Bearing Rigidity Maintenance for Formations of Quadrotor UAVs

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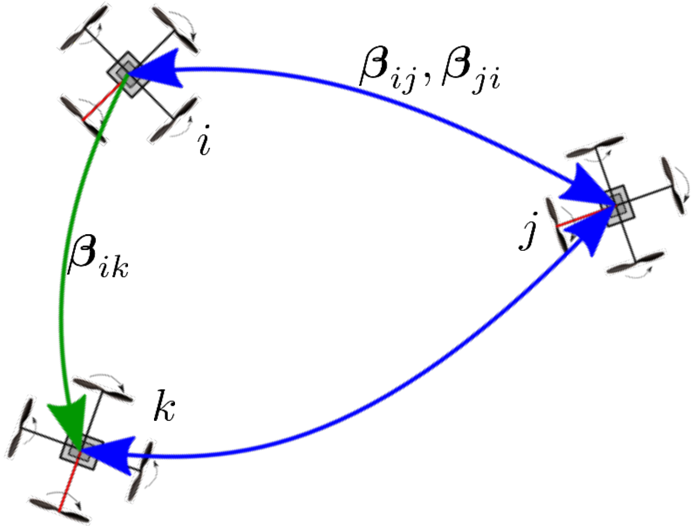
Goals of the Formation Control Algorithm

$$\boldsymbol{q}_i = (\boldsymbol{p}_i, \psi_i) \qquad \boldsymbol{p}_i \in \mathbb{R}^3, \quad \psi_i \in \mathcal{S}$$

- Control of a formation of quadrotor UAVs equipped with onboard cameras



- Taking into account several sensing constraints
  - Maximum/minimum range of the camera
  - Limited fov of the camera
  - Possible occlusions between the agents

- **Bearing information** is **low cost**

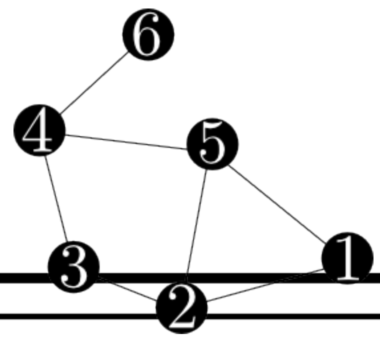
$$\boldsymbol{\beta}_{ij} = \boldsymbol{R}^T(\psi_i)\frac{\boldsymbol{p}_j - \boldsymbol{p}_i}{\|\boldsymbol{p}_j - \boldsymbol{p}_i\|} \in \mathbb{S}^2$$

$$\begin{pmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\psi}_i \end{pmatrix} = \begin{pmatrix} \boldsymbol{R}(\psi_i) & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_i \\ w_i \end{pmatrix}$$

Outline

- Brief introduction on the theory of rigidity
- Details of the bearing rigidity maintenance strategy
- Decentralized gradient-based controller
- Experimental results
- Conclusions and future works

