Active Decentralized Scale Estimation for Bearing-Based Localization

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Lagadic group

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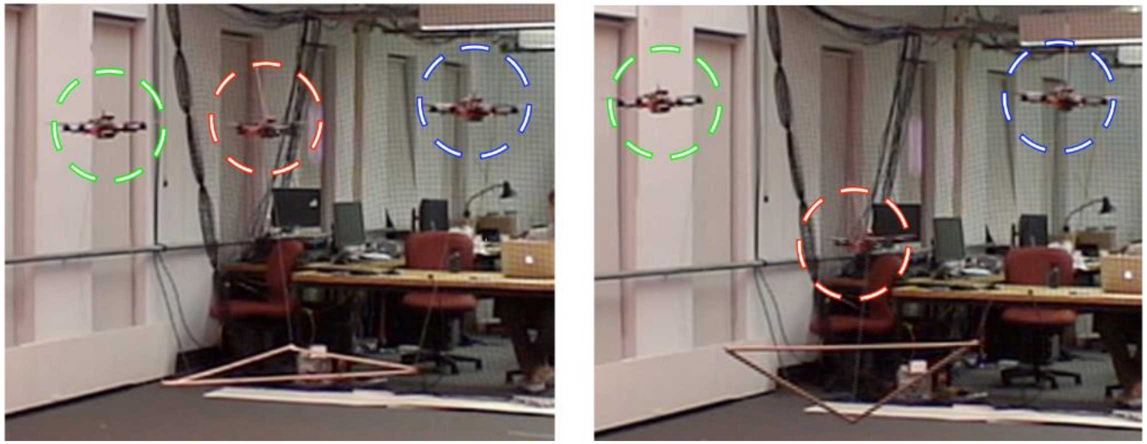
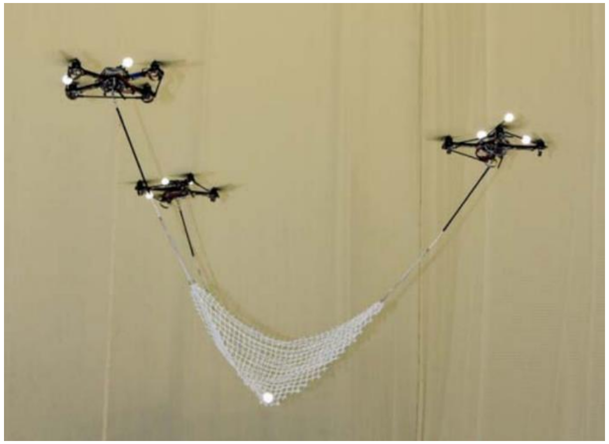
http://www.irisa.fr/lagadic

Introduction

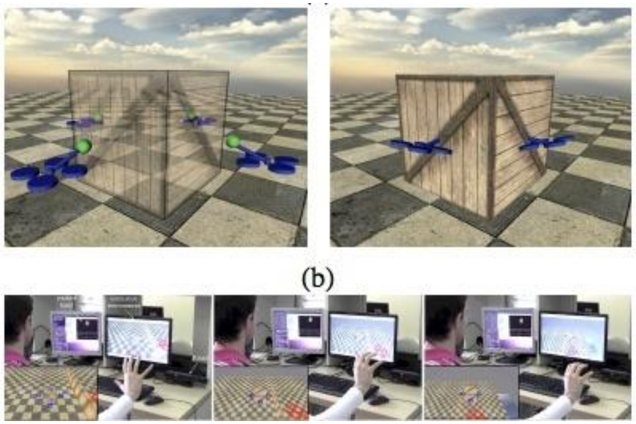


Michael et al. 2011



Ritz et al. 2012

Mellinger et al. 2013



Gioioso et al. 2014

Hausman et al. 2016

Setup

- N UAVs: $\boldsymbol{q} = (\boldsymbol{p}, \boldsymbol{\psi}) \in (\mathbb{R}^3 \times \mathcal{S}^1)^N$

$$\begin{bmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_i(\psi_i) & 0 \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_i \\ w_i \end{bmatrix}$$

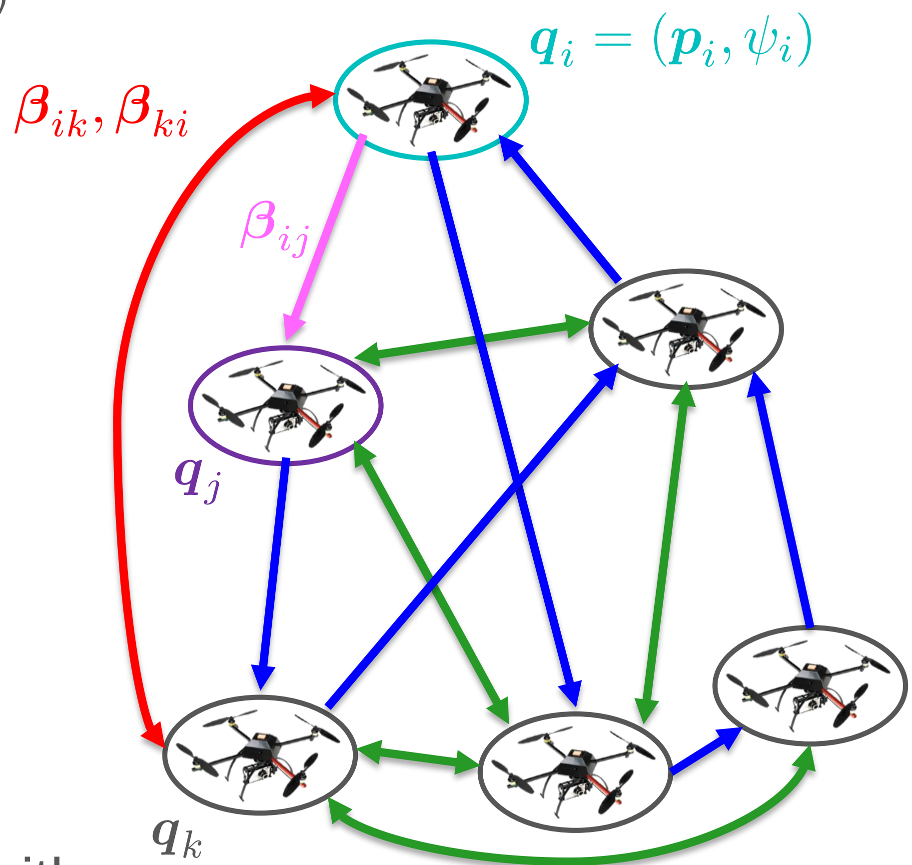
- camera measurements (if $(i, j) \in \mathcal{E}$)

$$\boldsymbol{\beta}_{ij} = \boldsymbol{R}_i^T \frac{\boldsymbol{p}_j - \boldsymbol{p}_i}{\|\boldsymbol{p}_j - \boldsymbol{p}_i\|} = \boldsymbol{R}_i^T \frac{\boldsymbol{p}_{ij}}{d_{ij}} \in \mathbb{S}^2$$

