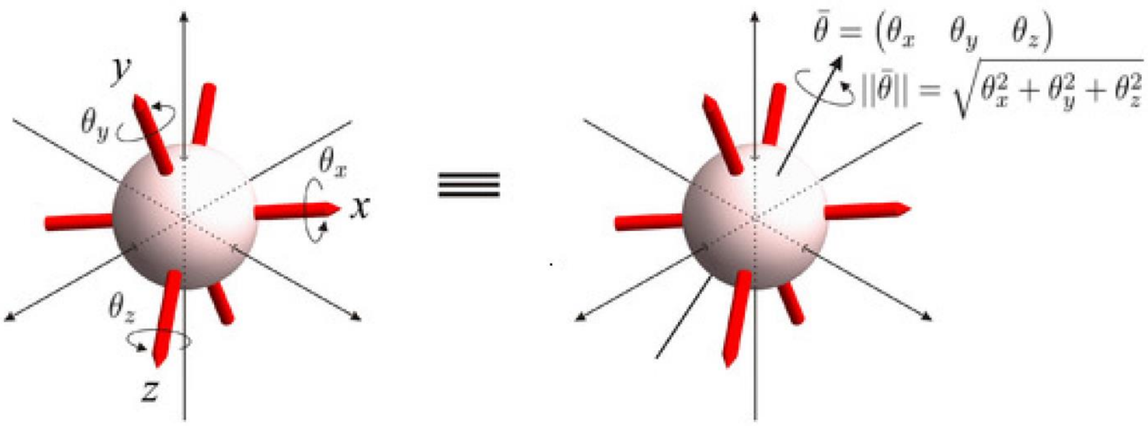
Quaternion techniques vs Classical Approaches applied to UAVs: a robustness question

