Invariant Kalman filtering for multi-sensor navigation

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GT UAV
Paris
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Introduction

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Reference paper for attitude estimation: Nonlinear Complementary Filters on the Special Orthogonal Group R. Mahony, Tarek Hamel, Jean-Michel Pflimlin. 2008.

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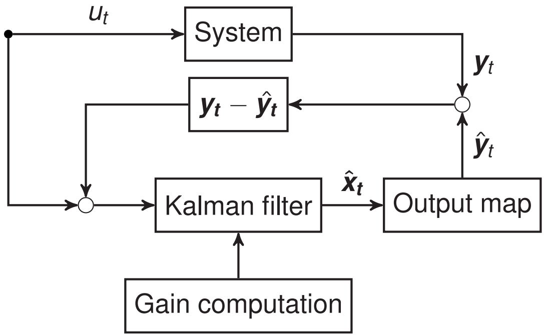
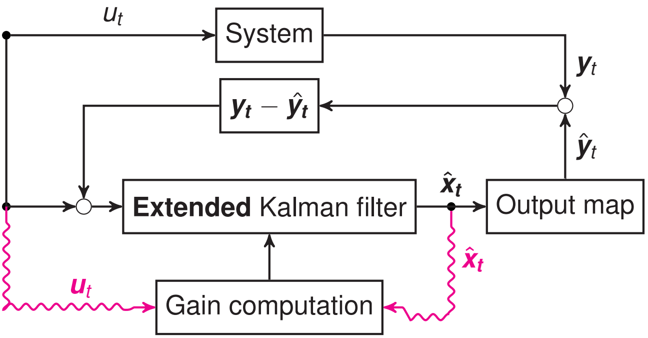
(Linear) Kalman filter



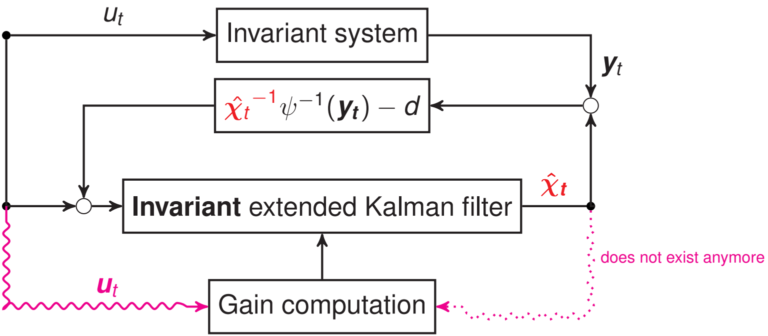
Remark The gain computation is independent from the rest.

Extended Kalman filter



Remark We can see the inner loop between gain and estimate.

Invariant EKF



Outline

1) Some recent industrial results

2) A simple theoretical case study to explain the results

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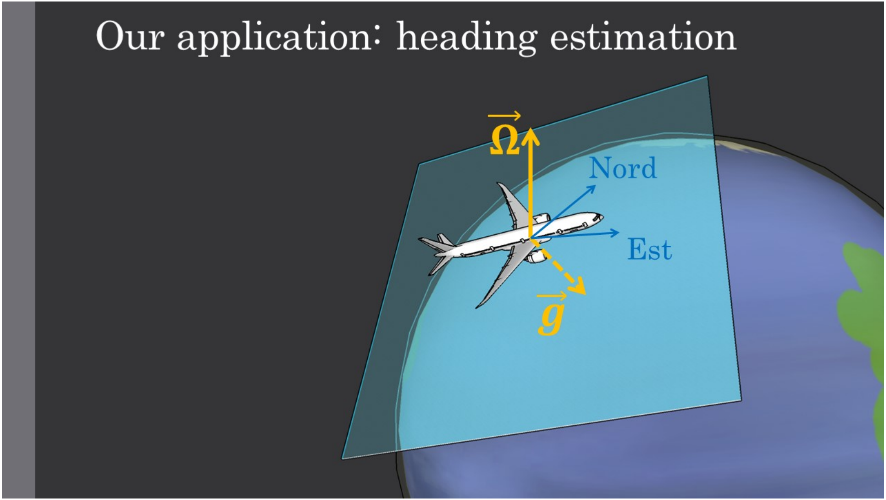
Remark: NO equations !

