

A GLOBAL EXPONENTIAL OBSERVER FOR VELOCITY-AIDED ATTITUDE ESTIMATION

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- 1 MOTIVATION
- 2 KINEMATIC MODEL
- 3 MEASUREMENTS
- 4 OBSERVER DESIGN
- 5 SIMULATIONS
- 6 CONCLUSIONS - EXTENSIONS

- 1 MOTIVATION
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- Attitude estimation is **crucial for control** purposes
- Estimation algorithms are required to be **robust** and **performant** in the presence of uncertainties and perturbations
- For lightweight, low-cost systems equipped with a MEMS Inertial Measurement Unit, “true” inertial navigation (i.e., based on the Schuler effect due to a rotating non-flat Earth) is excluded due to **limited accuracy** in the long run
- Inertial sensors must be “**aided**” by incorporating
 - a **model of the forces** acting on the vehicle, in which case the estimator is specific to the vehicle
 - an **additional sensor**, providing information in inertia or body axes (Sun tracker, Doppler radar, camera); in this case the estimator can be generic, but at the cost of the extra sensor

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EXISTING WORKS

- Almost globally asymptotically converging solutions available
- Design/stability on the natural geometry $SO(3) \times \mathbb{R}^3$ more involved ([Hua,Martin,Hamel AUT2016],[Mahony,Hamel TAC2008])
- Known gravity direction
- Global decoupling of pitch and roll estimation from magnetic measurements

PROPOSED SOLUTION

- Globally exponentially converging estimate
- Simple Design/stability established on $\mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3$
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ATTITUDE KINEMATICS

We consider a moving rigid body subjected to the angular velocity ω (in body axes). Its orientation (from inertial to body axes) matrix $R \in \text{SO}(3)$ is related to ω by

$$\dot{R} = RS(\omega), \quad (1)$$

where $S(\omega)$ is the skew-symmetric matrix defined by $S(\omega)x := \omega \times x$ whatever the vector x .

REDUCED ATTITUDE KINEMATICS

Define as gE_3 the gravity vector (in Earth axes). The dynamics of the gravity vector (in body axes) $\gamma := gR^T E_3$ is then

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MEASUREMENTS

- $v_m = v$ – Linear velocity in body frame
- $\omega_m = \omega$ – Angular velocity in body frame
- $a_m = a$ – Specific acceleration
- $\beta_m = \beta$ – Magnetic field in body frame
- $y := \text{col}(v_m, \beta_m), \quad u := \text{col}(a_m, \omega_m)$

REMARK

If B is the known constant magnetic field vector (in Earth axes), $\beta := R^T B$ and

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MEASUREMENTS' MODEL (CONT'D)

DYNAMICAL MODEL

$$\dot{v}_m = v_m \times \omega + \gamma + a_m \quad (4)$$

$$\dot{\beta}_m = \beta_m \times \omega_m \quad (5)$$

APPROACH FOR ATTITUDE ESTIMATION

For $B \times E_3 \neq 0$, R is completely specified by γ and β since

$$\begin{aligned} R^T &= R^T (gE_3 \quad B \quad gE_3 \times B) \cdot (gE_3 \quad B \quad gE_3 \times B)^{-1} \\ &= (\gamma \quad \beta \quad \gamma \times \beta) \cdot (gE_3 \quad B \quad gE_3 \times B)^{-1}, \end{aligned} \quad (6)$$

with $R^T(x \times y) = R^T x \times R^T y \quad \forall x, y$.

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MODEL FOR ESTIMATOR DESIGN

$$\dot{v}_m = v_m \times \omega_m + \gamma + a_m \quad (7)$$

$$\dot{\gamma} = \gamma \times \omega_m \quad (8)$$

$$\dot{\beta}_m = \beta_m \times \omega_m. \quad (9)$$

Notice the two subsystems (7)-(8) and (9) are completely independent.