Rigidity for distance constraints in the plane

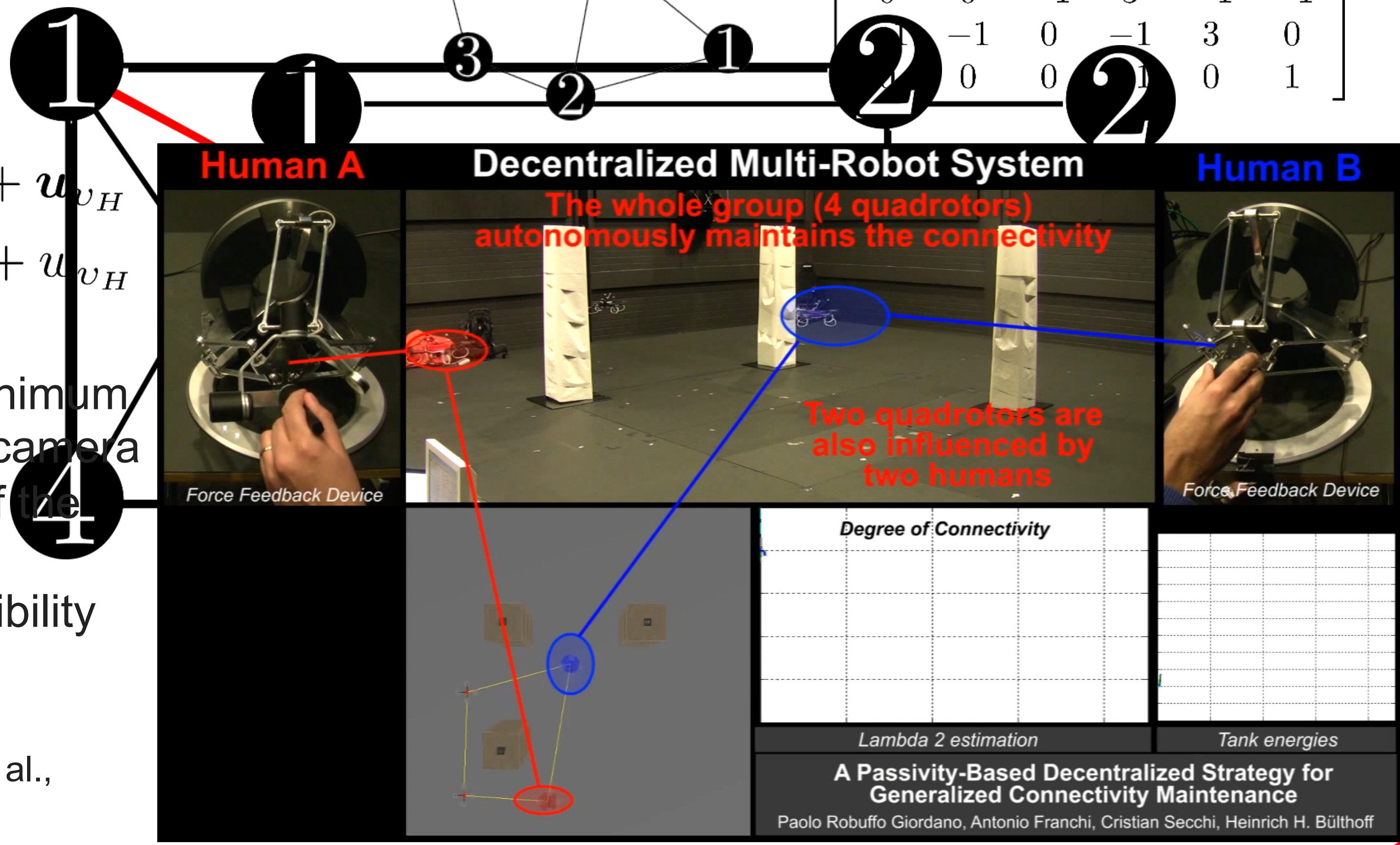


Laplacian Matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} \boldsymbol{u}_v = \boldsymbol{u}_{v_M} + \boldsymbol{u}_{v_H} \\ w_v = w_{v_M} + w_{v_H} \end{cases}$$

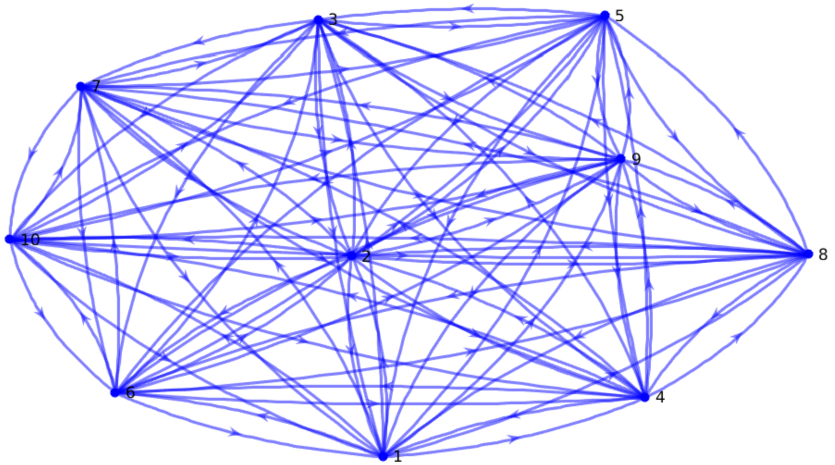
- Maximum/minimum range of the camera
- Limited fov of the camera
- Occluded visibility



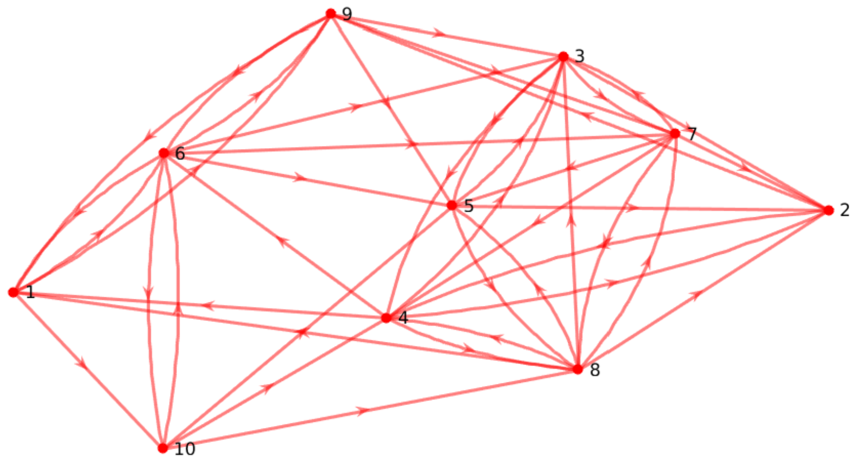
Robuffo Giordano et al., IJRR 2013

Why Rigidity?

- **Localization problem**
  - If the framework is rigid, it is possible to reconstruct the pose of all the agents w.r.t. a common frame
- **Formation Control problem**
  - No need of all possible pairwise geometrical constraints (i.e. no need of the complete graph).