Industrial application



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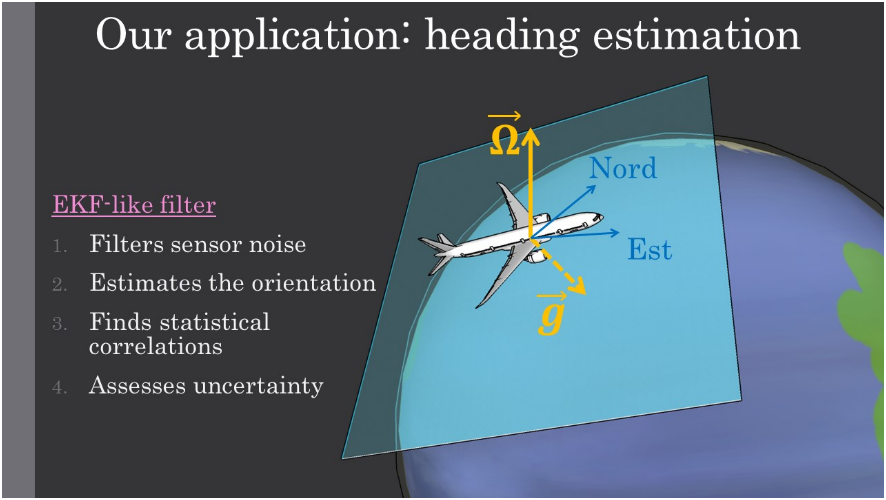

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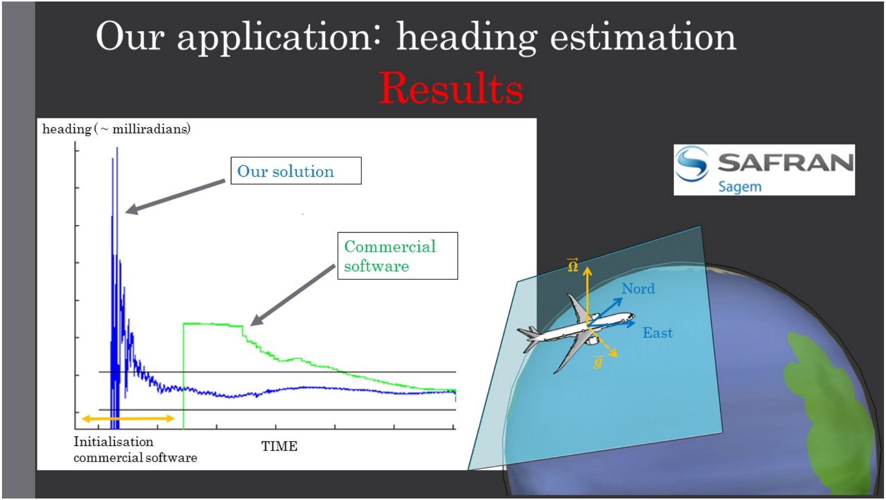

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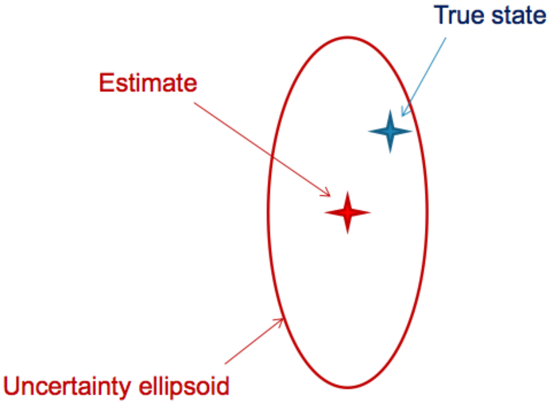
Preliminary

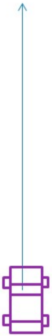
Graphical representation: The EKF computes an estimate $\hat{X}_t$ with a covariance matrix P_t. This is the confidence ellipsoid.



A simple terrestrial navigation problem

Introduction : Extended Kalman Filter

- Accurate odometer
- Initial position known
- Heading unknown



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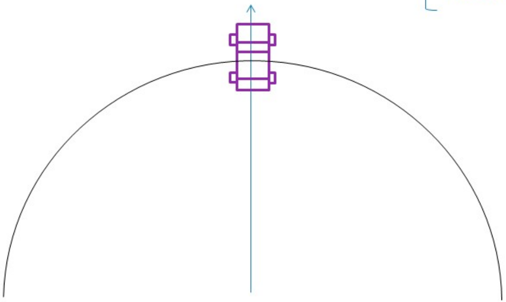

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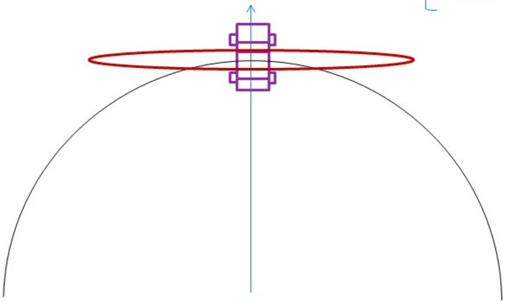

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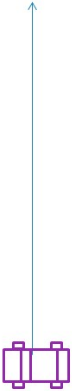
Non-linear system : estimate



