{L} \dot{\mathbf{x}} + (\mathbf{B}_x \mathbf{J} \mathbf{B}_j^t) \mathbf{u} = \mathbf{0}$$

8 velocity constraints



10 degrees  
mobilities

Operational space velocity  $\dot{\mathbf{x}}$  6 state parameters

Internal parameters  $\mathbf{e}$  4 state parameters

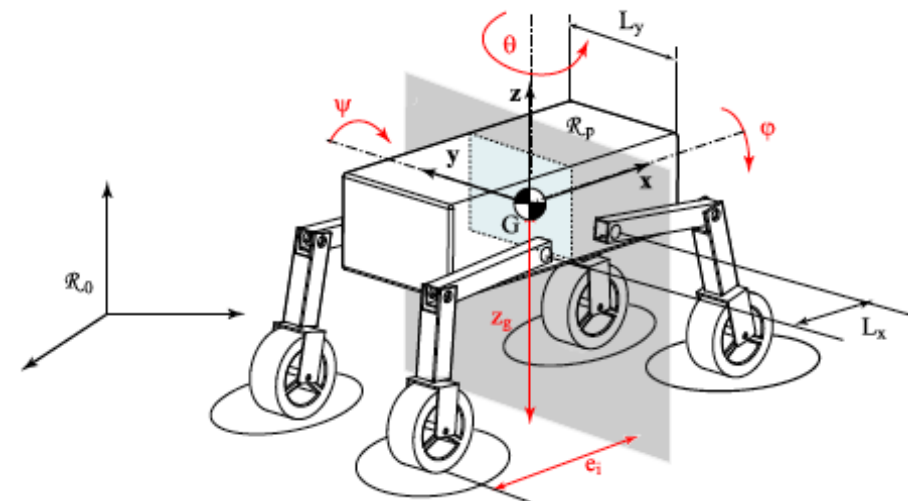
$$\dot{\mathbf{e}} = \mathbf{J}_e(\Theta) \mathbf{u}$$

$$\xi^t = (\mathbf{x}^t, \mathbf{e}^t)$$

$$\begin{bmatrix} \mathbf{B}_x \mathbf{L} \\ -\mathbf{I} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_x \mathbf{J} \mathbf{B}_j^t \\ \mathbf{J}_e \end{bmatrix} \mathbf{u} = \mathbf{0}$$



$$\tilde{\mathbf{L}} \dot{\xi} + \tilde{\mathbf{J}} \mathbf{u} = \mathbf{0}$$



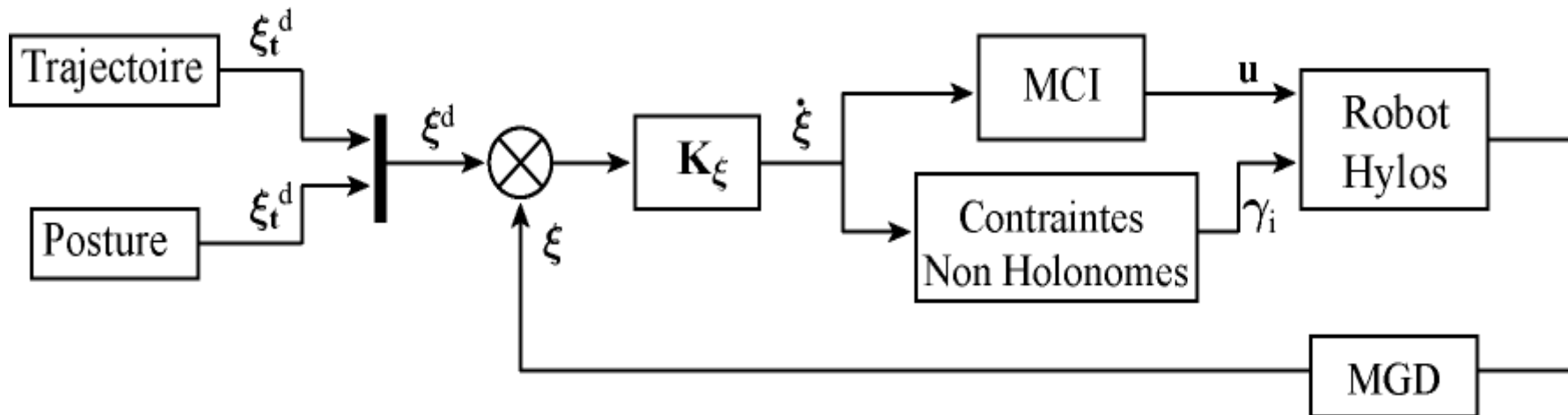
# Contrôle de posture

$\tilde{\mathbf{J}}$  is a square 12x12 matrix and it is singular only for few identified cases:

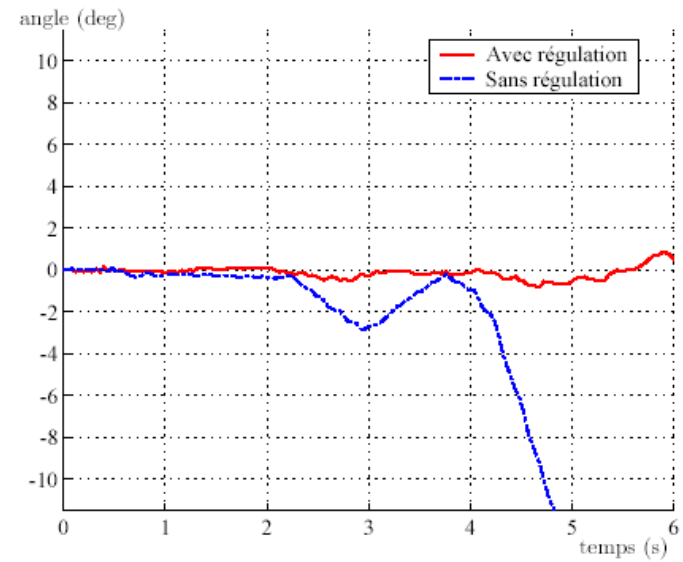
Joint velocities:  $\mathbf{u} = -\tilde{\mathbf{J}}^{-1}\tilde{\mathbf{L}}\dot{\boldsymbol{\xi}}$

$$\boldsymbol{\xi}^t = (\mathbf{x}^t, \mathbf{e}^t) \quad \rightarrow \quad \begin{cases} \boldsymbol{\xi}_p = (z, \varphi, \psi, e_1, e_2, e_3, e_4)^t \\ \boldsymbol{\xi}_t = (x, y, \theta)^t \end{cases}$$

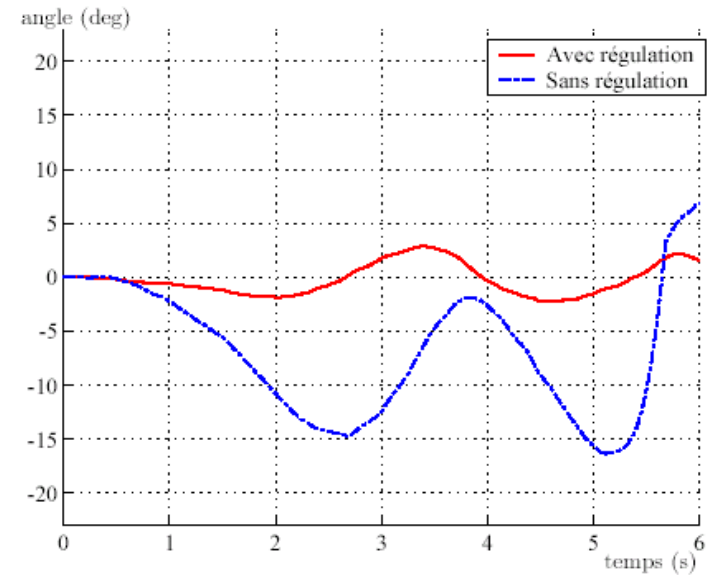
State-feedback control law:  $\dot{\boldsymbol{\xi}} = \mathbf{K}_p(\boldsymbol{\xi}_p^d - \boldsymbol{\xi}_p) + \mathbf{K}_t(\boldsymbol{\xi}_t^d - \boldsymbol{\xi}_t)$