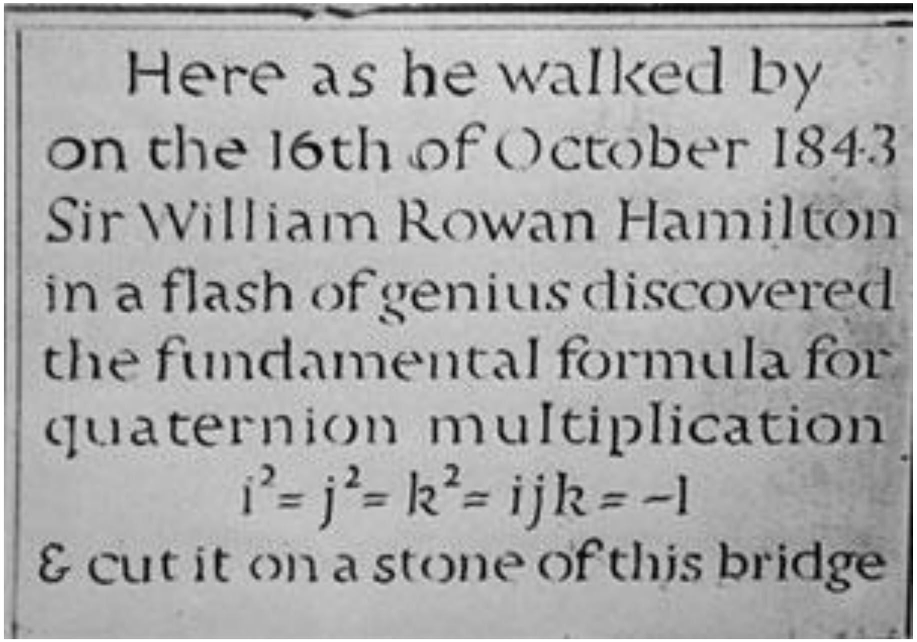
DRONES - HDS

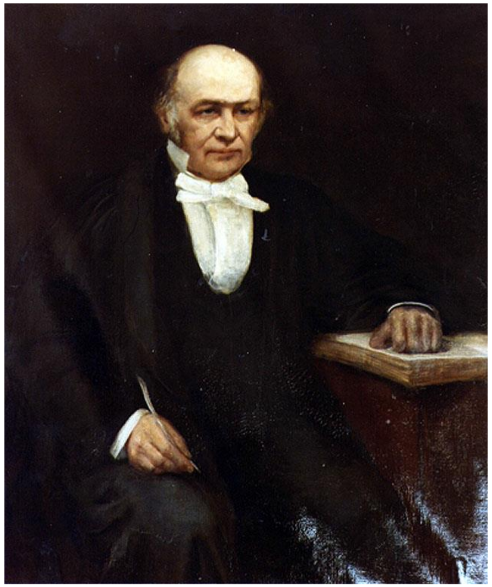
Euler's Rotation Theorem (1776)

"When a sphere is moved around its centre it is always posible to find a diameter whose direction in the displaced position is the same as in the intial position"

Leonhard Paul Euler







William Rowan Hamilton

- Euler-Rodrigues Formula:
  - $q_0 = \cos\frac{\theta}{2}$
  - $q_1 = e_x \cos\frac{\theta}{2}$
  - $q_2 = e_y \cos\frac{\theta}{2}$
  - $q_3 = e_z \cos\frac{\theta}{2}$

  - $\boldsymbol{q} = q_0 + q_1\hat{\imath} + q_2\hat{\jmath} + q_3\hat{k}$

  - With $e = \left[e_x, e_y, e_z\right]^T = \frac{\bar{\theta}}{||\bar{\theta}||}$

Olinde Rodrigues

- Quaternions belong to the space $\mathbb{H}$:
  - $\boldsymbol{q} \in \mathbb{H}; \bar{q} \in \mathbb{R}^3; q_0 \in \mathbb{R}$, then
    - $\boldsymbol{q} = q_0 + \bar{q} = q_0 + \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$
- Some operations with quaternions:
  - $\boldsymbol{q}, \boldsymbol{r} \in \mathbb{H}$
  - $\boldsymbol{q} \otimes \boldsymbol{r} = (q_0 r_0 - \bar{q} \cdot \bar{r}) + (r_0 \bar{q} + q_0 \bar{r} + \bar{q} \times \bar{r})$
  - $\|\boldsymbol{q}\| = \sqrt{\boldsymbol{q} \otimes \boldsymbol{q}^*} = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2}$
  - $\boldsymbol{q}^{-1} = \frac{\boldsymbol{q}^*}{\|\boldsymbol{q}\|}$
  - $\ln \boldsymbol{q} = \begin{cases} \ln\|\boldsymbol{q}\| + \frac{\bar{q}}{\|\bar{q}\|} \cos^{-1} \frac{q_0}{\|\boldsymbol{q}\|}, & \|\bar{q}\| \neq 0 \\ \ln\|\boldsymbol{q}\| , & \|\bar{q}\| = 0 \end{cases}$

- Quaternion derivative:
  - $\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{q} \otimes \omega$
- Quaternion Rotation:
  - A 3D vector can be rotated from one reference frame to another:
  - $\|\boldsymbol{q}\| = 1; \quad v, v' \in \mathbb{R}^3$
    - v is considered to be in the inertial frame
    - v' is in the body's reference.
  - $v' = L(\boldsymbol{q}, v) = \boldsymbol{q}^* \otimes v \otimes \boldsymbol{q}$

- Quadrotor Classical model:
  - Translational dynamics:

$$m\ddot{p} = F_{th}\begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

  - Rotational dynamics:

$$\begin{bmatrix} \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

$$p, \ddot{p} \in \mathbb{R}^3, \qquad F_{th} \in \mathbb{R}, \qquad g = -9{,}81\ m/s^2$$

- Non-linear
- Linearization involves
  - Singularities
  - Small-angles considerations
- Discontinuities