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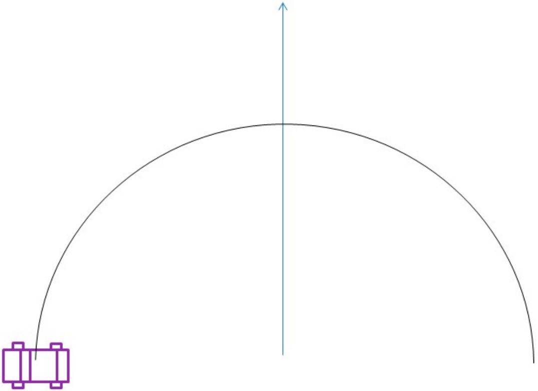

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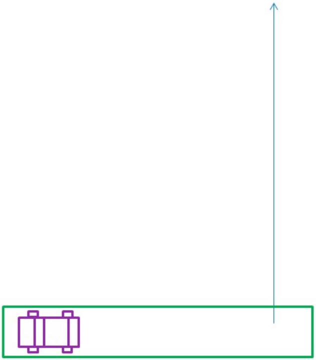
This is what the EKF **should** do



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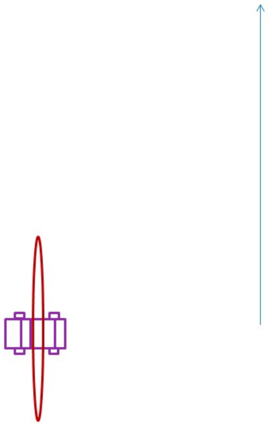

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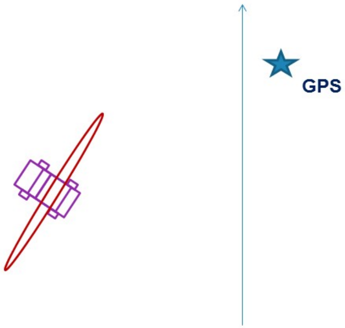


Reminder: EKF equations

Consider continuous time deterministic dynamics with discrete time measurements $t_0, t_1, \cdots$

$$\frac{d}{dt}X_t = f(X_t, U_t), \quad Y_n = h(X_t) + V_n$$

The EKF is based on a prior distribution with $\hat{X}_0,\ P_0$.

Propagation step: for $t_{n-1} < t < t_n$

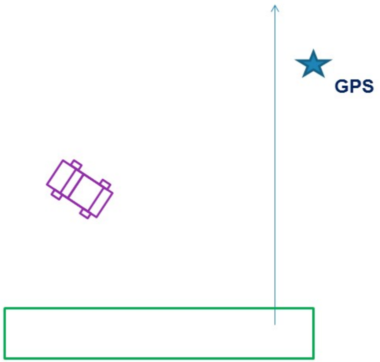
$$\frac{d}{dt}\hat{X}_t = f(\hat{X}_t, U_t) \quad \frac{d}{dt}P_t = A_t P_t + P_t A_t^T$$

Update step: at $t = t_n$,

$$K_n = P_{t_n} H_n^T \left(H_n P_{t_n} H_n^T + N \right)^{-1}$$
$$\hat{X}_{t_n}^+ = \hat{X}_{t_n} + K_n(Y_n - \hat{X}_{t_n})$$
$$P_{t_n}^+ = (I - K_n H_n)\, P_{t_n}.$$

A simple terrestrial navigation problem

Unfortunately the EKF is based on **linearizations**



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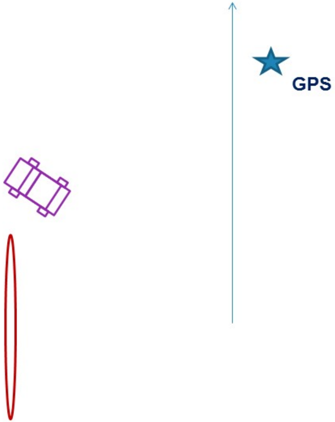

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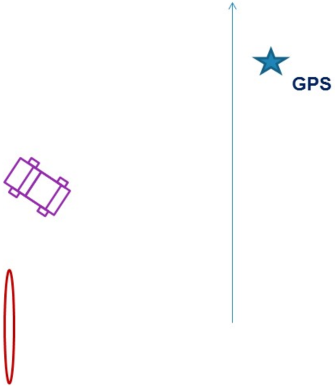

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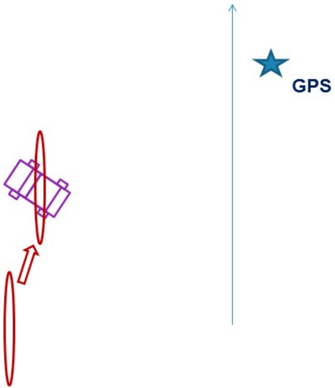
And the EKF **does this**



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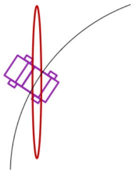

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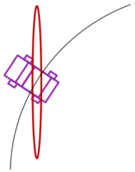

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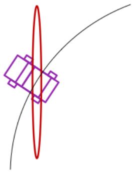

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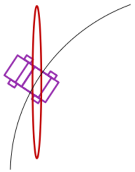

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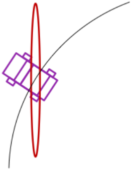

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Namely: we use an Invariant (I)-EKF [3].