Complete (rigid) graph - $O(N^2)$



Non-Complete (rigid) graph - $O(N)$

Directed Bearing Rigidity in $\mathbb{R}^3 \times \mathcal{S}^1$

- Directed Bearing Rigidity Function associated to a framework $(\mathcal{G}, \boldsymbol{q})$:

$$\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}) : (\mathbb{R}^3 \times \mathcal{S}^1)^N \to (\mathbb{S}^2)^{|\mathcal{E}|}$$
$$\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}) = \left[\boldsymbol{\beta}_{e_1}^T \dots \boldsymbol{\beta}_{e_{|\mathcal{E}|}}^T\right]^T$$

- Directed Bearing Rigidity Matrix (BRM):

$$\boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{W}}(\boldsymbol{q}) = \frac{\partial \boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q})}{\partial \boldsymbol{q}} \in \mathbb{R}^{3|\mathcal{E}| \times 4N} \qquad \dot{\boldsymbol{\beta}}_{\mathcal{G}} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{W}}(\boldsymbol{q}) \begin{bmatrix} \dot{\boldsymbol{p}} \\ \dot{\psi} \end{bmatrix}$$

$$\dot{\boldsymbol{\beta}}_{\mathcal{G}} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{W}}(\boldsymbol{q}) \begin{bmatrix} \mathrm{diag}(\boldsymbol{R}_i) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{bmatrix} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{W}} \boldsymbol{T}(\psi) \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{bmatrix} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q}) \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{bmatrix}$$

Infinitesimal Bearing Rigidity (IBR)

- Infinitesimal rigidity: study of the flexibility of a framework under instantaneous motions of its nodes
- Assume a smooth time dependence $\boldsymbol{q} = \boldsymbol{q}(t)$: the instantaneous motions of $\boldsymbol{q}(t)$ which preserve the constraints $\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}(t)) = const.$ are:

$$\dot{\boldsymbol{\beta}}_{\mathcal{G}}(\boldsymbol{q}(t)) = 0 \implies \frac{\partial \boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}(t))}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q})\dot{\boldsymbol{q}} = 0$$

- The infinitesimal motions consistent with the constraints are

$$\dot{\boldsymbol{q}} \in ker(\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q}))$$

- A framework $(\mathcal{G}, \boldsymbol{q})$ is infinitesimally bearing rigid if:

$$ker(\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q})) = ker(\boldsymbol{\mathcal{B}}_{\mathcal{K}_N}(\boldsymbol{q}))$$

- Or equivalently:

$$rank(\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q})) = rank(\boldsymbol{\mathcal{B}}_{\mathcal{K}_N}(\boldsymbol{q})) = 4N - 5$$

- Where $\mathcal{K}_N$ is the complete graph

Body-Frame Bearing Rigidity Matrix

- In $\mathbb{R}^3 \times \mathbb{S}^1$ the row of the *Body-Frame*-BRM associated to edge *(i,j)* is:

$$\left[-\mathbf{0}- \quad \underbrace{-\frac{\boldsymbol{P}_{ij}}{d_{ij}}}_{i} \quad -\mathbf{0}- \quad \underbrace{\frac{\boldsymbol{P}_{ij}\boldsymbol{R}(\psi_j - \psi_i)}{d_{ij}}}_{j} \quad -\mathbf{0}- \quad \cdots \quad \underbrace{-\boldsymbol{S}\boldsymbol{\beta}_{ij}}_{3N+i} \quad -\mathbf{0}- \right]$$

$$d_{ij} = \|\boldsymbol{p}_i - \boldsymbol{p}_j\|, \; \boldsymbol{P}_{ij} = \boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T \;, \boldsymbol{S} = \left[[0\ 0\ 1]^T\right]_\times$$

- It contains measured bearings and relative quantities

Cooperative localization from relative bearings
[Schiano et al., IROS 2016 – Zelazo et al., CDC 2015, ECC 2014]

- In general, formation control/localization schemes need the relative ${}^{i}\boldsymbol{R}_j = \boldsymbol{R}(\psi_j - \psi_i)$ orientation and distance d_{ij}

  ⇒ Estimation through bearing-based localization scheme
- $\hat{\boldsymbol{q}}_i = (\hat{\boldsymbol{p}}_i, \hat{\psi}_i)$: estimation of the true configuration $\boldsymbol{q}_i$ of agent i
- The proposed algorithm is capable of:

$$\begin{cases} \hat{\boldsymbol{p}} & = & (\boldsymbol{I}_N \otimes \boldsymbol{R}(\bar{\psi}))\boldsymbol{p} + \boldsymbol{1}_N \otimes \boldsymbol{t} \\ \hat{\boldsymbol{\psi}} & = & \boldsymbol{\psi} + \boldsymbol{1}_N \bar{\psi} \end{cases}$$

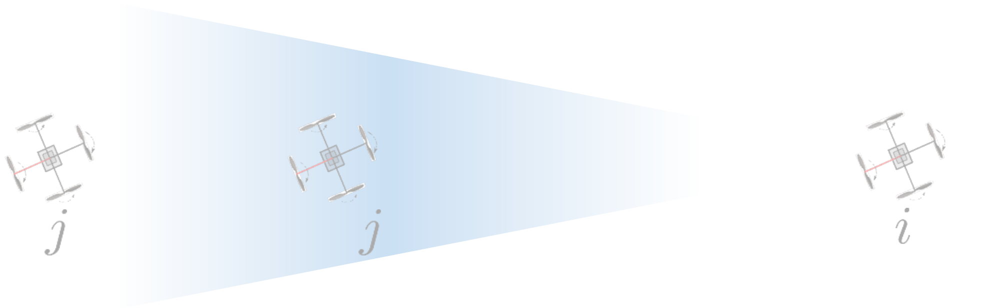
- Neighboring agents can exchange $\hat{\boldsymbol{q}}_i$ and $\hat{\boldsymbol{q}}_j$ to replace (d_{ij}, ψ_{ij}) with $(\hat{d}_{ij}, \hat{\psi}_{ij})$