MEASUREMENTS

$$y := \text{col}(v, m), \quad u := \text{col}(a, \omega)$$

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OBSERVER

$$\dot{\hat{v}} = \hat{v} \times \omega_m + a_m + \hat{\gamma} - (L + K)(\hat{v} - v_m) \quad (10)$$

$$\dot{\hat{\gamma}} = \hat{\gamma} \times \omega_m - (LS(\omega_m) - S(\omega_m)L + LK)(\hat{v} - v_m) \quad (11)$$

$$\dot{\hat{\beta}} = \hat{\beta} \times \omega_m - M(\hat{\beta} - \beta_m), \quad (12)$$

where the 3×3 matrices K, L, M are tuning parameters. The observer is a copy of (7)–(9) with (time-varying) correction terms, which respects the decoupled structure of (7)–(9).

$\hat{\gamma}$ IS INDEPENDENT OF MAGNETIC MEASUREMENTS

If R is parametrized by the Euler angles (ϕ, θ, ψ) , γ reads

$$\gamma = g \begin{pmatrix} -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{pmatrix}^T$$

γ thus encodes the roll and pitch angles, which are crucial for stabilization purposes (whereas the yaw angle matters far less).

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ERROR DYNAMICS (PERFECT MEASUREMENTS)

■ Error variables

$$e_v := \hat{v} - v$$

$$e_\gamma := \hat{\gamma} - \gamma - L e_v$$

$$e_\beta := \hat{\beta} - \beta$$

■ Error dynamics

$$\dot{e}_v = e_v \times \omega + e_\gamma - K e_v \quad (13)$$

$$\dot{e}_\gamma = e_\gamma \times \omega - L e_\gamma \quad (14)$$

$$\dot{e}_\beta = e_\beta \times \omega - M e_\beta \quad (15)$$

THEOREM

*If the symmetric parts of K, L, M are positive definite, then the equilibrium point $(\bar{e}_v, \bar{e}_\gamma, \bar{e}_\beta) := (0, 0, 0)$ of the error system (13)–(15) is **globally exponentially stable**.*

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PROOF SKETCH – LYAPUNOV FUNCTION

$$V(e_\gamma, e_v, e_\beta) := \underbrace{\frac{\rho_1}{2} |e_\gamma|^2 + \frac{\rho_1 \epsilon^2}{2} |e_v|^2}_{=: V_\gamma(e_\gamma, e_v)} + \underbrace{\frac{1}{2} |e_\beta|^2}_{=: V_\beta(e_\beta)}, \quad \rho_1, \epsilon > 0$$

$$\dot{V}_\beta = -\langle e_\beta, \frac{M + M^T}{2} e_\beta \rangle \leq -\underline{\sigma}_M |e_\beta|^2$$

$$\dot{V}_\gamma \leq -\rho_1 \left(\underline{\sigma}_L - \frac{\epsilon}{2} \right) |e_\gamma|^2 - \rho_1 \epsilon^2 \left(\underline{\sigma}_K - \frac{\epsilon}{2} \right) |e_v|^2,$$

with $\underline{\sigma}_M$ the smallest eigenvalue of $\frac{M+M^T}{2}$.

PROOF SKETCH (CONT'D) – ROTATION MATRIX ESTIMATE

Under the assumptions of Theorem 1,

$$\tilde{R} := \begin{pmatrix} \frac{(\hat{\gamma} \times \hat{\beta}) \times \hat{\gamma}}{|(gE_3 \times B) \times gE_3|} & \frac{\hat{\gamma} \times \hat{\beta}}{|gE_3 \times B|} & \frac{\hat{\gamma}}{|gE_3|} \end{pmatrix}^T$$

globally exponentially converges to the orientation matrix R .

PROPOSITION

Considerer the polar decomposition of $\tilde{R}^T$

$$\tilde{R}^T = \hat{R}^T (\tilde{R} \tilde{R}^T)^{\frac{1}{2}}.$$

Then $\hat{R}$, which is by construction the best approximation of $\tilde{R}$ among all orthogonal matrices, is a rotation matrix that globally exponentially converges to R . When $\hat{\gamma}$ and $\hat{\beta}$ are not collinear, $\hat{R}$ is uniquely defined by

$$\hat{R} := \begin{pmatrix} \frac{(\hat{\gamma} \times \hat{\beta}) \times \hat{\gamma}}{|(\hat{\gamma} \times \hat{\beta}) \times \hat{\gamma}|} & \frac{\hat{\gamma} \times \hat{\beta}}{|\hat{\gamma} \times \hat{\beta}|} & \frac{\hat{\gamma}}{|\hat{\gamma}|} \end{pmatrix}^T.$$