# Contrôle de posture

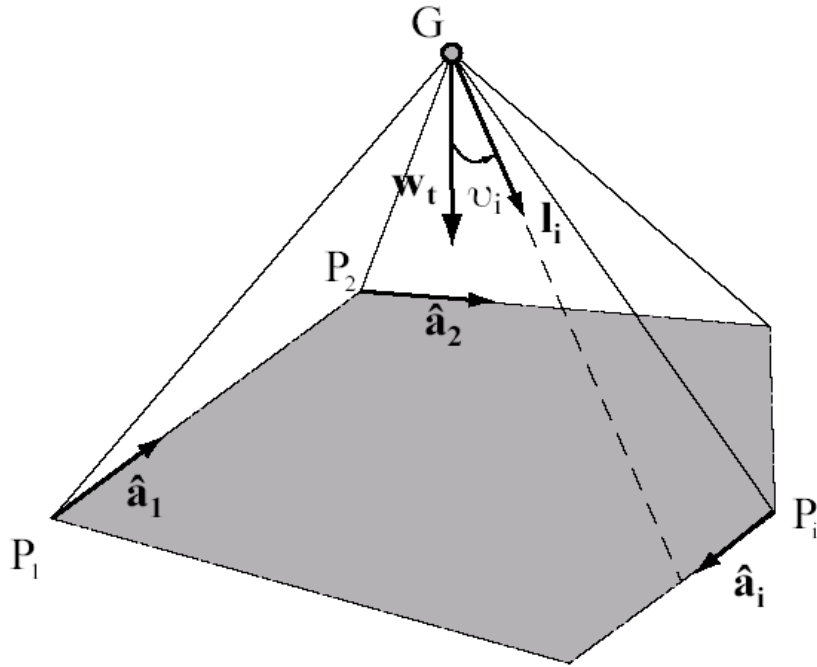


(b) Angle de roulis

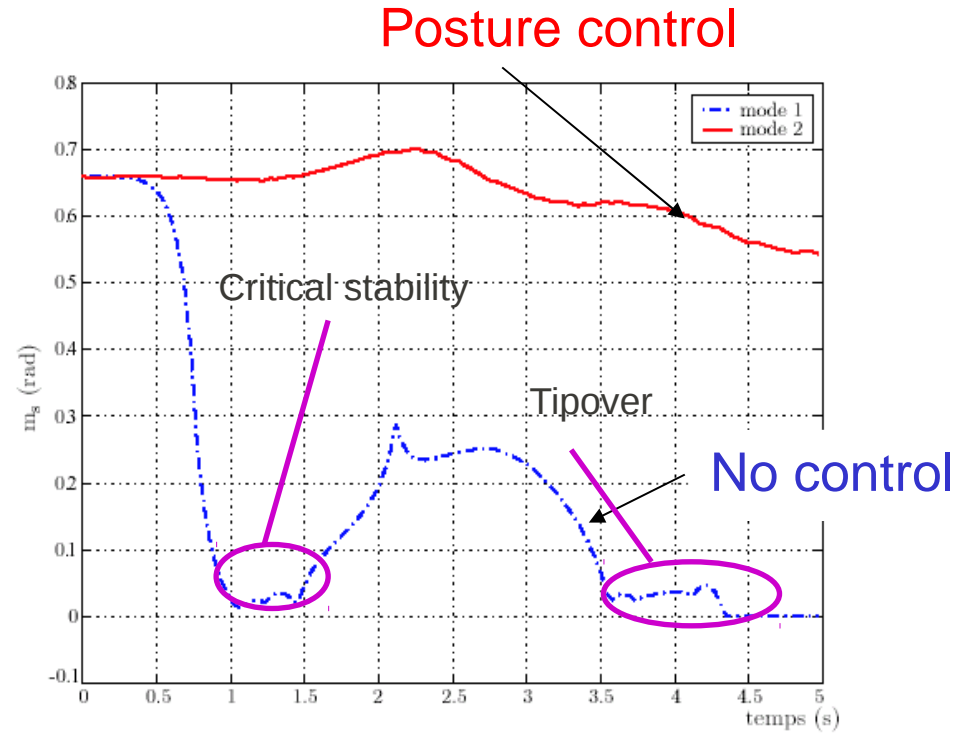


(a) Angle de tangage

# Contrôle de posture

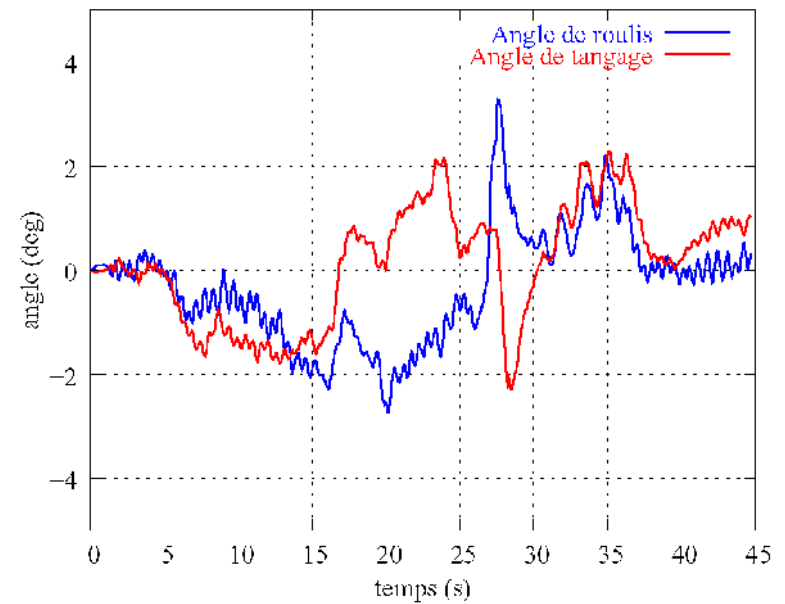
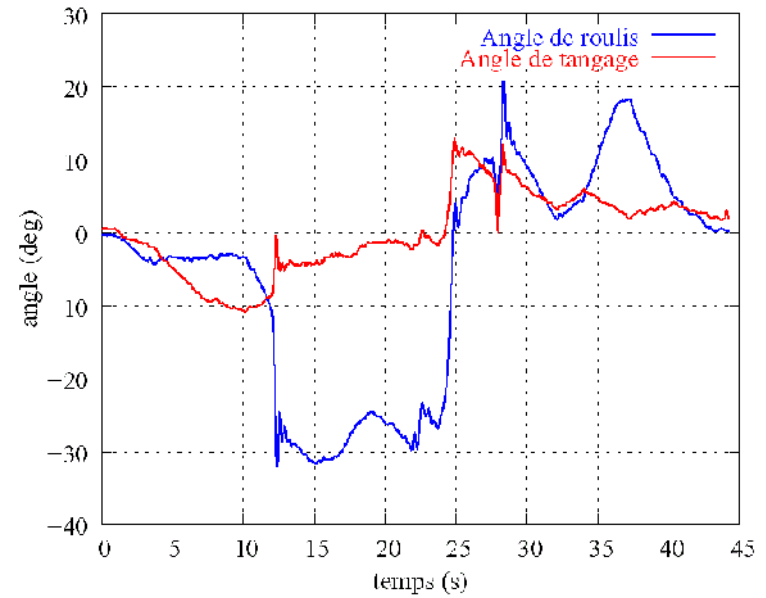