- directed measurement graph

$$(i, j) \in \mathcal{E} \not\Rightarrow (j, i) \in \mathcal{E}$$

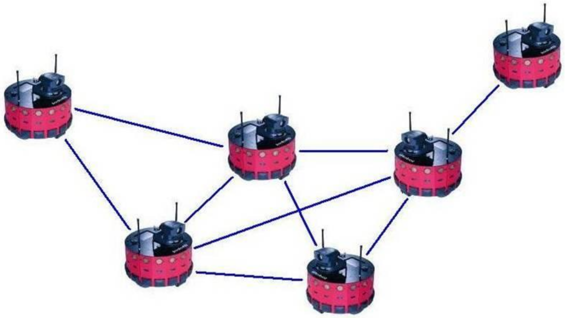
- no shared reference frame
- goal: decentralized localization with correct scale as fast as possible



Control and localization using local measurements

- formation control

$$\boldsymbol{\beta}_{ij}(\boldsymbol{q}) = \boldsymbol{\beta}_{ij}(\boldsymbol{q}^*), \forall (i,j) \in \mathcal{E} \iff \boldsymbol{q} \cong \boldsymbol{q}^*$$

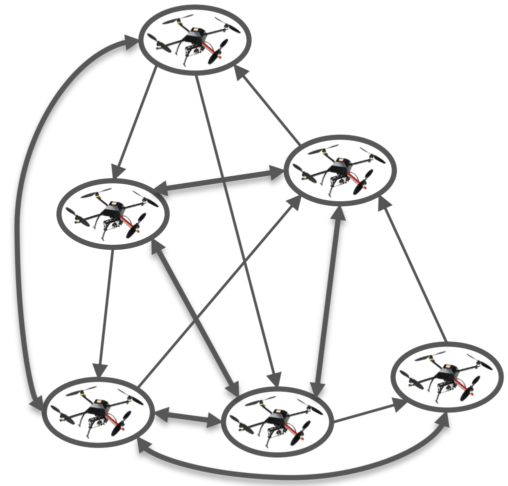


- relative localization (in a common frame)

$$\boldsymbol{\beta}_{ij}(\widehat{\boldsymbol{q}}) = \boldsymbol{\beta}_{ij}(\boldsymbol{q}), \forall (i,j) \in \mathcal{E} \iff \widehat{\boldsymbol{q}} \cong \boldsymbol{q}$$

- [Anderson et al. 2008; Tron et al. 2015; Montijano et al. 2014; Zelazo et al. 2014/2015]

Bearing rigidity



- framework (~formation) =
  - (measurement) graph $\mathcal{G}$ + configuration $\boldsymbol{q}$
- bearing function associated to

$$\boldsymbol{\beta}_{\mathcal{G}} : (\mathbb{R}^3 \times \mathcal{S}^1)^N \mapsto (\mathbb{S}^2)^{|\mathcal{E}|}, \quad \boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}) = (\boldsymbol{\beta} e_1, \ldots, \boldsymbol{\beta} e_{|\mathcal{E}|})$$

$$\dot{\boldsymbol{\beta}}_{\mathcal{G}} = \frac{\partial \boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q})}{\partial \boldsymbol{q}} \begin{bmatrix} \mathrm{diag}\,(\boldsymbol{R}_i) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_N \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{bmatrix} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{A}}(\boldsymbol{q}) \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{w} \end{bmatrix}$$

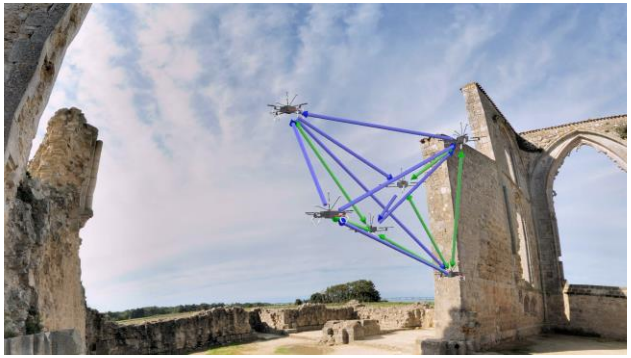
bearing rigidity matrix

- let $\mathcal{K}_N$ be the complete graph, $(\mathcal{G}, \boldsymbol{q})$ is said to be infinitesimally bearing rigid (IBR) if [Anderson et al. 2008]

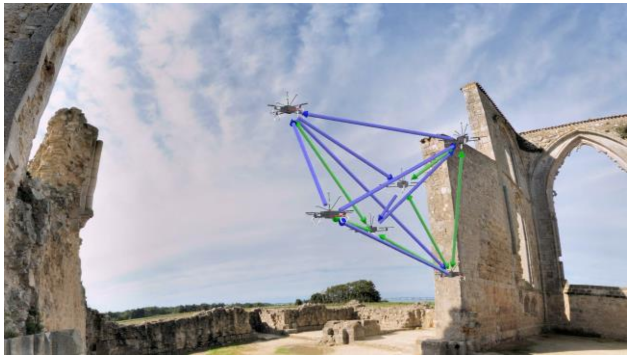
$\mathcal{N}(\boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{A}}(\boldsymbol{q})) = \mathcal{N}(\boldsymbol{\mathcal{B}}_{\mathcal{K}_N}^{\mathcal{A}}(\boldsymbol{q}))$ ➡ no need of a complete graph for control/estimation ($O(N)$ vs $O(N^2)$)