LINEAR TIME-VARYING ERROR DYNAMICS

Rotated error coordinates: $(E_v, E_\gamma, E_\beta) := (Re_v, Re_\gamma, Re_\beta)$

$$\dot{E}_v = E_\gamma - RK R^T E_v \quad (16)$$

$$\dot{E}_\gamma = -RL R^T E_\gamma \quad (17)$$

$$\dot{E}_\beta = -RM R^T E_\beta, \quad (18)$$

$(K, L, M) := (kI, lI, mI)$ — TIME-INVARIANT DYNAMICS

$$\dot{E}_v = E_\gamma - kE_v \quad (19)$$

$$\dot{E}_\gamma = -lE_\gamma \quad (20)$$

$$\dot{E}_\beta = -mE_\beta; \quad (21)$$

LOCAL TUNING (CONT'D)

For different gains on the components of each correction term, as for instance when the velocity sensor has for technological reasons a “privileged” direction (in general the vertical axis).

$$K := \begin{pmatrix} k_x & -k_y & 0 \\ k_y & k_x & 0 \\ 0 & 0 & k_z \end{pmatrix} \quad L := \begin{pmatrix} l_x & -l_y & 0 \\ l_y & l_x & 0 \\ 0 & 0 & l_z \end{pmatrix},$$

with $k_x, k_z, l_x, l_z > 0$; K (resp. L) has a pair of complex conjugate eigenvalues when $k_y \neq 0$ (resp. $l_y \neq 0$). For a trajectory of the system such that

$$R := \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where ψ arbitrary function of time; $RKR^T = K$ and $RLR^T = L$.

$$\begin{aligned} \dot{E}_v &= E_\gamma - KE_v \\ \dot{E}_\gamma &= -LE_\gamma \end{aligned}$$

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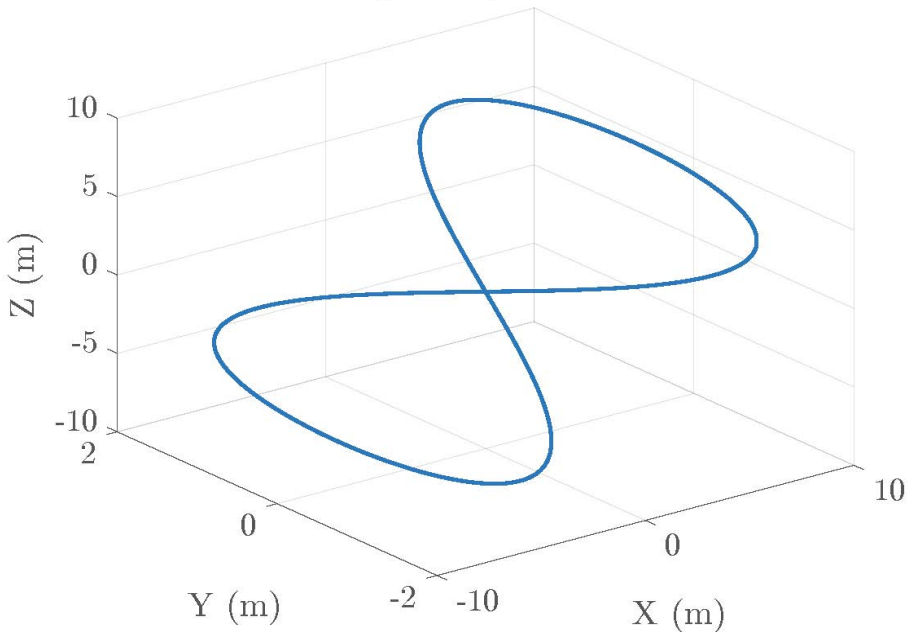
SCENARIO

- Aggressive trajectory
- Robustness wrt corrupted measurements
- Observer initialized with no error, but suddenly reinitialized at $t = 50$
- Magnetic vector B set to the nominal value $(1/\sqrt{2}, 0, 1/\sqrt{2})^T$, but subjected to a violent disturbance for $t \in [80, 100]$