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Main difference the estimation error it linearizes is not

$$\begin{pmatrix} \hat{\theta} - \theta \\ \hat{x}_t - x_t \end{pmatrix}, \quad \text{where} \quad \theta \in S^1,\ x \in \mathbb{R}^2$$

Invariant Kalman filtering

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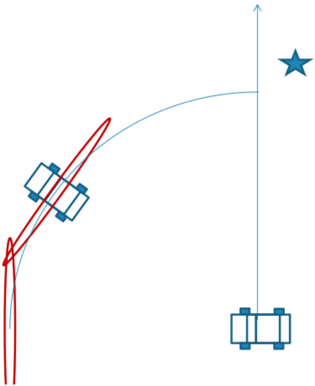
It is the estimation error

$$\begin{pmatrix} \hat{\theta} - \theta \\ R(\theta)^T (\hat{x}_t - x_t) \end{pmatrix}$$

where $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

Invariant Kalman filtering



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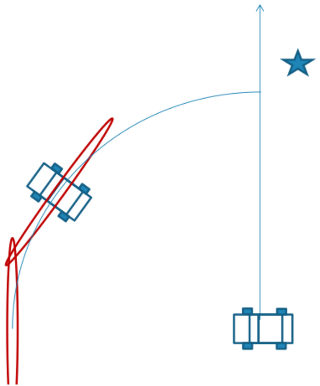

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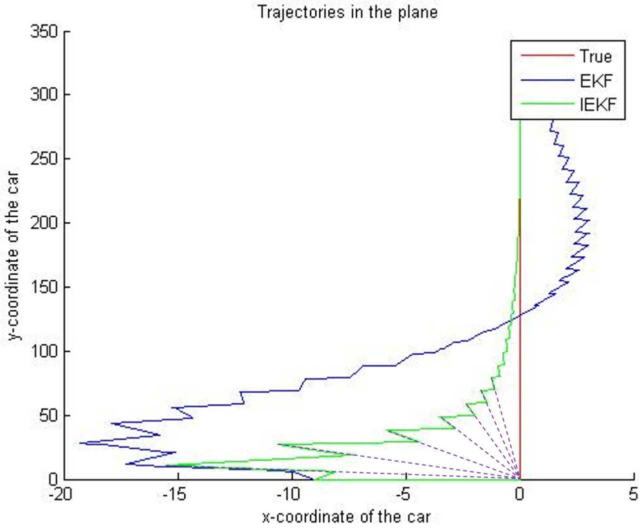
Theorem: The estimate belongs only to sets “physically" reachable.

Invariant Kalman filtering

Illustration with simulations for a straight line motion.



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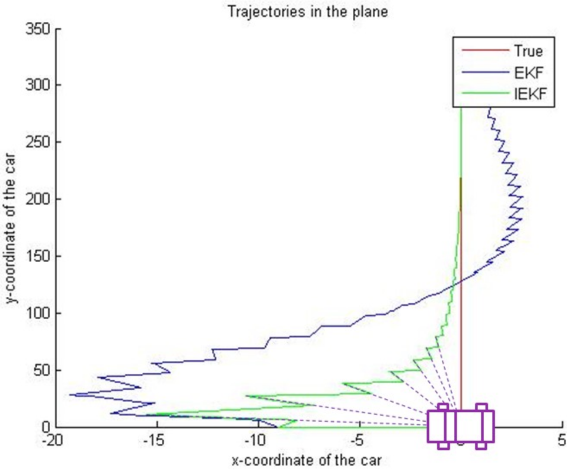

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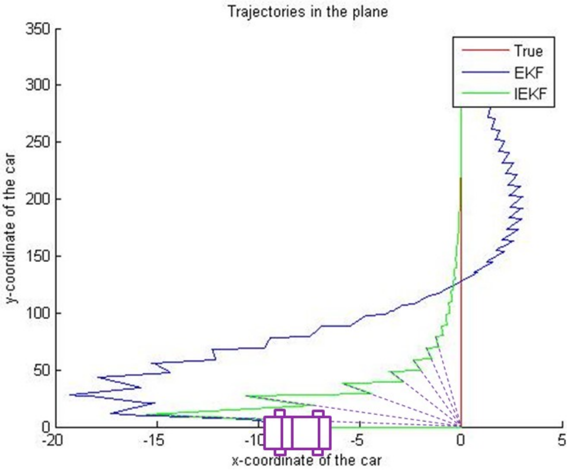