$$M_s = \min(v_i) \quad i = \{1, \dots, n\}$$



3 dimensional pseudo-dynamic stability measure<sup>1</sup>

# Contrôle de posture



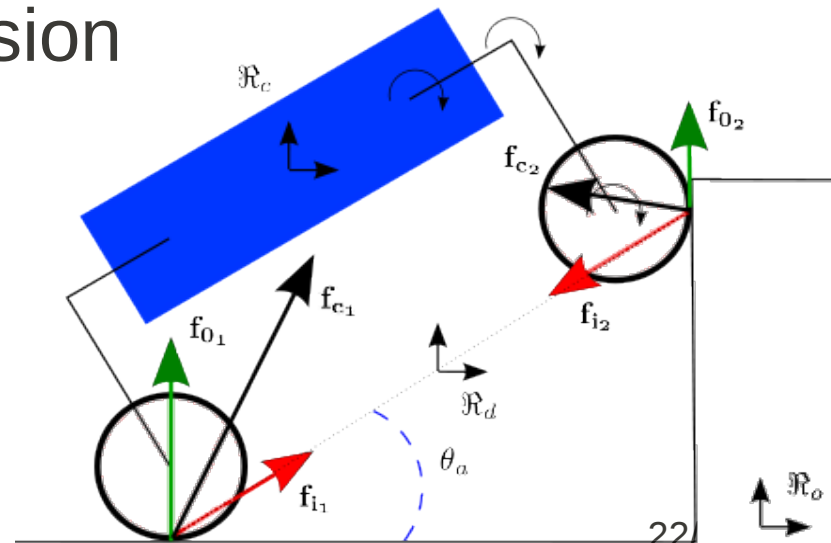
# Distribution des forces

## Objectifs

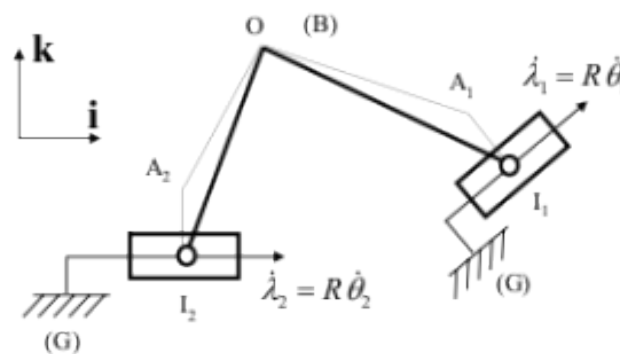
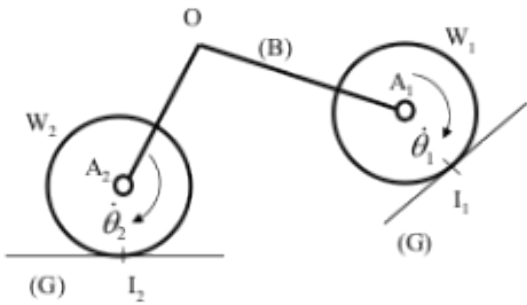
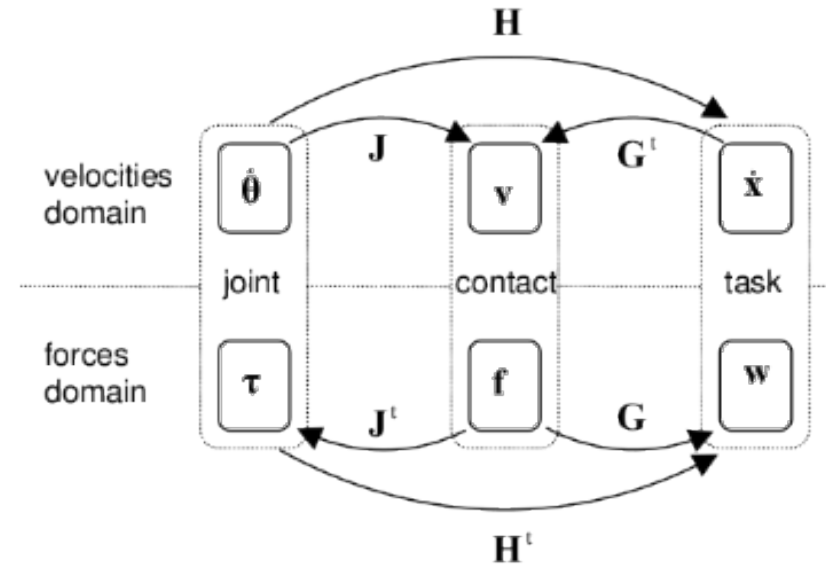
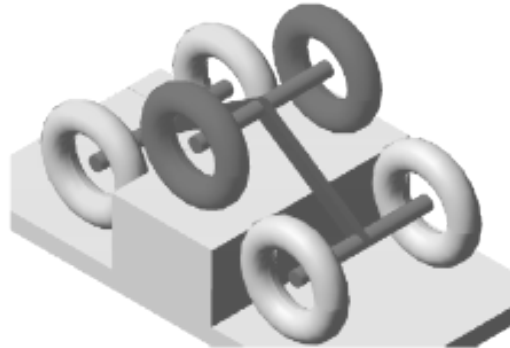
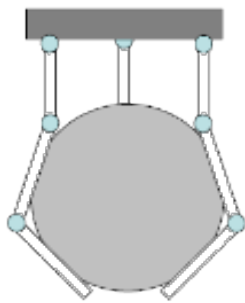
- Franchissement d'obstacle de type marche
- Utilisation des efforts internes pour optimiser la distribution des efforts de contact
- Maximiser les frottements de Coulomb