GAINS

$$(K, L, M) := (5I, 5I, 0.5I)$$

Eight-shaped Path



CORRUPTED MEASUREMENTS

$$v_m = v + b_v + \nu_v \quad (22)$$

$$\omega_m = \omega + b_\omega + \nu_\omega \quad (23)$$

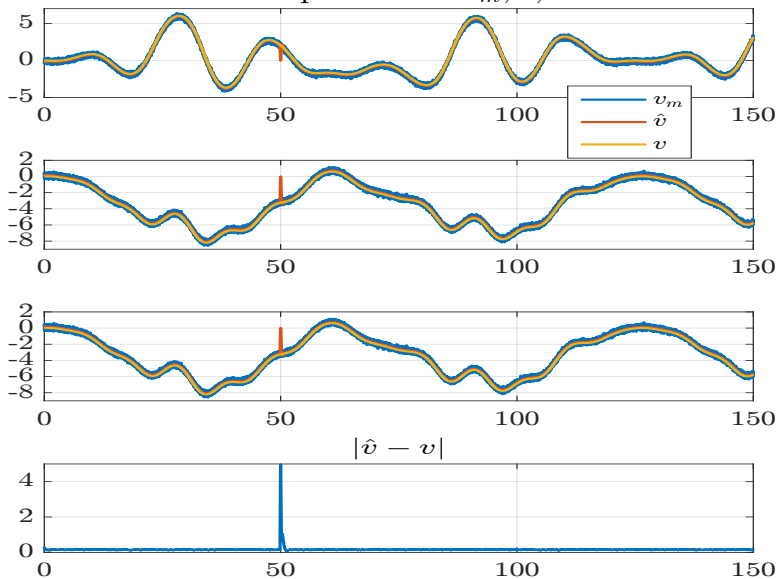
$$a_m = a + b_a + \nu_a \quad (24)$$

$$\beta_m = \beta + b_\beta + \nu_\beta, \quad (25)$$

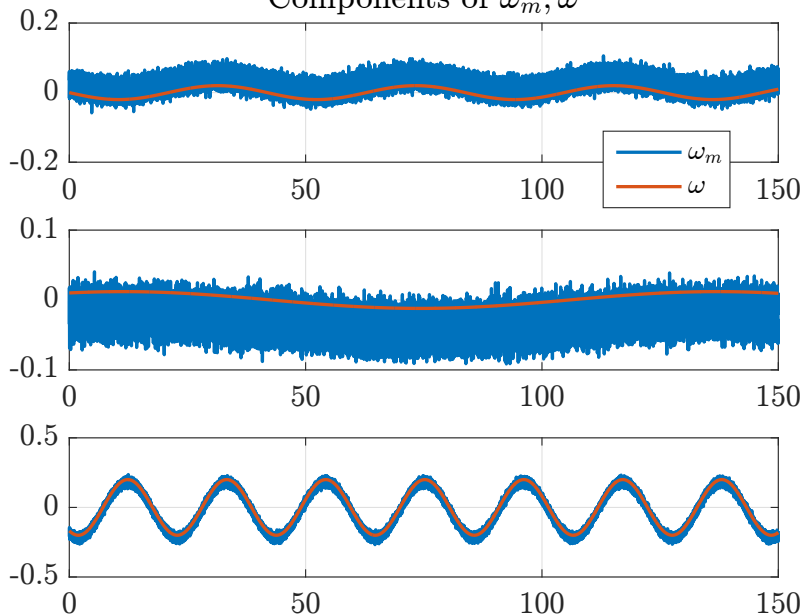
b_v	$(-0.10 \ 0.30 \ -0.05)^T$	σ_v^2	$10^{-5}(2 \ 2 \ 2)^T$
b_ω	$(0.0250 \ -0.0300 \ -0.0175)^T$	σ_ω^2	$10^{-7}(2 \ 2 \ 2)^T$
b_a	$(0.05 \ 0.04 \ -0.02)^T$	σ_a^2	$10^{-5}(1 \ 1 \ 1)^T$
b_β	$(0.024 \ -0.020 \ -0.018)^T$	σ_β^2	$10^{-5}(1 \ 1 \ 1)^T$

TABLE: (Constant) Biases and (white Gaussian independent noises with intensities σ_i) noises in the actual measurements.