✓ It is fully decentralized, it can cope with time-varying bearings $\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}(t))$
- Necessary condition for the localization convergence: IBR formation
- Assumption: one distance measurement $d_{\iota\kappa}$ is known
  - The scale could be estimated [Spica et al., IROS 2016]

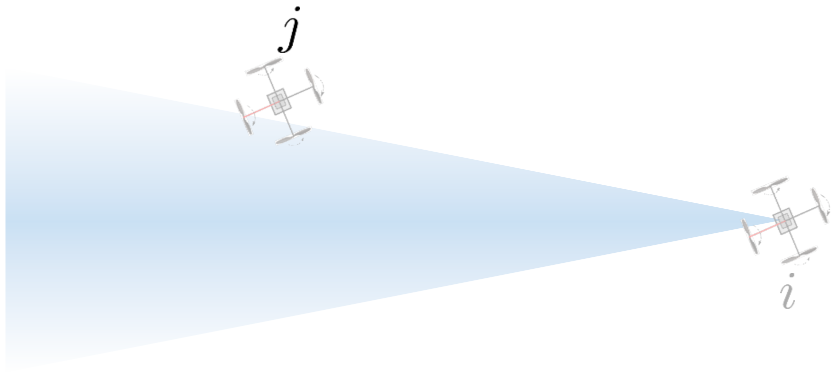
Sensing Constraints

$w_{ij} \in [0, 1]$

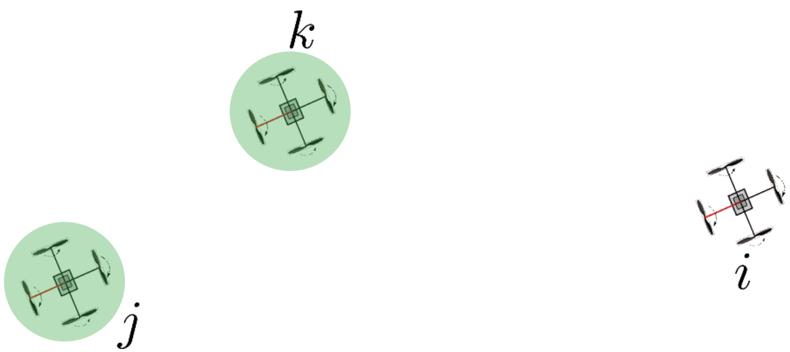
$w_{ik} = 1$



Maximum/minimum range of the camera



Limited fov of the camera



Occluded visibility

Weighted bearing rigidity matrix

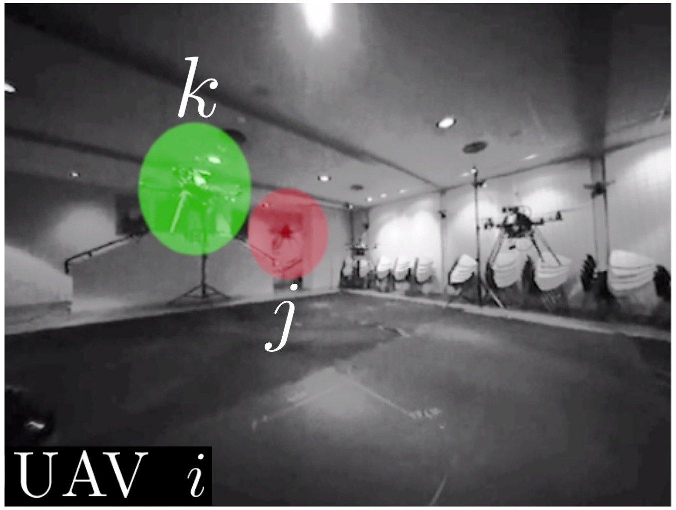
- Sensing constraints took into account
  - Maximum/minimum range of the camera $w_{R_{ij}}$
  - Limited FOV of the camera $w_{F_{ij}}$
  - Occluded visibility $w_{V_{ij}}$
- The sensing constraints affecting a pair of agents (i,j) are encoded together in a suitable scalar weight $w_{ij} = w_k$ ((i,j) is the $k-th$ edge)