## Approche

- Analogie locomotion / préhension
- Optimisation sous la forme *minimax*



# Distribution des forces



$$v = -G^t \dot{x} = J \dot{\theta}$$

$$\begin{cases} Gf = w \\ J^t f = \tau \end{cases}$$

# Distribution des forces

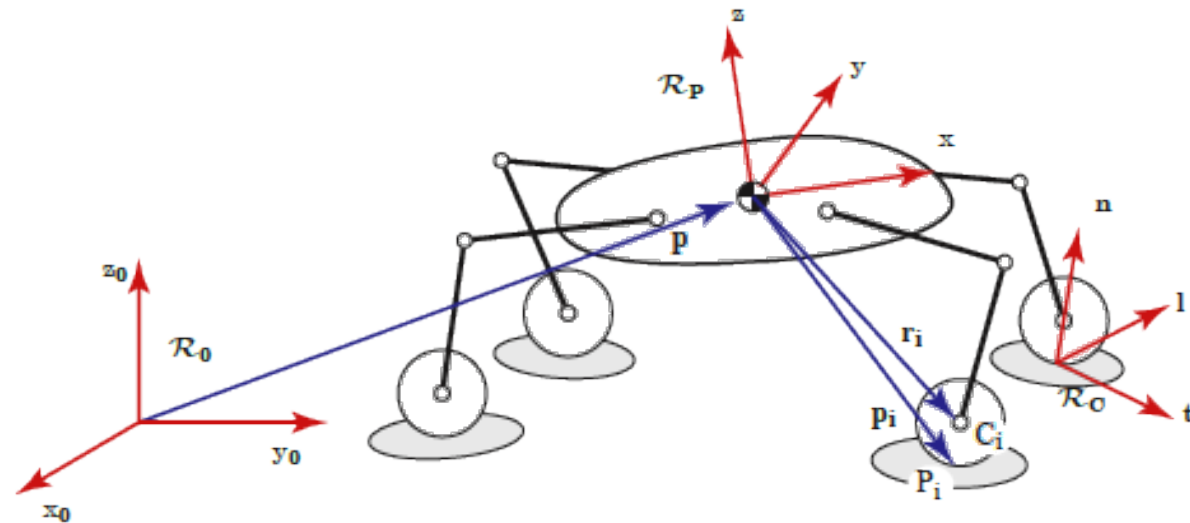
$$G \mathbf{f} = \mathbf{F}$$

$$\mathbf{G} = \begin{bmatrix} I_{3 \times 3} & R_{c_1} & \dots & I_{3 \times 3} & R_{c_n} \\ \tilde{\mathbf{p}}_1 & R_{c_1} & \dots & \tilde{\mathbf{p}}_n & R_{c_n} \end{bmatrix}$$

$$J^t \mathbf{f} = \boldsymbol{\tau}$$

where

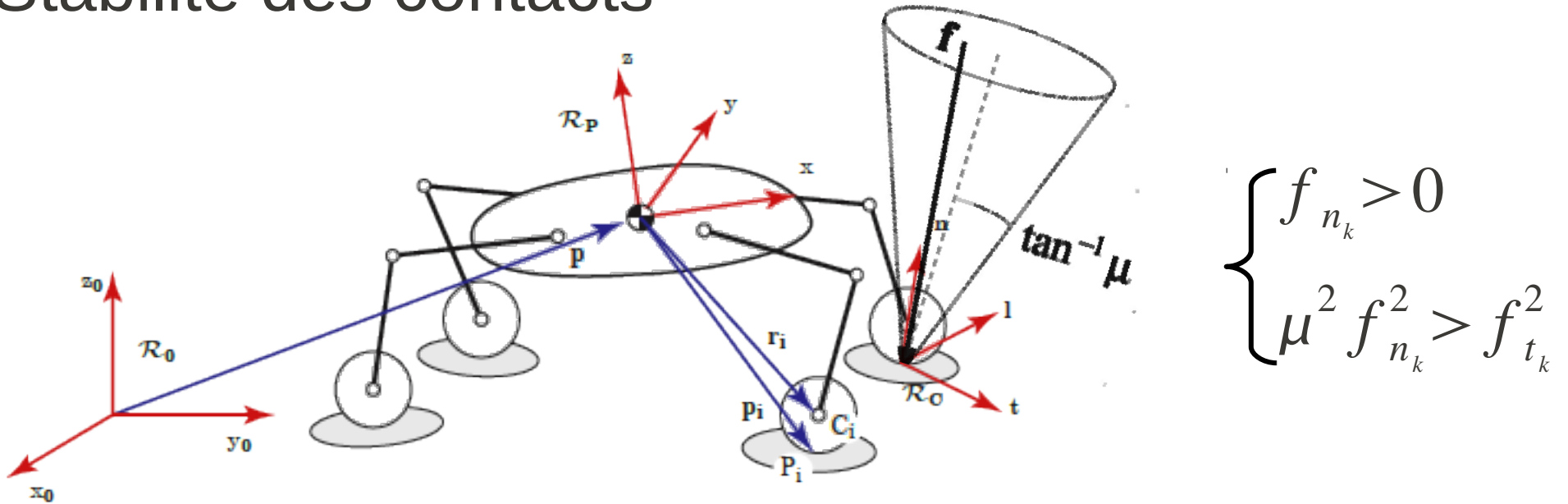
- $\mathbf{f}$  is the column vector of contacts forces
- $\mathbf{F}$  is the 6x1 vector of total wrench applied to the robot CoG
- $\mathbf{p}_i$  is position vector of each contact point  $P_i$
- $\mathbf{G}$  is the equivalent grasping matrix