- Quadrotor quaternion dynamic model:

$$\frac{d}{dt}\begin{bmatrix} p \\ \dot{p} \\ \boldsymbol{q} \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{p} \\ \boldsymbol{q} \otimes \frac{F_{th} b}{m} \otimes \boldsymbol{q}^* + bg \\ \frac{1}{2}\boldsymbol{q} \otimes \omega \\ J^{-1}(\tau - \omega \times J\omega) \end{bmatrix},$$

$$b = [0,0,1]^T$$

- Exact linearization:

$$F_u = \boldsymbol{q} \otimes \frac{F_{th} b}{m} \otimes \boldsymbol{q}^* + bg, \qquad \omega = \frac{d}{dt}, \qquad \bar{\theta} = \frac{d}{dt} 2 \ln \boldsymbol{q},$$

$$\tau_u = J^{-1}(\tau - \omega \times J\omega)$$

- Computing a desired attitude trajectory:

$$q_d = \frac{(b \cdot F_u + \|F_u\|) + b \times F_u}{\|(b \cdot F_u + \|F_u\|) + b \times F_u\|}, \qquad F_{th} = \|F_u\|$$

- Linearized equivalent model

$$\frac{d}{dt}\begin{bmatrix} p \\ \dot{p} \\ \bar{\theta} \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{p} \\ F_u \\ \omega \\ \tau_u \end{bmatrix}$$

✓ Linear
✓ Simple
✓ Cascade Integrator
✓ No Singularities
✓ No discontinuities

Current Results:

- Aggressive maneuvers
- Resistance to large disturbances
- Quadrotor Aerial Manipulator
- Energy- based control
- Passivity-based control
- Path planning using PSO technique
- Circular trajectory tracking, following an UGV

Publications:

[1] J. Cariño, H. Abaunza, and P. Castillo Quadrotor Quaternion Control, International Conference on Unmanned Aircraft Systems (ICUAS), Denver, USA, 2015.

[2] H. Abaunza, J. Cariño, P. Castillo, R. Lozano, Quadrotor Dual Quaternion Control, In the Workshop on Research, Education and Development of Unmanned Aerial Systems (REDUAS), Cancún, México 2015.

[3] H. Abaunza, P. Castillo, R. Lozano, and A. Victorino Quadrotor Aerial Manipulator based on Dual Quaternions, International Conference on Unmanned Aircraft Systems (ICUAS), Arlington, VA, USA, 2016.

[4] M.E. Guerrero, H. Abaunza, P. Castillo, R. Lozano, and C.D. García, Energy Based Control for a Quadrotor using Unit Quaternions, International Conference on Unmanned Aircraft Systems (ICUAS), Arlington, VA, USA, 2016.

[5] H. Abaunza, P. Castillo, R. Lozano, and A. Victorino Dual Quaternion Modeling and Control of a Quad-rotor Aerial Manipulator, Journal of Intelligent & Robotic Systems. *Paper waiting for revision

[6] M.E. Guerrero, H. Abaunza, P. Castillo, R. Lozano, and C.D. García, Quadrotor Energy-Based Control Laws: A Unit-Quaternion Approach, Journal of Intelligent & Robotic Systems. *Paper waiting for revision

[7] M.E. Guerrero, H. Abaunza, P. Castillo, and R. Lozano, Passivity-Based Control for a Micro Air Vehicle using Unit Quaternions, Applied Sciences - Open Access Journal. *Paper waiting for revision

[8] H. Abaunza, E. Ibarra, P. Castillo, and A. Victorino, Quaternion based control for circular UAV trajectory tracking, following a ground vehicle: Real-time validation, 20th IFAC World Congress, Toulouse, France, 2017. *Paper waiting for revision

[9] A. Belkadi, H. Abaunza, L. Ciarletta, P. Castillo, and D. Theilliol, Distributed Path Planning for Controlling a Fleet of UAVs : Application to a Team of Quadrotors, 20th IFAC World Congress, Toulouse, France, 2017. *Paper waiting for revision

VIDEOS

Future Work

- UAV-UGV cooperative navigation
- Fleet formation
- Obstacle avoidance
- More aggressive maneuvers