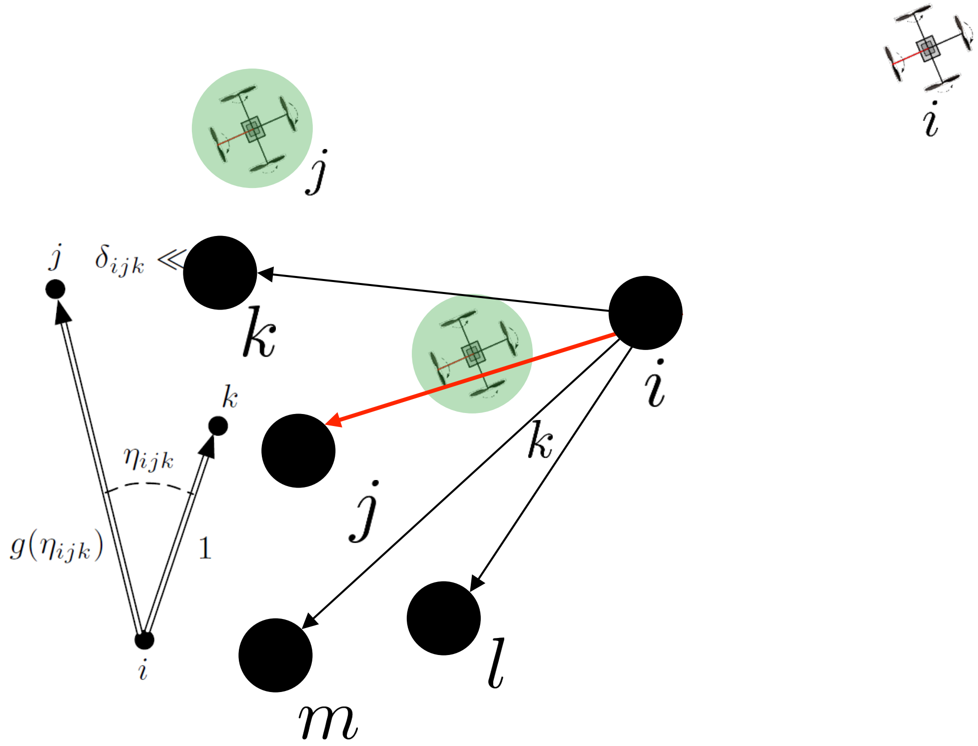
$$w_{ij} = w_k = w_{R_{ij}} w_{F_{ij}} w_{V_{ij}}$$

- Let $\boldsymbol{W} = \mathrm{diag}(w_k \boldsymbol{I}_3) \in \mathbb{R}^{3|\mathcal{E}| \times 3|\mathcal{E}|}$ be a diagonal matrix which collects all the weights associated to the all the edges $|\mathcal{E}|$

  ⇒ Weighted body-frame rigidity matrix: $\boldsymbol{W}\boldsymbol{\mathcal{B}}_{\mathcal{G}}$

Occluded Visibility





- Where

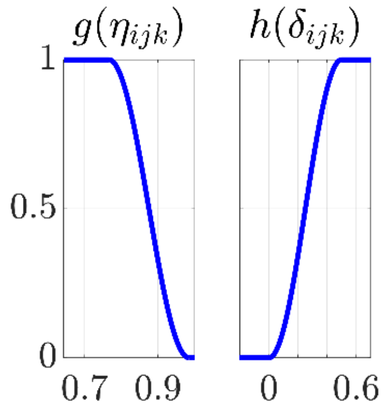
$$\eta_{ijk} = \boldsymbol{\beta}_{ij}^{T}\boldsymbol{\beta}_{ik} \qquad \delta_{ijk} = d_{ik} - d_{ij}$$

- Occluded visibility weight

$$w_{V_{ijk}}(\delta_{ijk}, \eta_{ijk}) = (1 - h(\delta_{ijk}))g(\eta_{ijk}) + h(\delta_{ijk})$$

$$w_{V_{ij}} = \prod_{k \in \mathcal{N}_i / \{j\}} w_{V_{ijk}}(\delta_{ijk}, \eta_{ijk})$$

$$w_{ij} = w_k = w_{R_{ij}}\, w_{F_{ij}}\, w_{V_{ij}}$$



<u>Symmetric</u> bearing rigidity matrix

- Remember that $\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q}) \in \mathbb{R}^{3|\mathcal{E}| \times 4N}$
- $\boldsymbol{B}_{\mathcal{G}}(\boldsymbol{q}) = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{T}(\boldsymbol{q})\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q}) \in \mathbb{R}^{4N \times 4N}$
- A framework is infinitesimally bearing rigid iff $\mathrm{rank}(\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q})) = 4N - 5$
- Which translates into $\lambda_6(\boldsymbol{q}) = \lambda_6(\boldsymbol{B}_{\mathcal{G}}(\boldsymbol{q})) > 0$

$$\Downarrow$$

$\lambda_6(\boldsymbol{q}) :=$ Bearing rigidity eigenvalue

- Proposition: $\lambda_6(\boldsymbol{q})$ does NOT depend on the agent orientations ψ