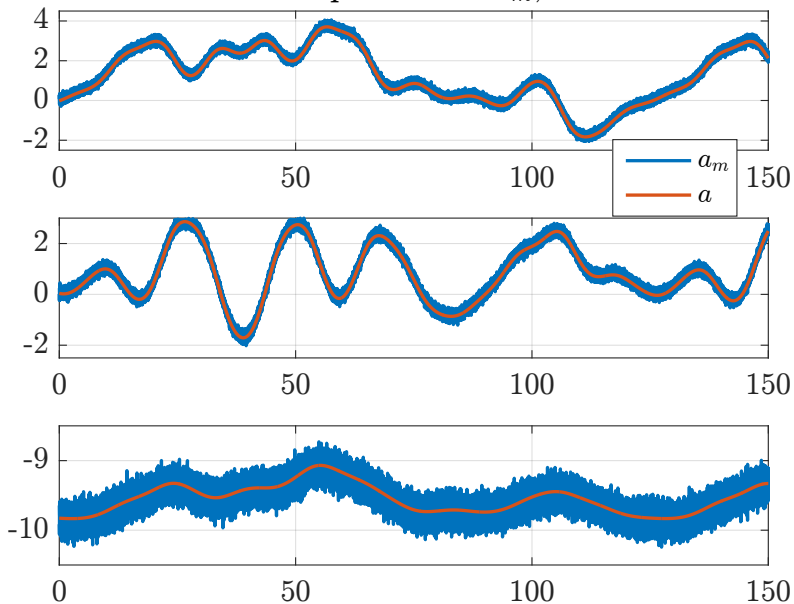
Components of $v_m, \hat{v}, v$



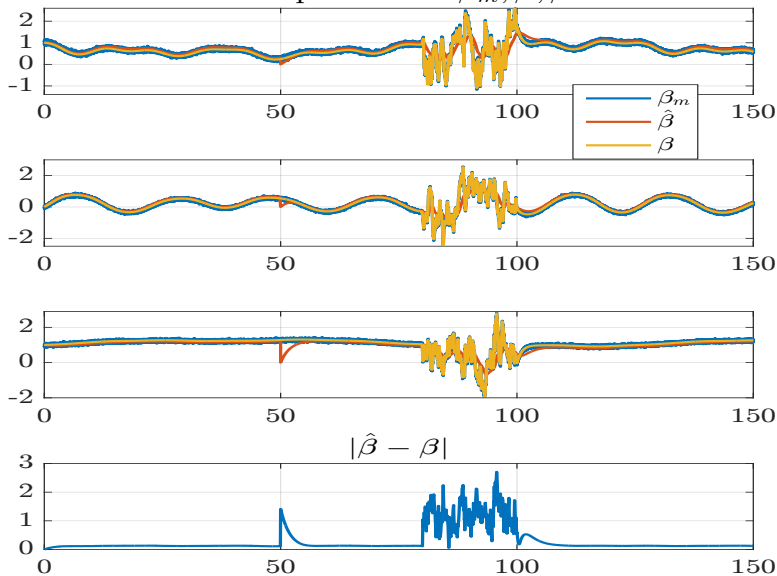
Components of ω_m, ω



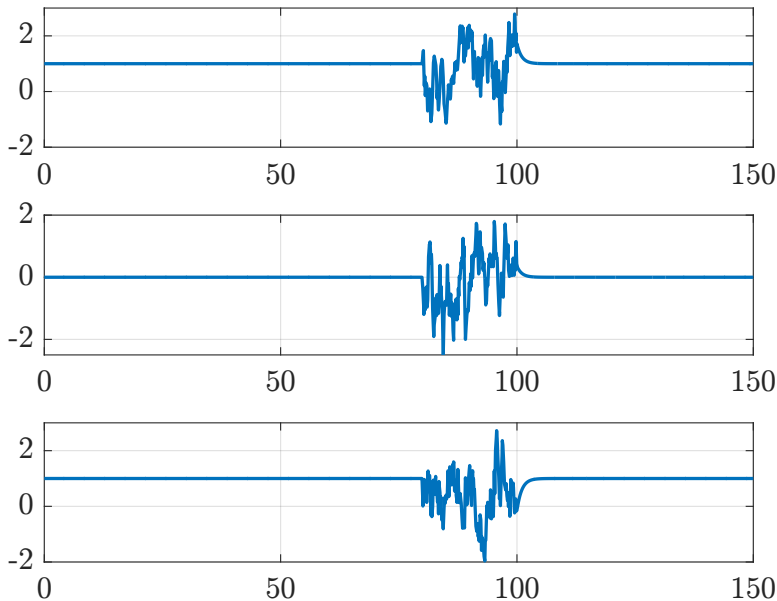
Components of a_m, a