Null space transformations for IBR formations

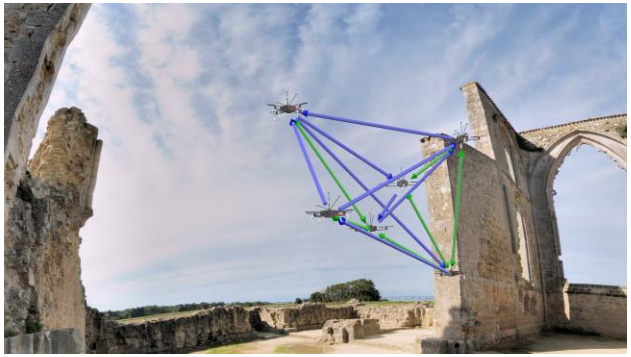
- translation



- rotation around Z



- scaling
  - one (at least) distance measurement can fix the scale [Zelazo et al. 2014]



Bearing-based localization

- $\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}) = const$ ⟹ if $\mathcal{G}$ is IBR, using $\dot{\widehat{\boldsymbol{q}}} = k_L \boldsymbol{\mathcal{B}}_{\mathcal{G}}(\widehat{\boldsymbol{q}})^T \boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q})$

$$\widehat{\boldsymbol{q}} \rightarrow \begin{cases} \widehat{\boldsymbol{p}} = s\left(\boldsymbol{I}_N \otimes \boldsymbol{R}(\bar{\psi})\right)\boldsymbol{p} + \mathbf{1}_N \otimes \boldsymbol{t} \\ \widehat{\psi} = \psi + \mathbf{1}_N \bar{\psi} \end{cases}$$

[Zelazo et al. 2014]

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[Zelazo et al. 2014]

- $\boldsymbol{\beta}_{\mathcal{G}}(\boldsymbol{q}) \neq const$ ⟹ need feedforward $\dot{\boldsymbol{\beta}}_{\mathcal{G}} = \boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{A}}(\boldsymbol{q}) \left[\begin{smallmatrix}\boldsymbol{u}\\ \boldsymbol{w}\end{smallmatrix}\right]$
  - row block associated to edge (i, j)

$$\begin{bmatrix} \boldsymbol{0} & -\frac{\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T}{d_{ij}} & \boldsymbol{0} & \frac{\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T}{d_{ij}} {}^{i}\boldsymbol{R}_j(\psi_j - \psi_i) & \boldsymbol{0} & -\boldsymbol{S}\boldsymbol{\beta}_{ij} & \boldsymbol{0} \end{bmatrix}$$

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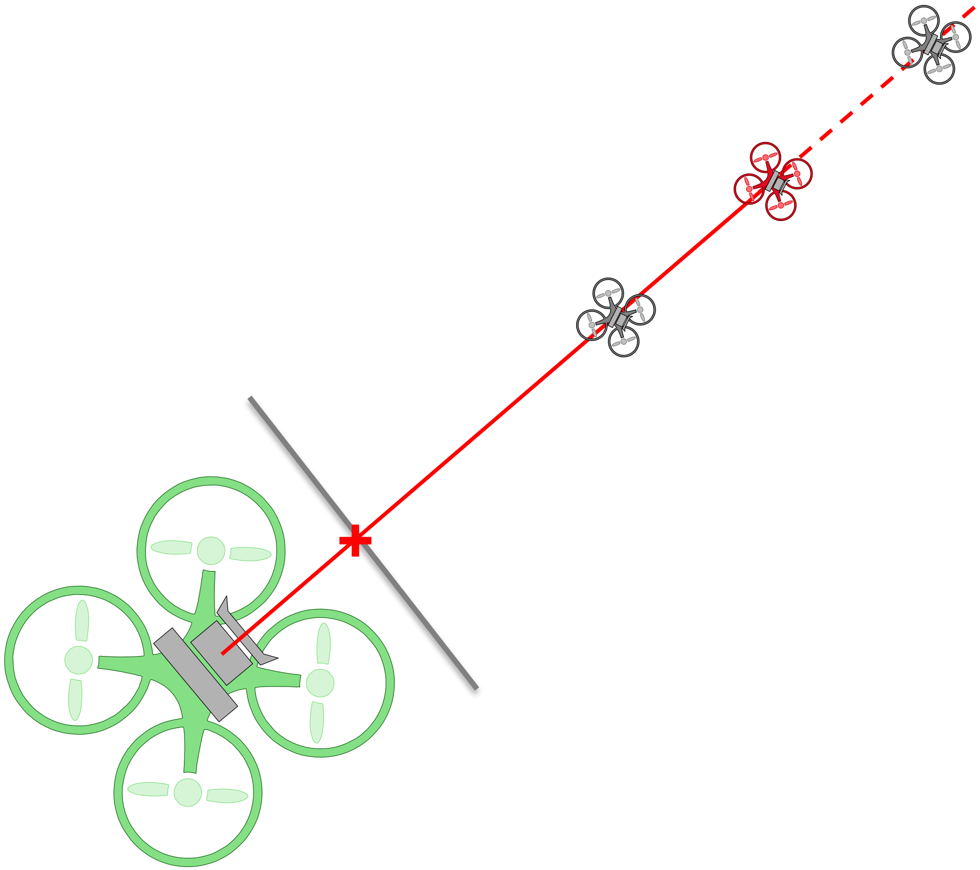
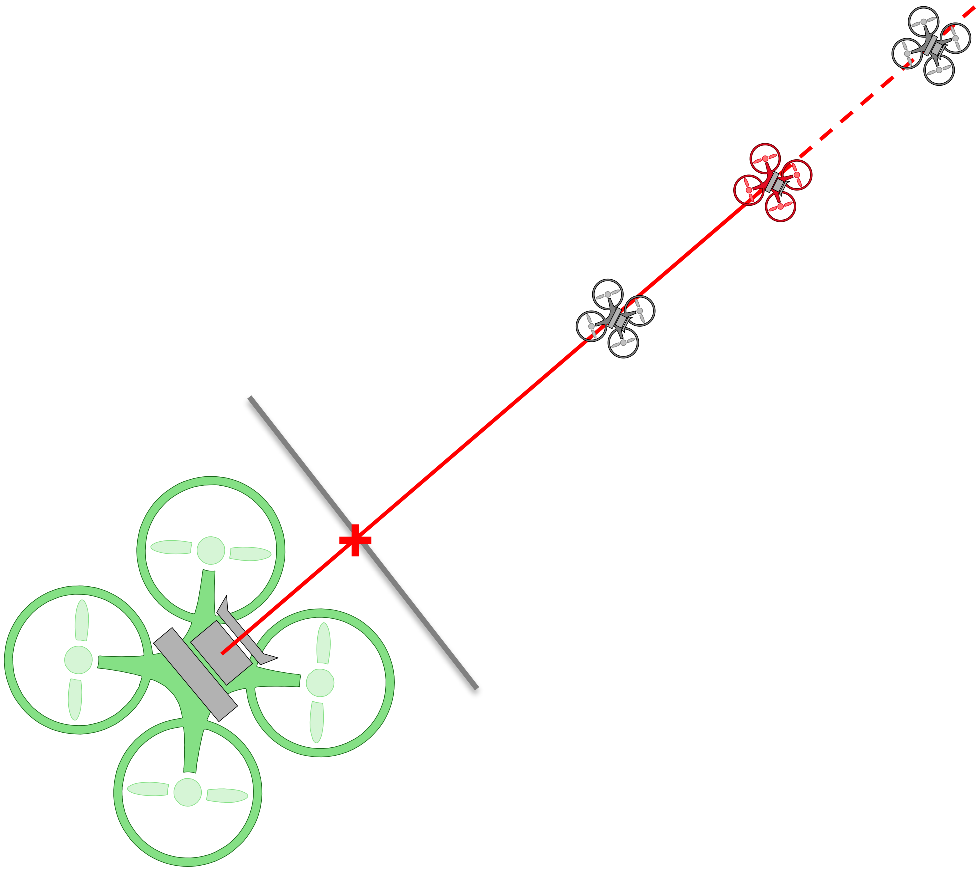
- a metric information is needed to recover the scale
  - one (at least) “special agent” measures its distance from a neighbor [Schiano et al. IROS 2016]
  - use the (already known) $\boldsymbol{u}$ to estimate relative distances