# Distribution des forces

## Stabilité des contacts

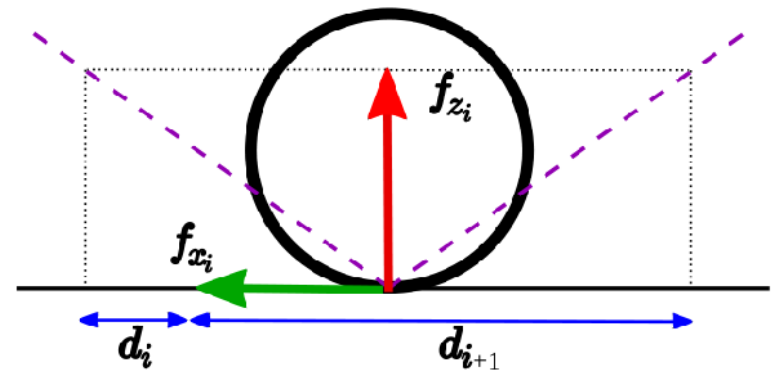


$$\begin{cases} f_{n_k} > 0 \\ \mu^2 f_{n_k}^2 > f_{t_k}^2 \end{cases}$$

Approximation:

$$\mathbf{d} = A \mathbf{f} \quad A_i = \begin{bmatrix} \frac{\mu_i}{\sqrt{2}} & 1 & 0 \\ \frac{\mu_i}{\sqrt{2}} & -1 & 0 \\ \frac{\mu_i}{\sqrt{2}} & 0 & 1 \\ \frac{\mu_i}{\sqrt{2}} & 0 & -1 \end{bmatrix}$$

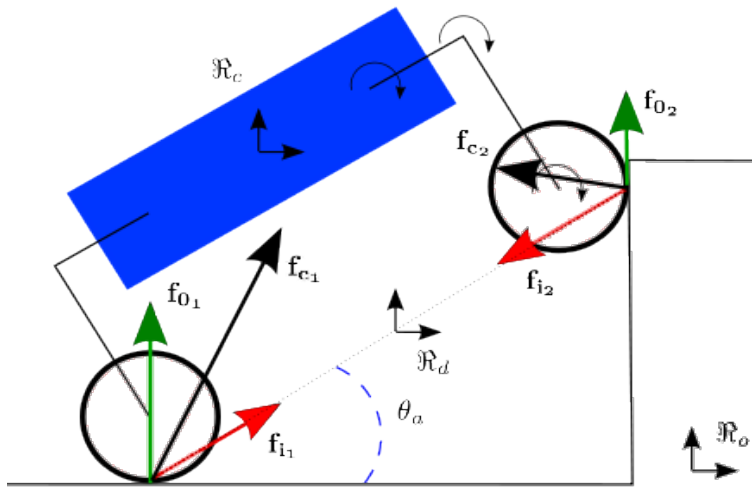
$$A = \text{blockdiag}(A_i)$$



$$\phi(\mathbf{f}) = \min(A \mathbf{f})$$

# Distribution des forces

Optimisation du franchissement :



Find  $\mathbf{f} \in \mathbb{R}^{3n}$  maximizing  
Subject to:

$\phi(\mathbf{f})$

$$\begin{aligned} G\mathbf{f} &= \mathbf{F} \\ J^T \mathbf{f} &< \tau_{max} \\ -J^T \mathbf{f} &< \tau_{max} \end{aligned}$$

# Distribution des forces

Gestion de la redondance:

$$G \mathbf{f} = \mathbf{F} \quad \text{avec } \dim(G) = 6 \times 12$$

Solution :

$$\mathbf{f} = \mathbf{f}_p + \mathbf{f}_h$$

$$\mathbf{f}_p = G_{Norm}^+ \mathbf{F} \quad \text{et} \quad G \mathbf{f}_h = \mathbf{0}$$

$$\text{Ker}(G) = \text{vect} \left( \{ \mathbf{b}_i \}_{i \in [1, m]} \right) \quad \rightarrow \quad N_f = [ \mathbf{b}_1 \quad \dots \quad \mathbf{b}_m ]$$

$$\mathbf{f} = G_{Norm}^+ \mathbf{F} + \underbrace{N_f \mathbf{x}_f}_{\text{Efforts internes}}$$

*Efforts internes*

# Distribution des forces

Optimisation des forces internes:

Find  $\mathbf{x}_f \in \mathbb{R}^m$  maximizing

$$\min (A[G^+ \mathbf{F} + N_f \mathbf{x}_f])$$

Subject to:

$$J_f^T \mathbf{x}_f < \mathbf{a}_f$$

with:

$$J_f = \begin{bmatrix} N_f^T & J & -N_f^T & J \end{bmatrix}^T$$

$$\mathbf{a}_f = \begin{bmatrix} (\tau_{max} - J^T G^+ \mathbf{F})^T & (\tau_{max} + J^T G^+ \mathbf{F})^T \end{bmatrix}^T$$