(proof omitted)

$$\lambda_6(\boldsymbol{q}) \equiv \lambda_6(\boldsymbol{p})$$

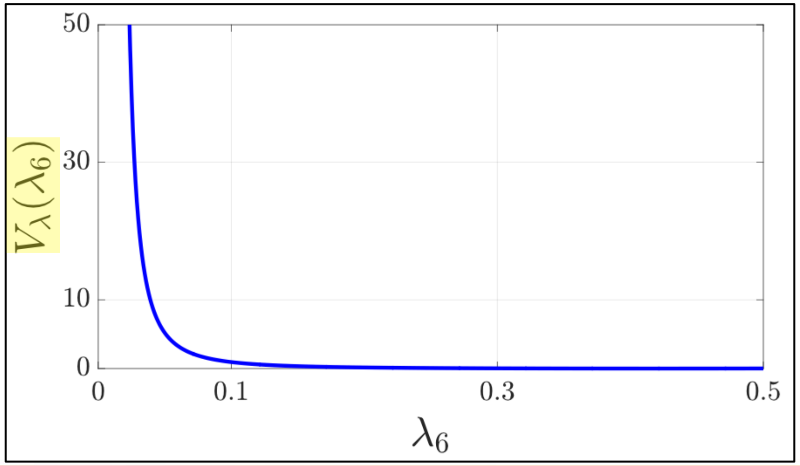
Controller

- The first design for the controller of the agent υ was:

$$\begin{cases} \boldsymbol{u}_\upsilon &= \nabla_{\boldsymbol{p}_\upsilon} \lambda_6 \\ w_\upsilon &= \nabla_{\psi_\upsilon} \lambda_6 \end{cases}$$

- Which evolved into:

$$\begin{cases} \boldsymbol{u}_\upsilon &= -\boldsymbol{R}_\upsilon^T \nabla_{\boldsymbol{p}_\upsilon} V_\lambda(\lambda_6) = -\boldsymbol{R}_\upsilon^T \nabla_{\lambda_6} V_\lambda \nabla_{\boldsymbol{p}_\upsilon} \lambda_6 \\ w_\upsilon &= -\nabla_{\psi_\upsilon} V_\lambda(\lambda_6) = -\nabla_{\lambda_6} V_\lambda \nabla_{\psi_\upsilon} \lambda_6 \end{cases}$$



$$\begin{cases} \boldsymbol{u}_\upsilon = \boldsymbol{u}_{\upsilon M} + \boldsymbol{u}_{\upsilon H} \\ w_\upsilon = w_{\upsilon M} + w_{\upsilon H} \end{cases}$$

Rigidity eigenvalue

- $\boldsymbol{\nu}_6 \in \mathbb{R}^{4N}$: body-frame normalized eigenvector associated to λ_6

$$\lambda_6 = \boldsymbol{\nu}_6^T \boldsymbol{B}_{\mathcal{G}} \boldsymbol{\nu}_6$$

- The rigidity eigenvalue has the following expression (unweighted case)

$$\lambda_6 = \sum_{(i,j)\in\mathcal{E}} \left(\boldsymbol{\nu}_{p_{ij}}^T \frac{\boldsymbol{P}_{ij}}{d_{ij}^2} \boldsymbol{\nu}_{p_{ij}} + 2\boldsymbol{\nu}_{p_{ij}}^T \boldsymbol{S} \frac{\boldsymbol{\beta}_{ij}}{d_{ij}} \nu_{\psi_i} - \nu_{\psi_i}^2 \boldsymbol{\beta}_{ij}^T \boldsymbol{S}^2 \boldsymbol{\beta}_{ij} \right) = \sum_{(i,j)\in\mathcal{E}} l_{ij}$$

- Where $\boldsymbol{P}_{ij} = \boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T, \;\; \boldsymbol{\nu}_{p_{ij}} = \boldsymbol{\nu}_{p_i} - \boldsymbol{\nu}_{p_j}$

- It contains only body-frame quantities relative to agents *i,j* ($\boldsymbol{\beta}_{ij}, d_{ij}, {}^i\boldsymbol{R}_j, \boldsymbol{\nu}_6$)
- Introducing the weights, $\boldsymbol{\mathcal{B}}_{\mathcal{G}}^T(\boldsymbol{q})\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q}) \rightarrow \boldsymbol{\mathcal{B}}_{\mathcal{G}}^T(\boldsymbol{q})\boldsymbol{W}\boldsymbol{\mathcal{B}}_{\mathcal{G}}(\boldsymbol{q})$

$$\Rightarrow \quad \lambda_6 = \sum_{(i,j)\in\mathcal{E}} w_{ij}\, l_{ij}$$

Controller of the $\upsilon - th$ agent

- Let $\mathcal{N}_i = \{j \in \mathcal{V} | \, (i,\, j) \in \mathcal{E}\}$: neighbors of an agent i
- Let $\mathcal{O}_i = \{j \in \mathcal{V} | \, (j,\, i) \in \mathcal{E}\}$: set of agents j for which i is a neighbor