Vision based reconstruction

- an inverse problem



Structure from known motion



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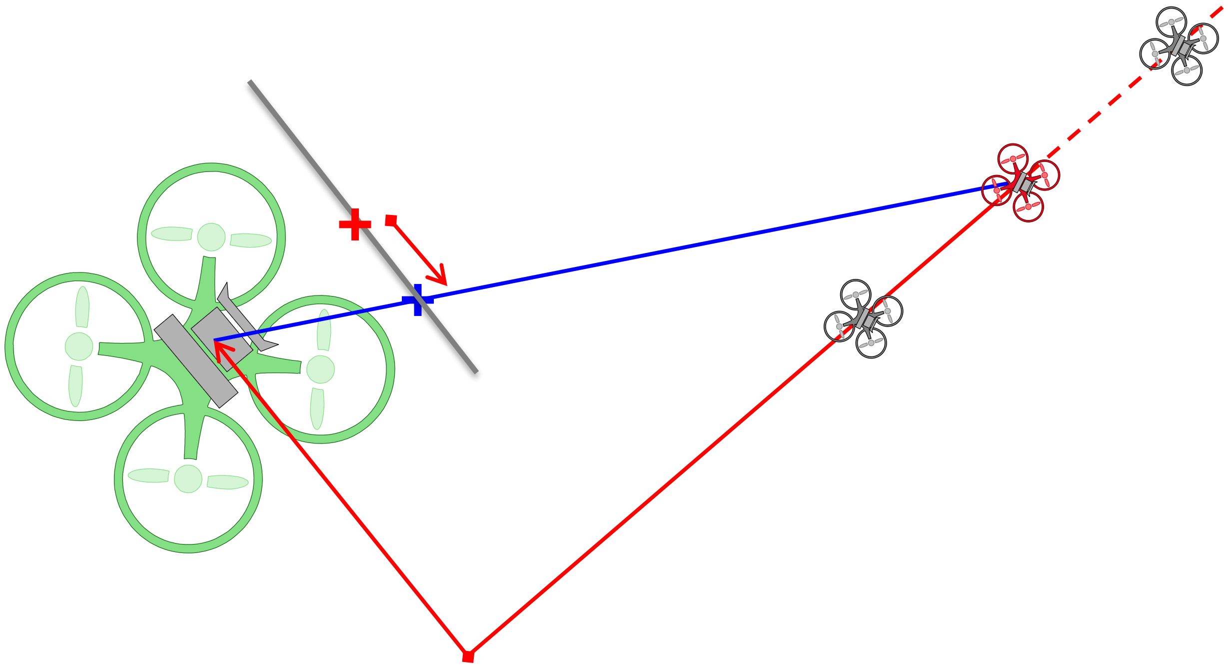
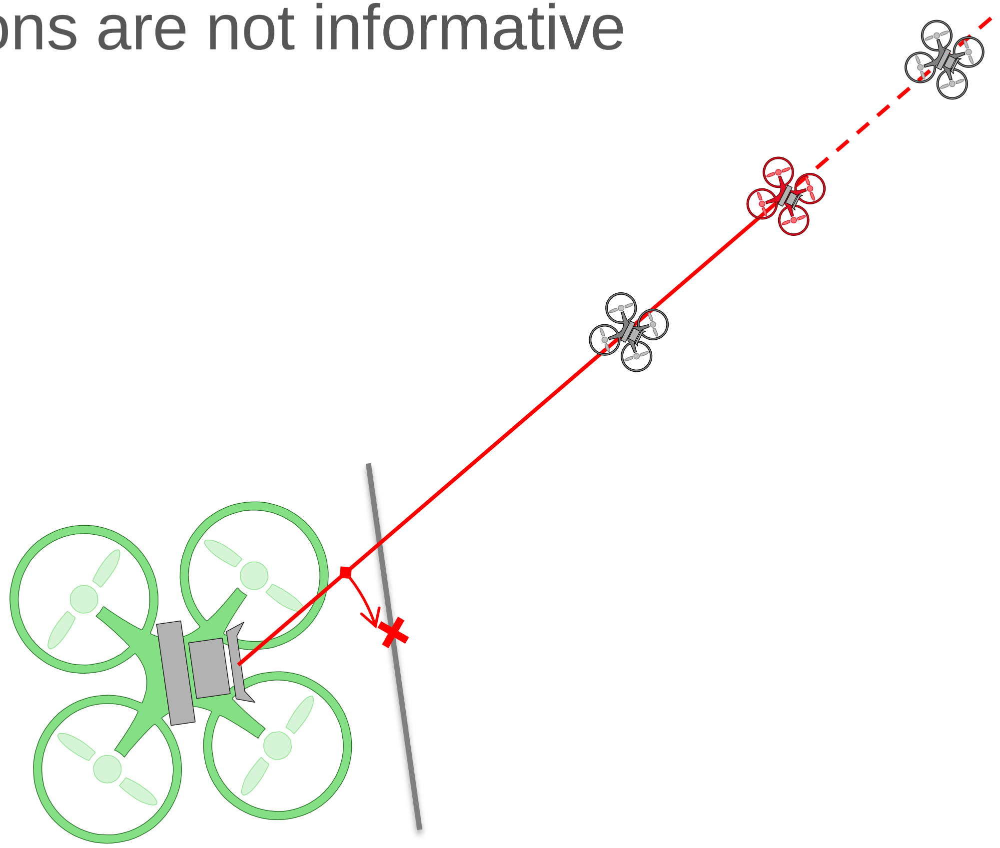