# Distribution des forces

Reconfiguration : position CdM

$$\mathbf{f}_p = \mathbf{c}_x X + \mathbf{c}_z Z + \mathbf{f}_0$$

$$\mathbf{f} = \mathbf{f}_0 + N \mathbf{x}$$

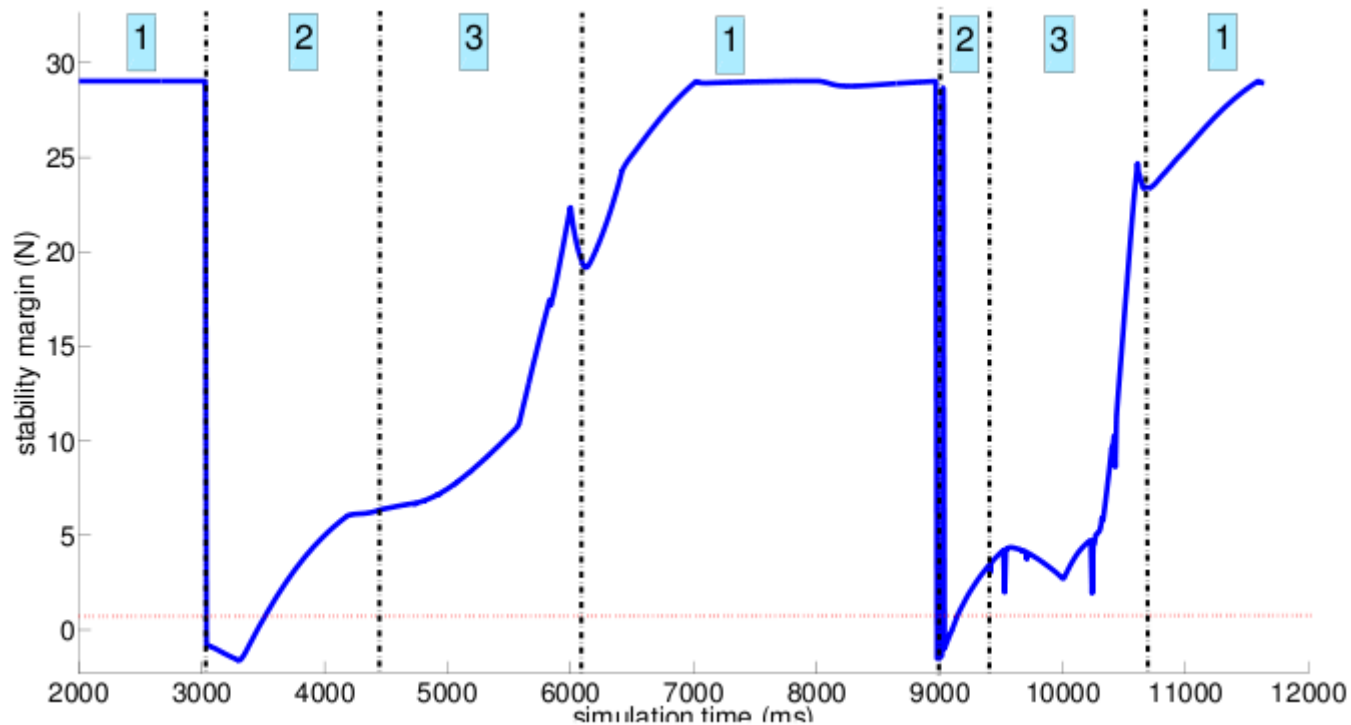
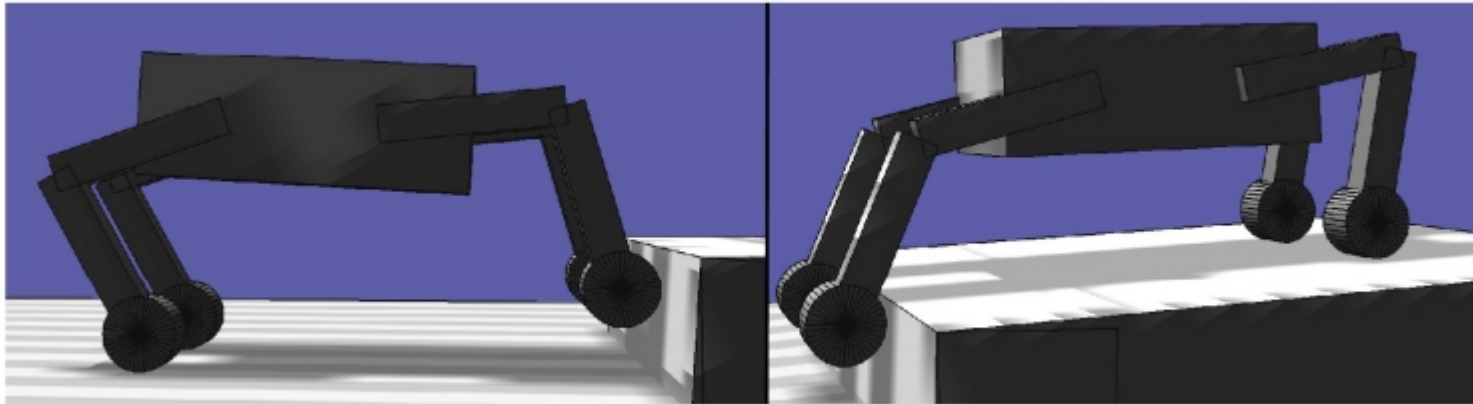
with  $N = \begin{bmatrix} N_f & \mathbf{c}_x & \mathbf{c}_z \end{bmatrix}$  and  $\mathbf{x} = \begin{bmatrix} \mathbf{x}_f^T & \mathbf{x}_p^T \end{bmatrix}^T$