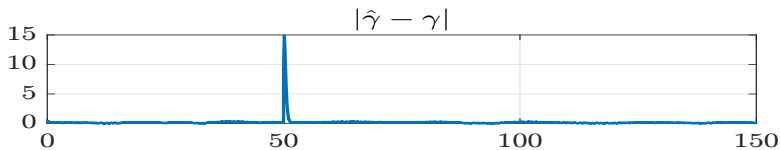
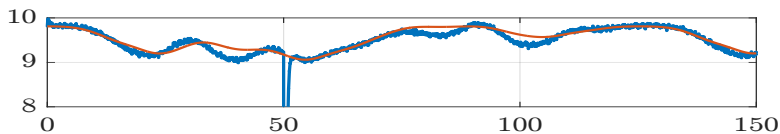
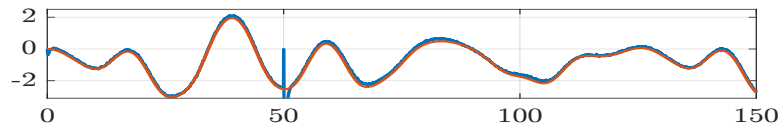
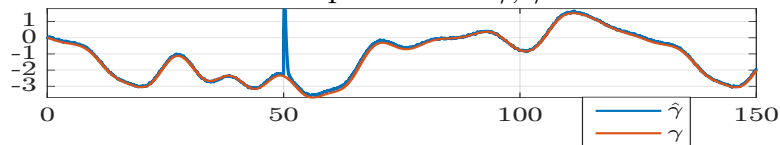
Components of $\beta_m, \hat{\beta}, \beta$