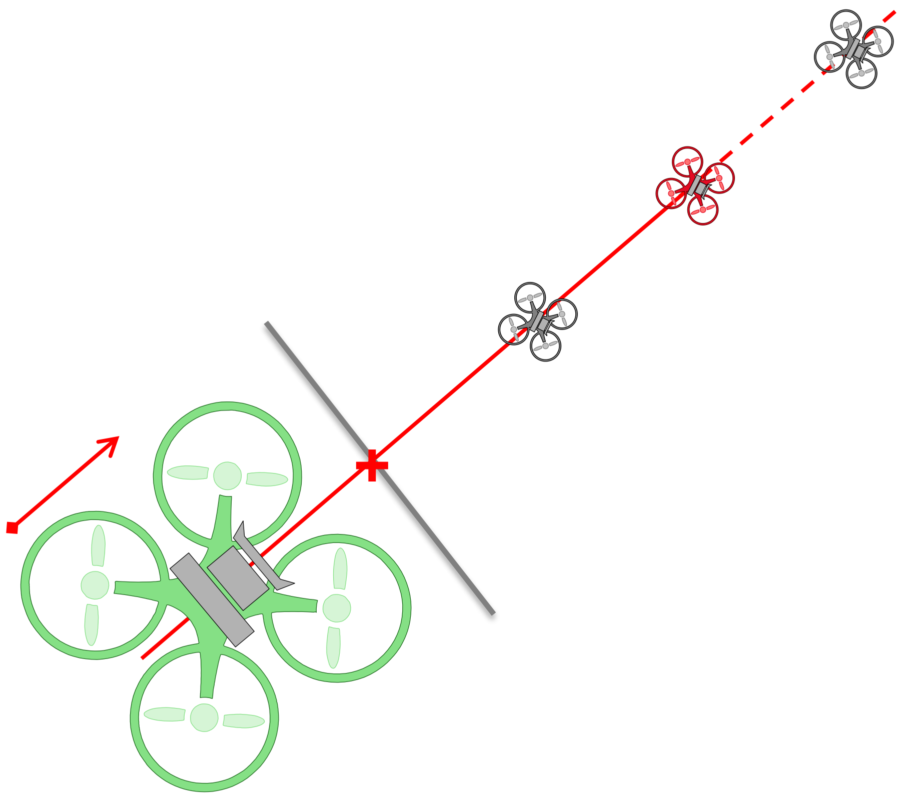
Effects of the camera/agent motion 1/2

- pure rotations are not informative



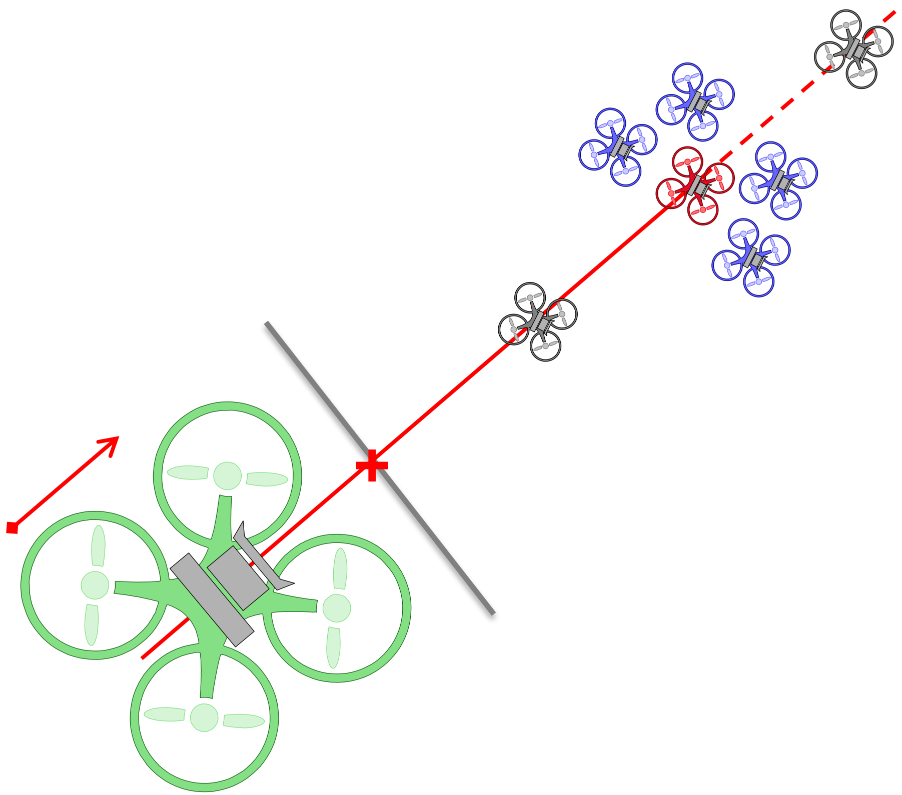
Effects of the camera/agent motion 2/2

- translations along the projection ray are not informative



Effects of the camera/agent motion 2/2

- translations along the projection ray are not informative
- or poorly informative



Distance estimation

- semi-globally exponentially stable observer for $\chi_{ij} = 1/d_{ij}$

$$\begin{cases} \dot{\widehat{\boldsymbol{\beta}}}_{ij} = -\boldsymbol{S}\boldsymbol{\beta}_{ij}w_i + (\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)\boldsymbol{u}_{ij}\widehat{\chi}_{ij} - \boldsymbol{H}(\widehat{\boldsymbol{\beta}}_{ij} - \boldsymbol{\beta}_{ij}) \\ \dot{\widehat{\chi}}_{ij} = -\widehat{\chi}_{ij}^2\boldsymbol{u}_{ij}^T\boldsymbol{\beta}_{ij} - \alpha\boldsymbol{u}_{ij}^T(\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)(\widehat{\boldsymbol{\beta}}_{ij} - \boldsymbol{\beta}_{ij}) \end{cases}$$

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- observer convergence rate depends of

$$\lambda_{ij} = \boldsymbol{u}_{ij}^T (\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)\boldsymbol{u}_{ij}$$

R. Spica, P. Robuffo Giordano, F. Chaumette, "Active Structure from Motion: Application to Point, Sphere and Cylinder." IEEE Trans. on Robotics, vol. 30, no. 6, pp. 5434-5440, 2014.