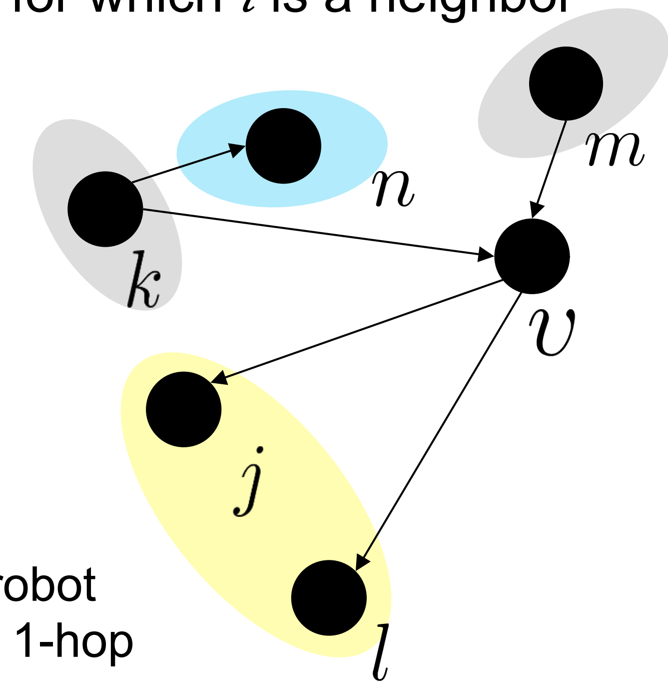
$$\begin{cases} \boldsymbol{u}_\upsilon = -\dfrac{\partial V_\lambda}{\partial \lambda_6} \left[\displaystyle\sum_{j \in \mathcal{N}_\upsilon} (l_{\upsilon j} \nabla_{\boldsymbol{p}_\upsilon} w_{\upsilon j} + 2\boldsymbol{L}_{\upsilon j} w_{\upsilon j}) + \right. \\ \left. \quad + \displaystyle\sum_{j \in \mathcal{O}_\upsilon} (-2^\upsilon \boldsymbol{R}_j 2 w_{j\upsilon} \boldsymbol{L}_{j\upsilon} + \displaystyle\sum_{m \in \mathcal{N}_j} {}^\upsilon\boldsymbol{R}_m l_{jm} \nabla_{\boldsymbol{p}_\upsilon} w_{jm}) \right] \\ w_\upsilon = -\dfrac{\partial V_\lambda}{\partial \lambda_6} \displaystyle\sum_{j \in \mathcal{N}_\upsilon} l_{\upsilon j} \nabla_{\psi_\upsilon} w_{\upsilon j} \end{cases}$$

1. $\boldsymbol{\beta}_{\upsilon j}, d_{\upsilon j}, \psi_{\upsilon j}, \boldsymbol{\nu}_{p_j}, \nu_{\psi_j}, \forall j \in \mathcal{N}_\upsilon$
2. $\boldsymbol{\beta}_{j\upsilon}, d_{j\upsilon}, \psi_{j\upsilon}, \boldsymbol{\nu}_{p_j}, \nu_{\psi_j}, \forall j \in \mathcal{O}_\upsilon$

Available to robot υ or through 1-hop communication

3. $\boldsymbol{\beta}_{jm}, d_{jm}, \psi_{jm}, \boldsymbol{\nu}_{p_m}, \nu_{\psi_m}, \forall j \in \mathcal{O}_\upsilon, \forall m \in \mathcal{N}_j / \{\upsilon\}$

2-hop communication



Final remarks

- The amount of information in the previous quantities is limited ⇒
  - Upper bound of communication complexity for agent υ :

$$O(|\mathcal{N}_\upsilon| + |\mathcal{O}_\upsilon| \cdot \max_{j \in \mathcal{O}_\upsilon} |\mathcal{N}_j|)$$

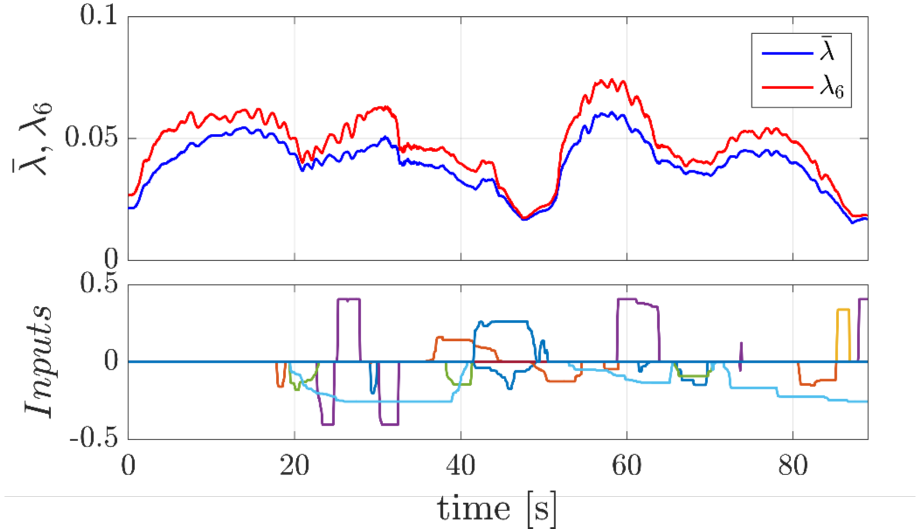
- λ_6 and $(\boldsymbol{\nu}_{p_\upsilon}, \nu_{\psi_\upsilon})$ could be estimated readapting [Robuffo Giordano et al., IJRR 2013]-[Yang et al., Automatica 2010]
- Derivative of an eigenvalue is not well-defined for repeated eigenvalues:
  - Possible *workaround*: $\lambda_6 \rightarrow \bar{\lambda} = \sum_{i=6}^{4N} \sqrt[p]{\lambda_i^p}$