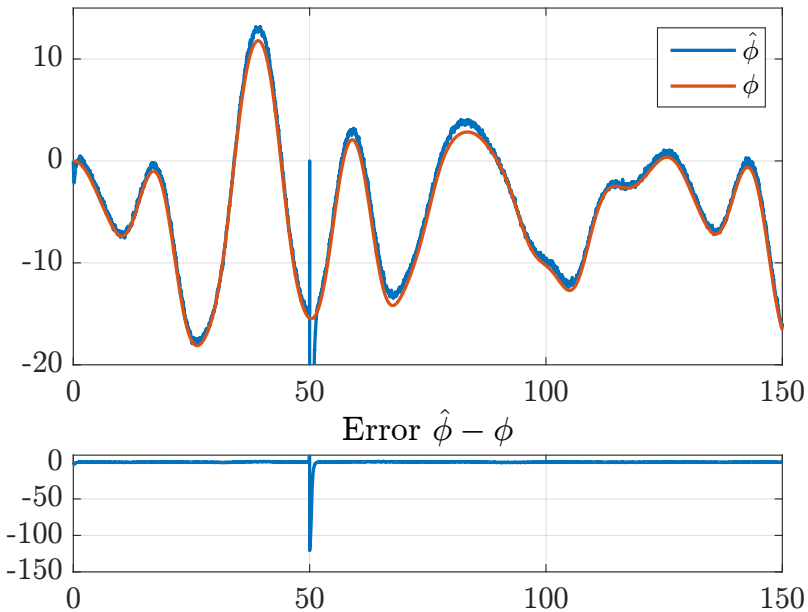


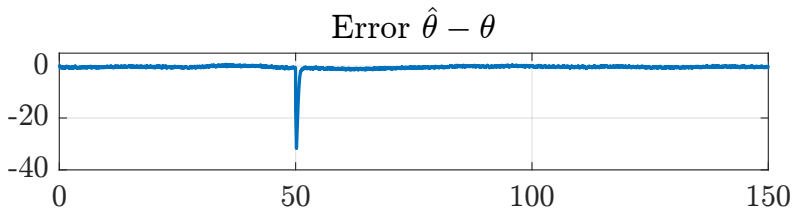
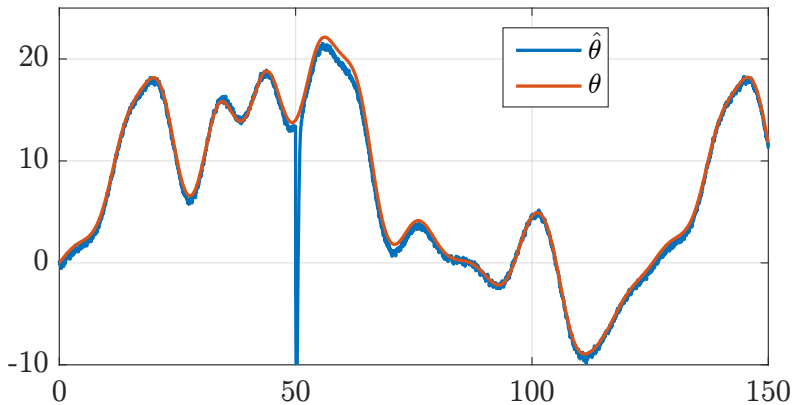
Components of B

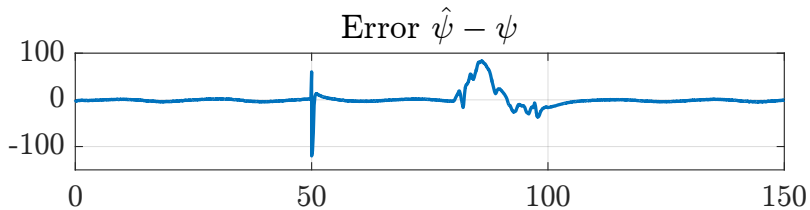
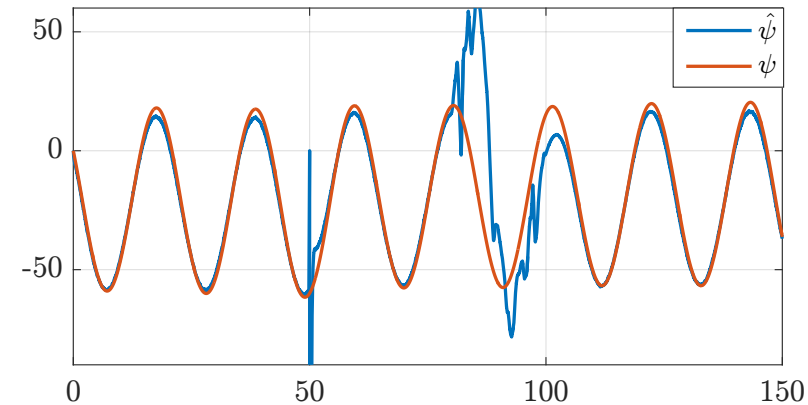


Components of $\hat{\gamma}, \gamma$









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CONCLUSIONS

- (Velocity,Magnetic field,Gyro)-measurement scenario for global attitude estimation
- Model parameterization => Linear Time-Varying dynamics with constraints
- Observer derived using the invariant-manifold-based approach ([Astolfi,Karagiannis,Ortega])
- Observer = Copy of system + (Linear/Nonlinear) Correction terms
- Constant tuning gains
- Global exponential convergence – Robustness wrt small perturbations
- Good observer performance in the presence of unaccounted bias and (additive) disturbances

EXTENSIONS

- Inertial velocity measurement instead of body velocity
- Account for gyro bias
- Account for vector bias
- Time-delay in measurements
- Experimental validation

- P. Martin, I. Sarras, M. -D. Hua, T. Hamel, “**A global exponential observer for velocity-aided attitude estimation**”, *Automatica*, 2016, submitted. (version arXiv available)
- P. Martin and I. Sarras, “A simple model-based estimator for the quadrotor using only inertial measurements”, *CDC*, 2016. (version arXiv available)
- P. Martin and I. Sarras, “A simple global observer for attitude and gyro biases”, *IFAC World Congress*, 2016, submitted. (version arXiv available)