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- convergence of $\widehat{\chi}_{ij}$ to χ_{ij} can be detected by measuring

$$E_{ij}(t) = \frac{1}{T}\int_{t-T}^{t} \left\|\widehat{\boldsymbol{\beta}}_{ij}(\tau) - \boldsymbol{\beta}_{ij}(\tau)\right\|^2 \mathrm{d}\tau = 0 \Leftrightarrow \widehat{\chi}_{ij} - \chi_{ij} = 0$$

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where $\boldsymbol{u}_{ij} = \boxed{{}^i\boldsymbol{R}_j(\psi_j - \psi_i)} \boldsymbol{u}_j - \boldsymbol{u}_i$ not known in general!

- observer convergence rate depends of

$$\lambda_{ij} = \boldsymbol{u}_{ij}^T (\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)\boldsymbol{u}_{ij} \Rightarrow \boxed{\boldsymbol{u}_{ij} = \arg\max\{\lambda_{ij}(\boldsymbol{u}_{ij})\}}$$

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Relative orientation

- no common frame of reference

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- however for reciprocal links $(i,j) \in \mathcal{E},\ (j,i) \in \mathcal{E}$
  - $\boldsymbol{\beta}_{ij} = -{}^i\boldsymbol{R}_j(\psi_{ij})\boldsymbol{\beta}_{ji} \Rightarrow \psi_{ij} = \mathrm{atan2}(\|\boldsymbol{\beta}_{ji} \times \boldsymbol{\beta}_{ij}\|, -\boldsymbol{\beta}_{ij}^T\boldsymbol{\beta}_{ji})$
  - $\boldsymbol{\beta}_{ij}, \boldsymbol{\beta}_{ji}$ can be exchanged over local communication

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  - $\boldsymbol{\beta}_{ij}, \boldsymbol{\beta}_{ji}$ can be exchanged over local communication

- $\boldsymbol{u}_{ij}$ is known for reciprocal links and d_{ij} can be estimated for all these links → need at least one reciprocal link

  (i.e. a pair of agents in mutual visibility)

ψ_{ij} → distance estimation → $\widehat{d}_{ij}$ → bearing-based localization → $\widehat{\boldsymbol{q}}$

Unconstrained active estimation

- the estimation of the formation scale is the primary task

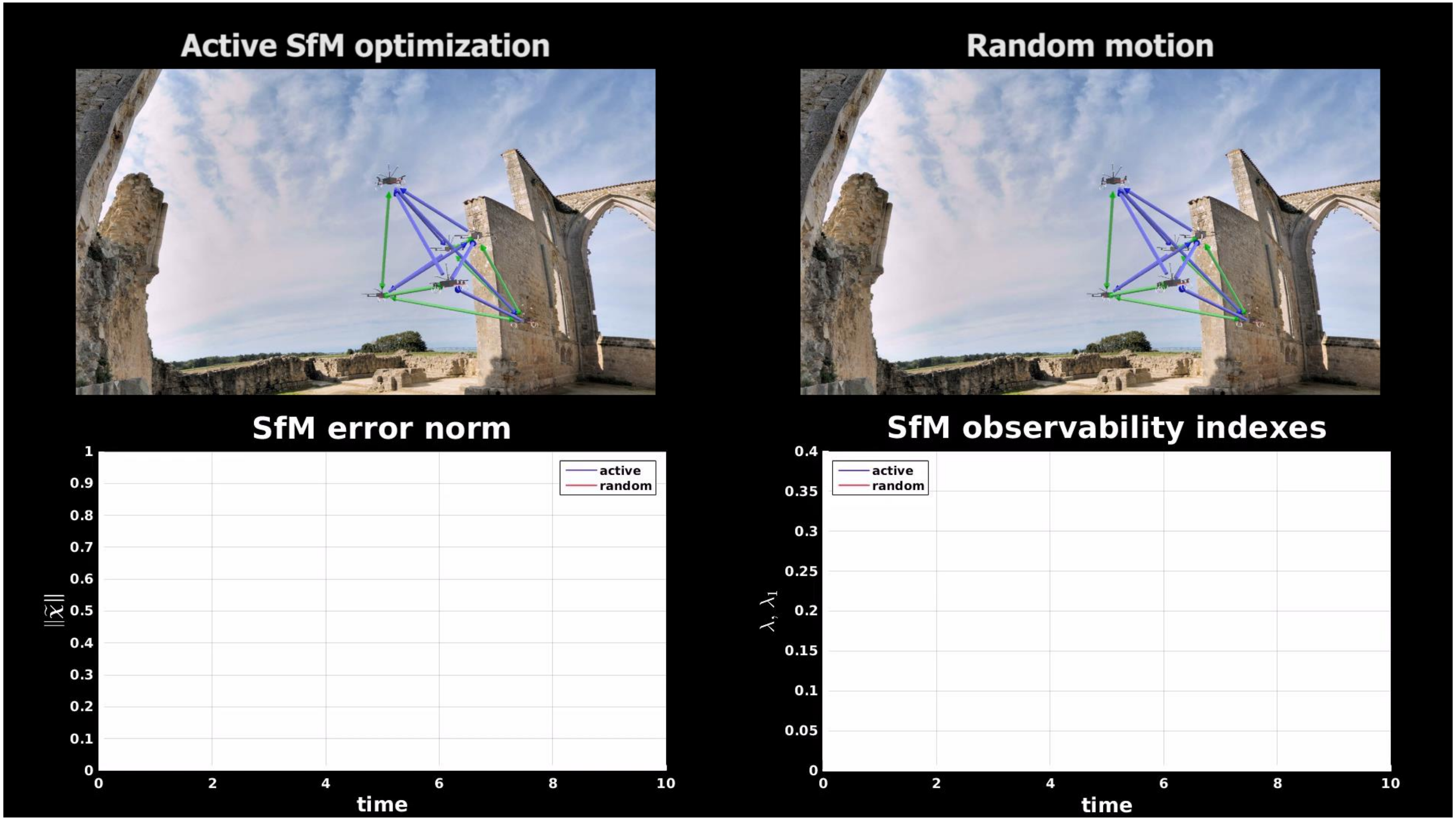
$$\dot{\boldsymbol{u}}_i = \underbrace{\boxed{-\frac{k_\nu}{2}\frac{\nu_i - \nu^*}{\nu_i}\boldsymbol{u}_i}}_{\text{constant } \|\boldsymbol{u}_i\|} + k_{\bar{\lambda}} \underbrace{\boxed{\left(\boldsymbol{I}_3 - \frac{\boldsymbol{u}_i\boldsymbol{u}_i^T}{2\nu_i}\right)}}_{\text{projector}} \underbrace{\boxed{\nabla_{\boldsymbol{u}_i}\bar{\lambda}}}_{\text{excitation}}$$