# Distribution des forces

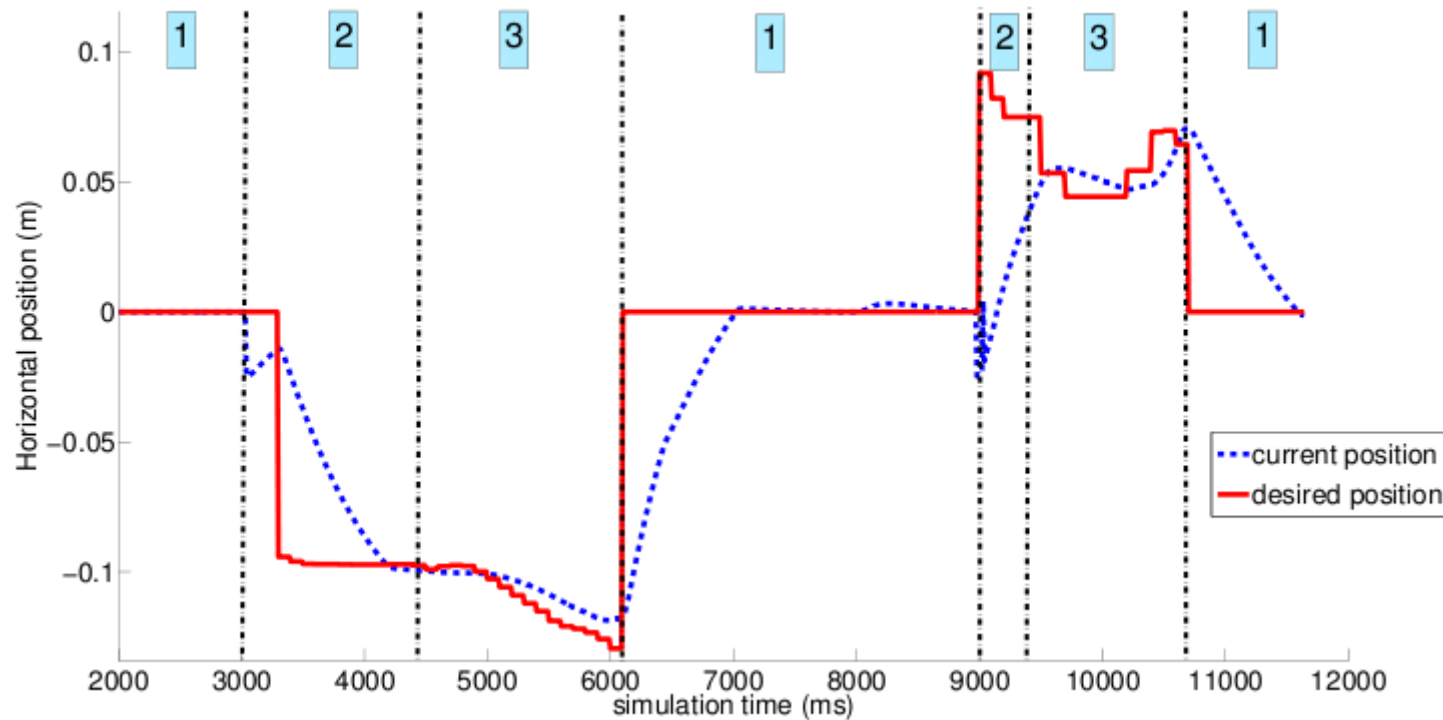
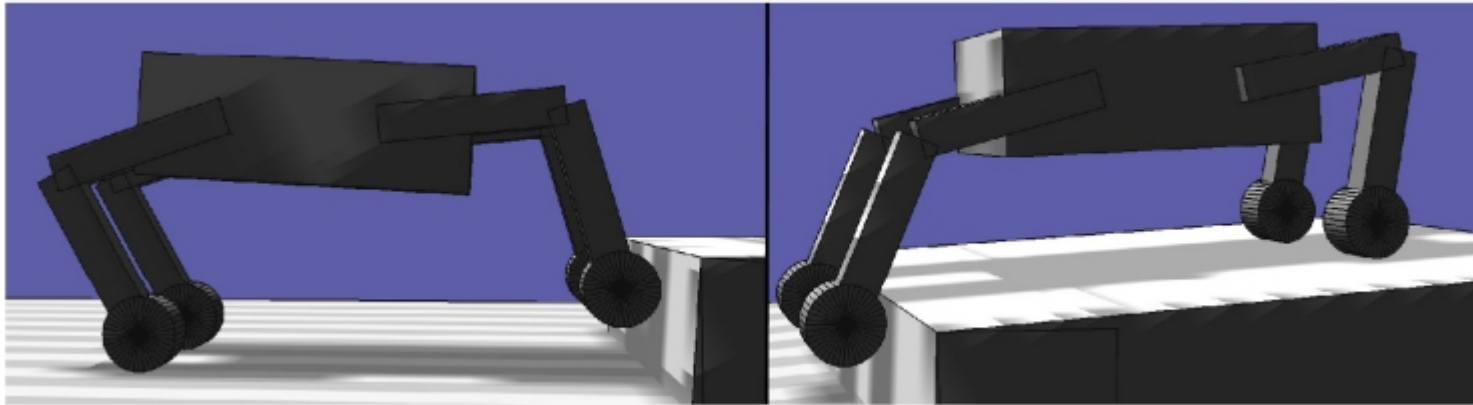
Simulation



# Distribution des forces



# Distribution des forces





# Distribution des forces

Simulation