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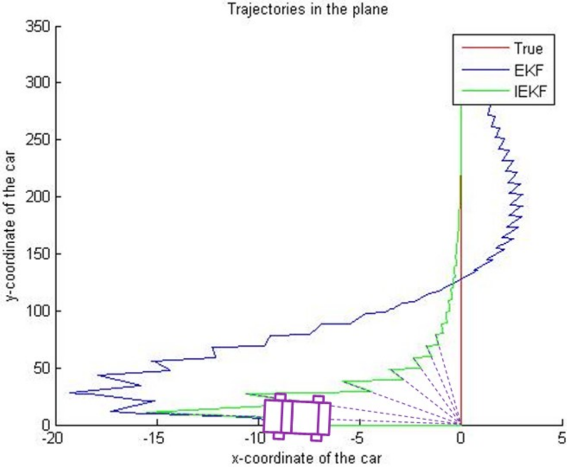

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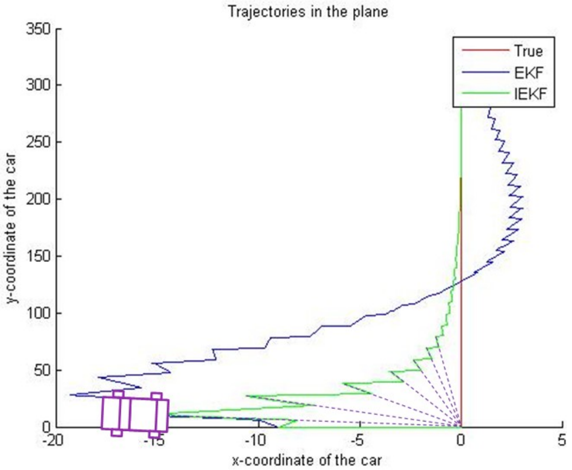

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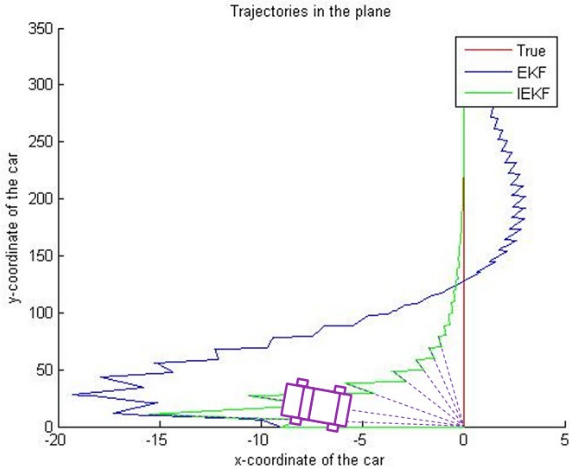

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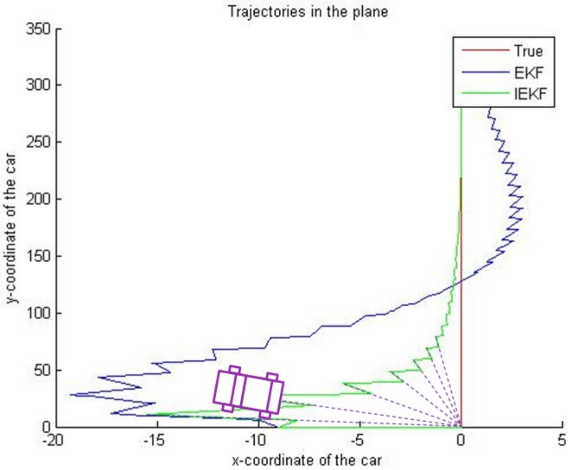

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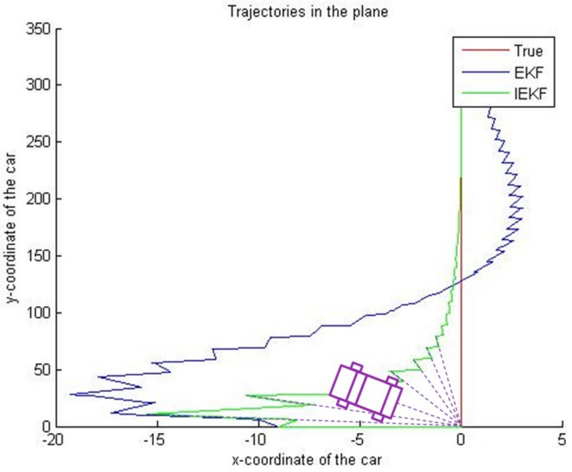