$$\nu_i = \frac{1}{2}\boldsymbol{u}_i^T\boldsymbol{u}_i \qquad \lambda_{ij} = \boldsymbol{u}_{ij}^T(\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)\boldsymbol{u}_{ij}$$

$$\bar{\lambda} = \sqrt[p]{\sum_{(i,j)\in\mathcal{E}_u} \lambda_{ij}^p} \xrightarrow{p\ll 0} \min_{(i,j)} \lambda_{ij} \quad \text{(worst case)}$$

- average consensus can be used to estimate $\bar{\lambda}$ in a decentralized way

Simulations of unconstrained active estimation



R. Spica, and P. Robuffo Giordano, “Active Decentralized Scale Estimation for Bearing-Based Localization,” In IEEE/RSJ IROS 2016, Daejeon, Korea, October 2016.

Bearing-constrained active perception

- observability maximization must be done while maintaining a desired bearing (IBR) formation

$$\begin{cases} \boldsymbol{u}_i = \boxed{\boldsymbol{u}_i^F} \\ w_i = \boxed{w_i^F} \end{cases} + \boxed{\mathcal{N}(\boldsymbol{\mathcal{B}}^{\mathcal{A}}_{\mathcal{K}_N}(\boldsymbol{q}))}$$

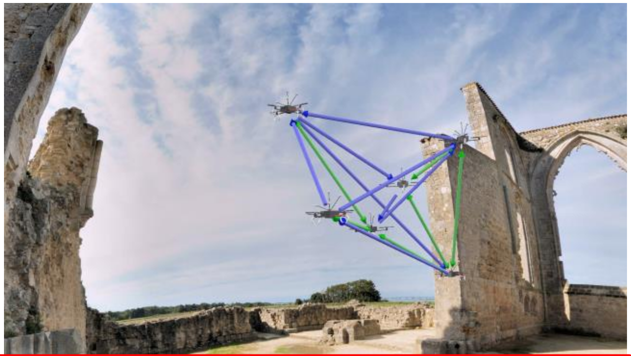
formation control
[Schiano et al. 2016]

observability maximization
in the null space of
the bearing constraints

Null space motions $\lambda_{ij} = \boldsymbol{u}_{ij}^T(\boldsymbol{I}_3 - \boldsymbol{\beta}_{ij}\boldsymbol{\beta}_{ij}^T)\boldsymbol{u}_{ij}$

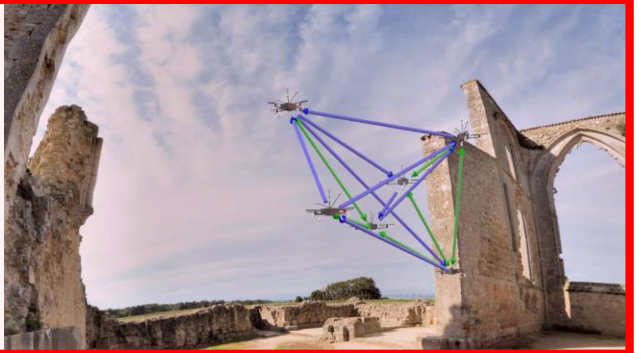
- translation

$$\boldsymbol{u}_{ij} = \boldsymbol{0} \implies \lambda_{ij} = 0$$



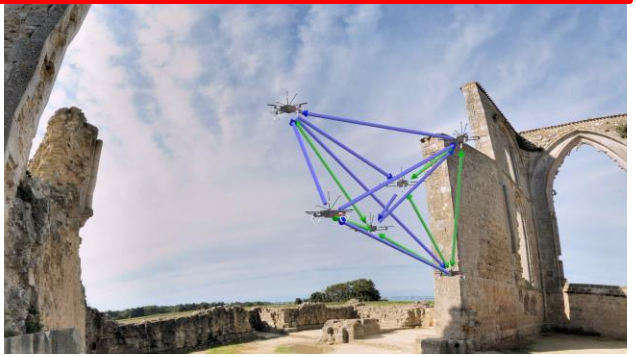
- rotation around Z

$$\lambda_{ij} \neq 0$$



- scaling

$$\boldsymbol{u}_{ij} \parallel \boldsymbol{\beta}_{ij} \implies \lambda_{ij} = 0$$

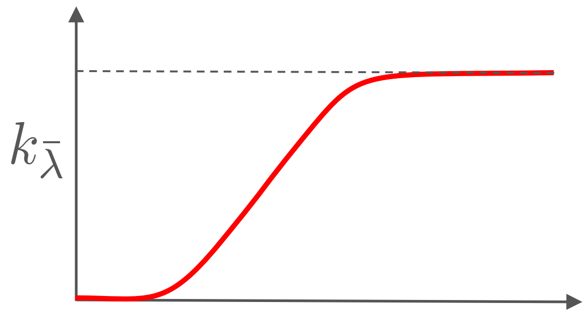


Bearing-constrained active perception

- observability maximization must be done while maintaining a desired bearing formation

$$\begin{cases} \boldsymbol{u}_i = \boldsymbol{u}_i^F + w_{\bar{\lambda}} \boldsymbol{R}_i^T \boldsymbol{S}(\boldsymbol{p}_i - \bar{\boldsymbol{p}}) \\ w_i = w_i^F + w_{\bar{\lambda}} \end{cases}$$

$$\dot{w}_{\bar{\lambda}} \propto k_{\bar{\lambda}}(E) \frac{\partial \bar{\lambda}}{\partial w_{\bar{\lambda}}}^T$$



$$E = \sum_{(i,j) \in e_u} E_{ij}(t)$$

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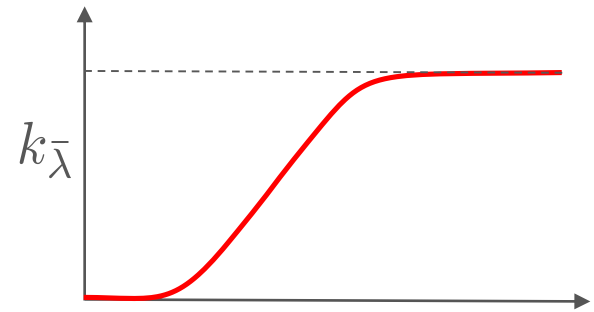