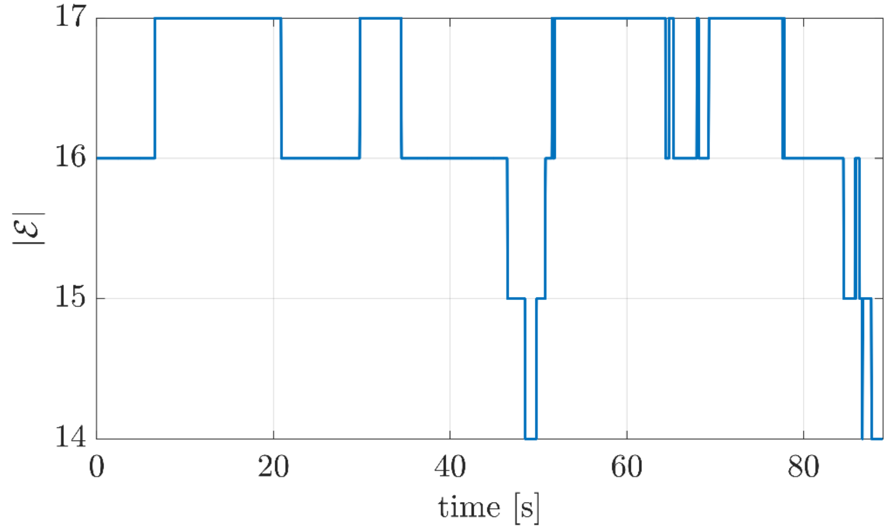
Video

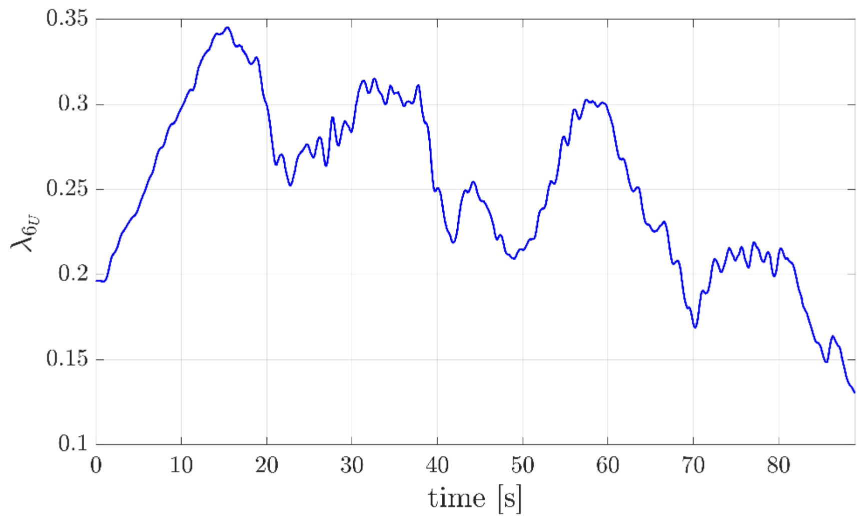
The video is submitted to the next ICRA 2017 conference and therefore is not yet shareable.

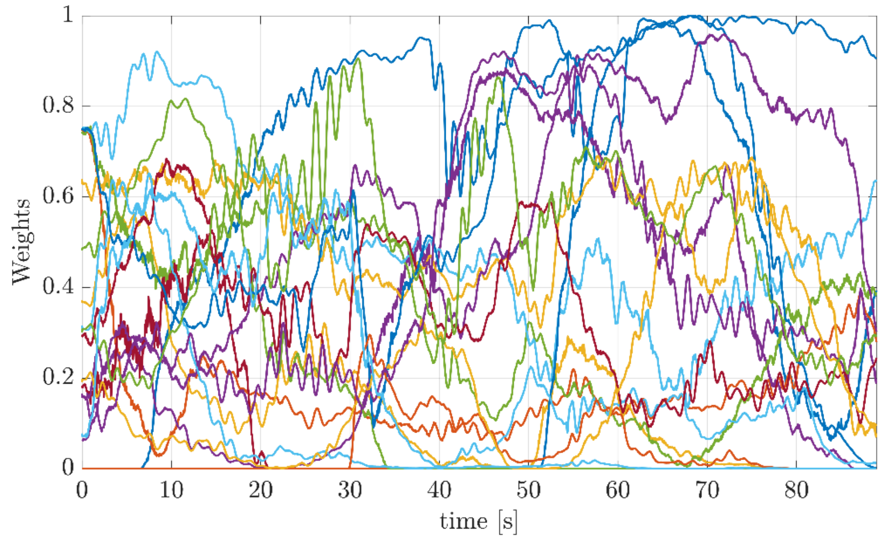
Sorry for the inconvenience.

Experimental Results









Conclusions

- Decentralized gradient-based controller which maintains bearing rigidity of a formation of UAVs
- Several sensing constraints were considered
- Closed-form expression of $\lambda_6(\boldsymbol{q})$ (and of its gradient)
- Experiments with 5 UAVs

Future Works

- Fully-onboard implementation through UAV cameras
- Estimation of $\lambda_6(\boldsymbol{q}),\ (\boldsymbol{\nu}_{p_v},\ \nu_{\psi_v})$
- Collision avoidance
- *'Anonymousity'* of the measurements
- Collaboration with Boston University (Prof. R. Tron)

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