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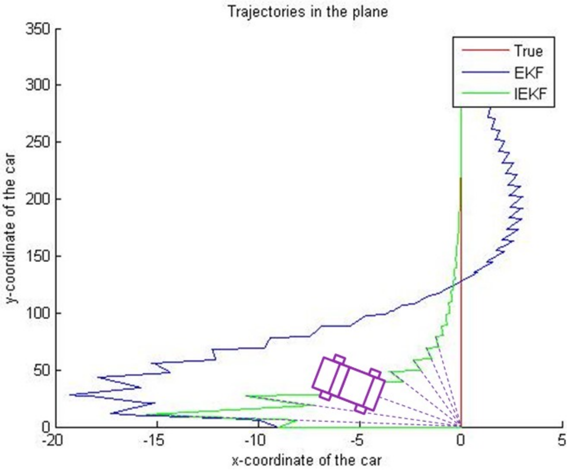

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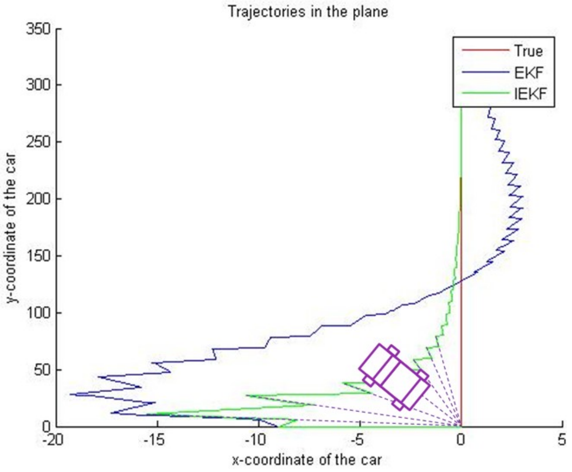

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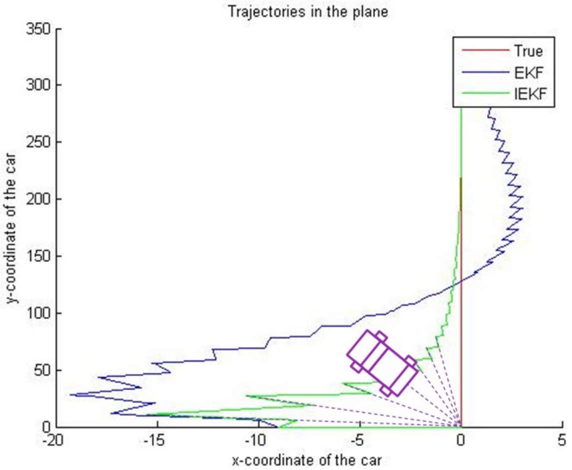

Invariant Kalman filtering

Illustration with simulations for a straight line motion.

What about the **EKF** ??

Invariant Kalman filtering

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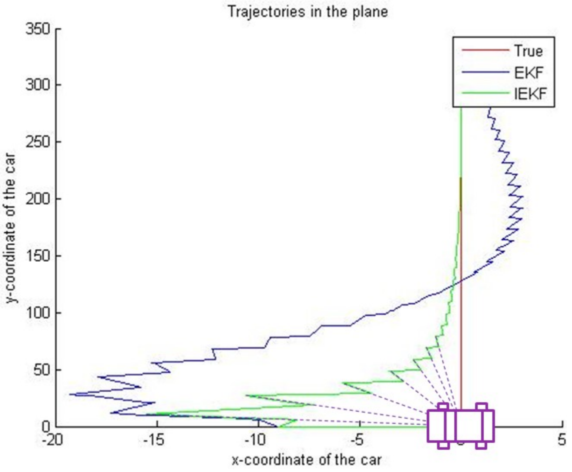

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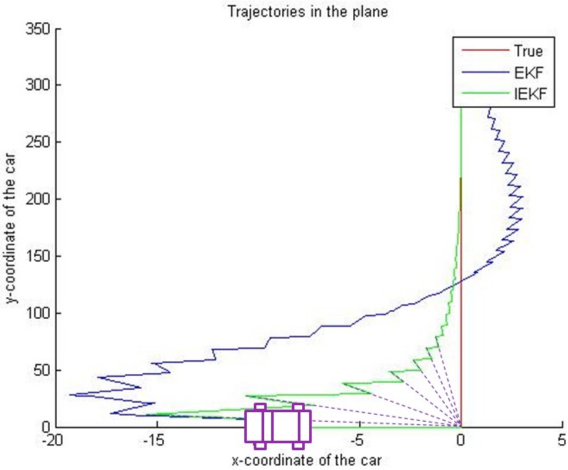

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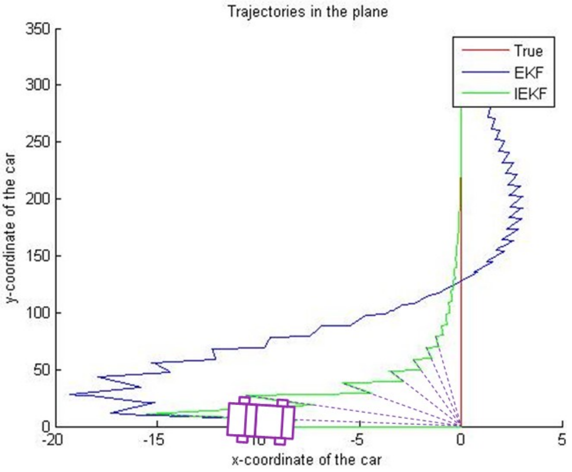

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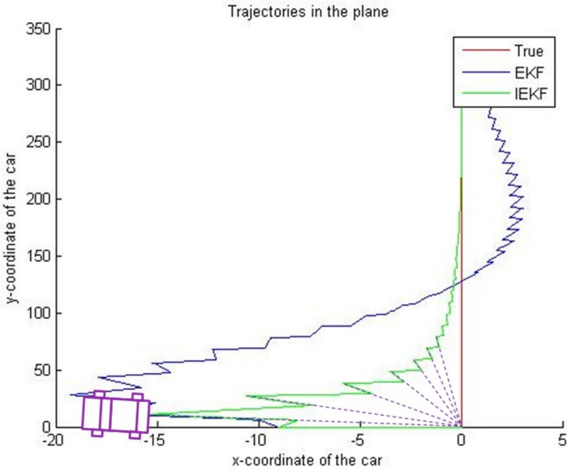

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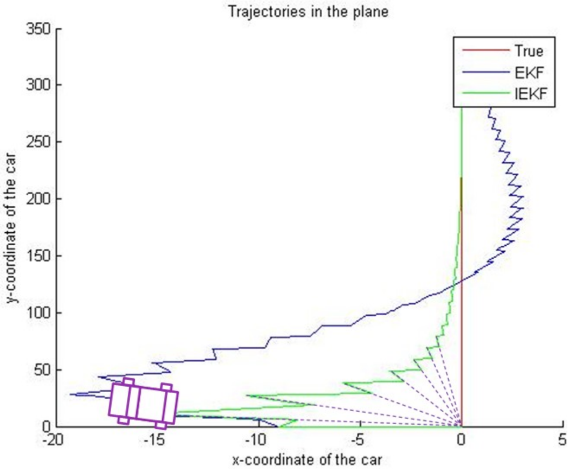

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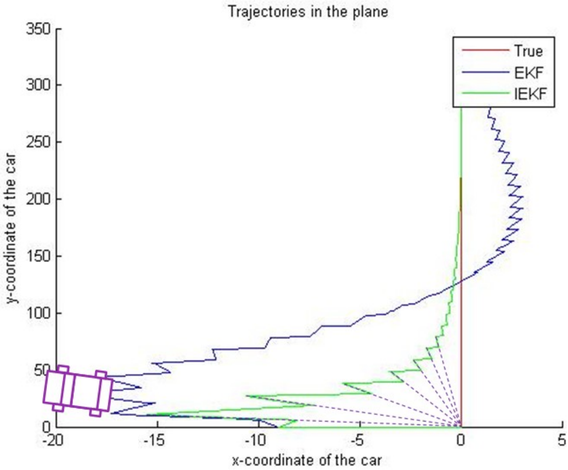

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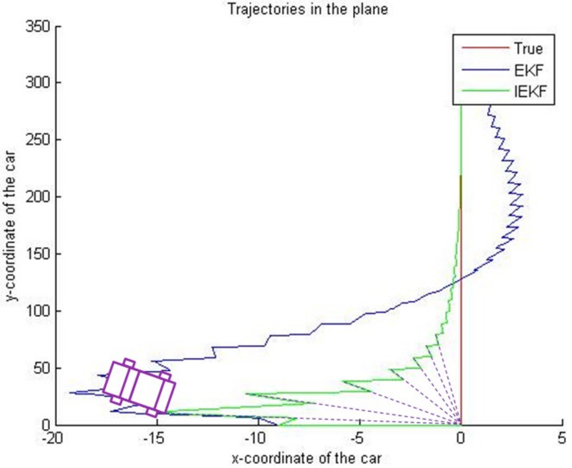

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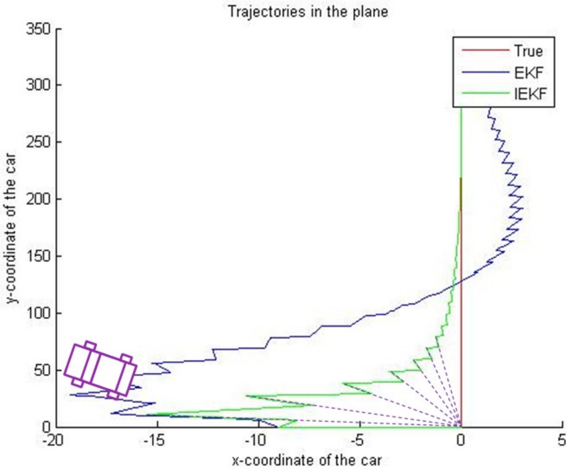

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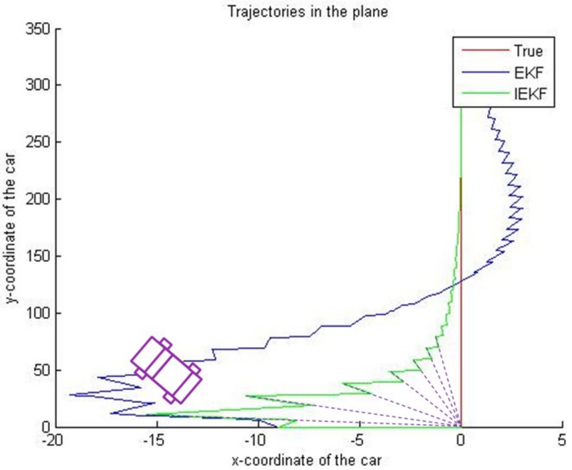

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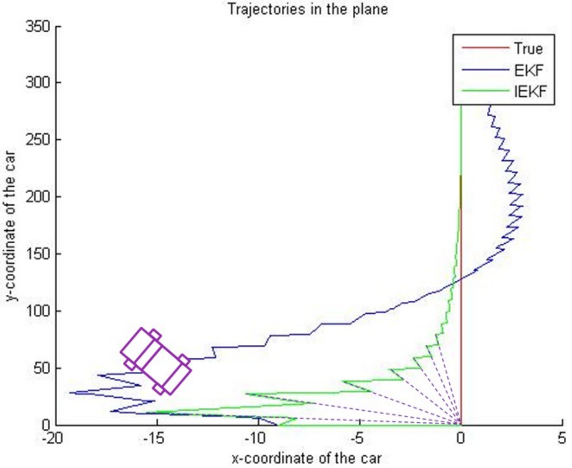

Invariant Kalman filtering

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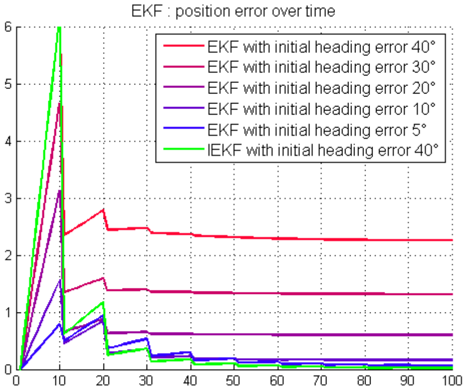

Figure: Position error of the EKF for GPS and perfect odometry, with initial position known and initial heading unknown. The IEKF initialized with a large error (40°) even beats the EKF initialized with a small error (5°).

Invariant Kalman filtering

Convergence: Does the EKF truly NOT converge ??

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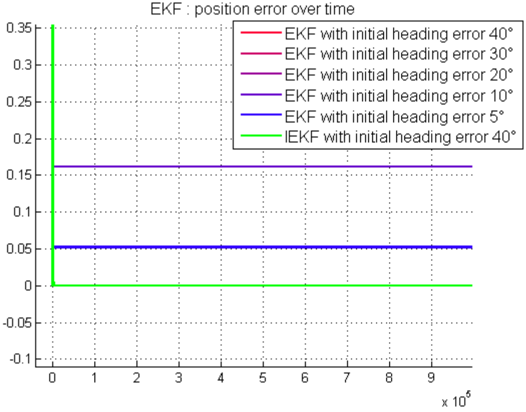

Figure: Extension to 1 million steps. The EKF does not converge.

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Convergence: Does the IEKF converge ??

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YES: for this simple problem we have proved a theorem.

Conclusion

Simple navigation problem where the EKF does not converge (!)

[4] the problem we considered serves as a simplified in flight alignment problem, an inevitable task for industrial high precision inertial navigation. See A. Barrau and S. Bonnabel. Alignment method for an inertial unit, 2014. SAGEM/ARMINES. EP 2014/075439, WO/2015/075248.

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Simple navigation problem where the EKF does not converge (!) whereas the IEKF provably globally converges.

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