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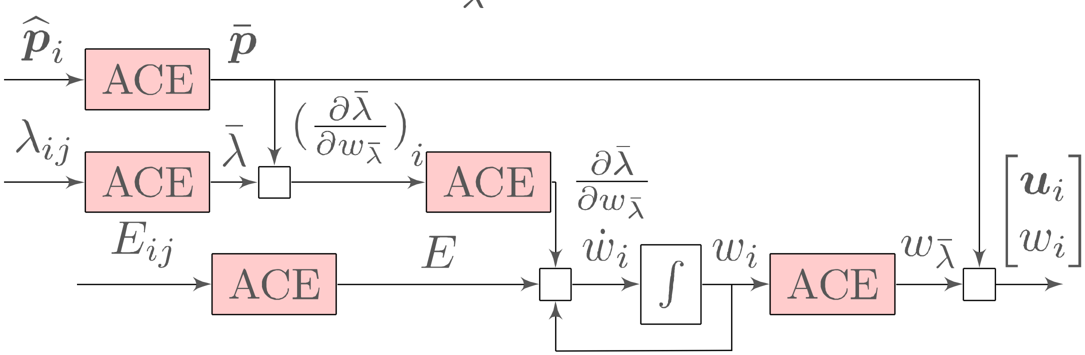
- consensus on $\bar{\boldsymbol{p}}, \bar{\lambda}, E, \frac{\partial \bar{\lambda}}{\partial w_{\bar{\lambda}}}, w_{\bar{\lambda}}$



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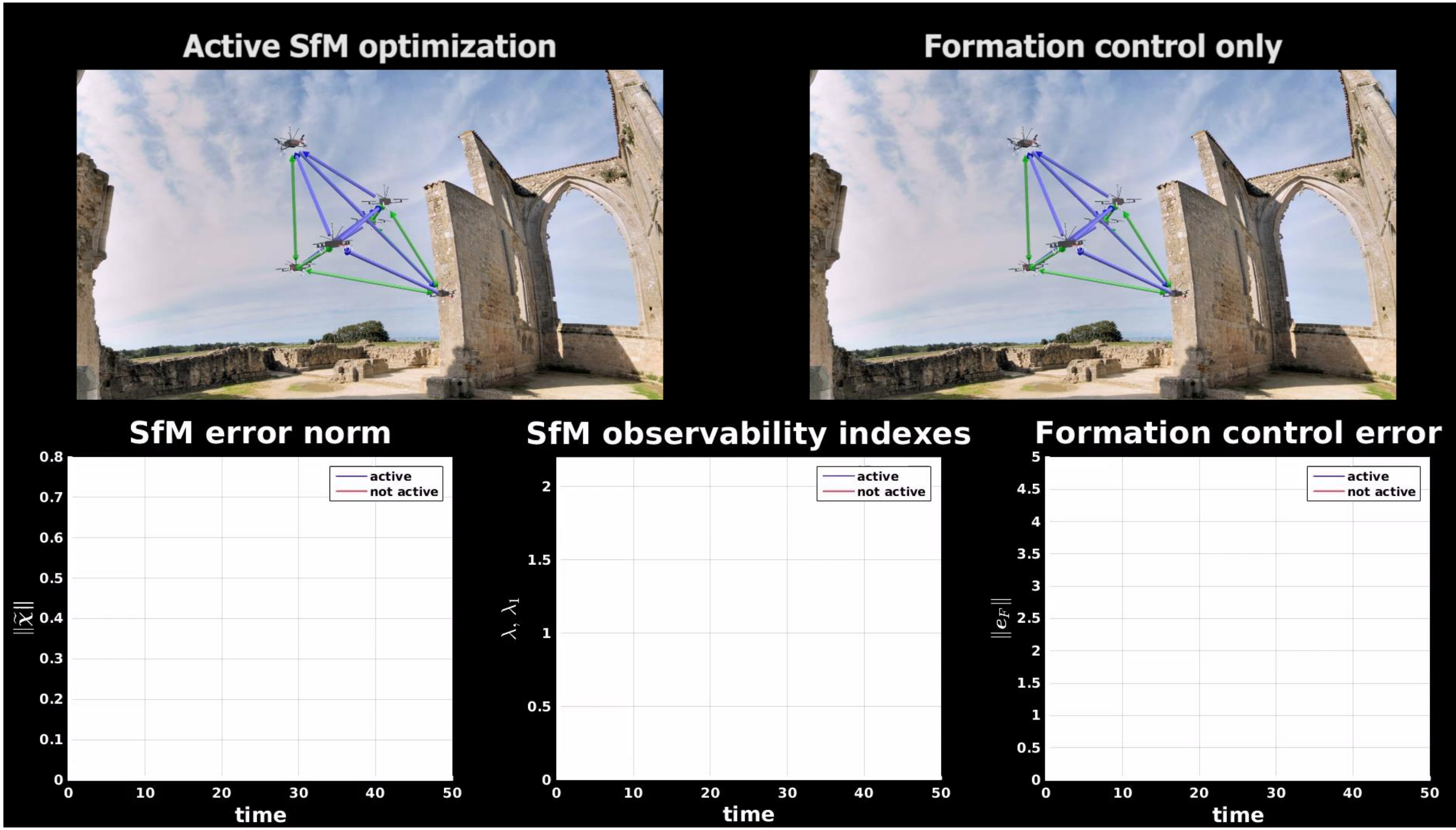

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Wrapping up

- we obtained
  - correctly scaled decentralized localization of the agents
  - decentralized maximization of distance estimation performance
  - active perception coupled with bearing formation control

- open points
  - localization performance also depends of formation rigidity

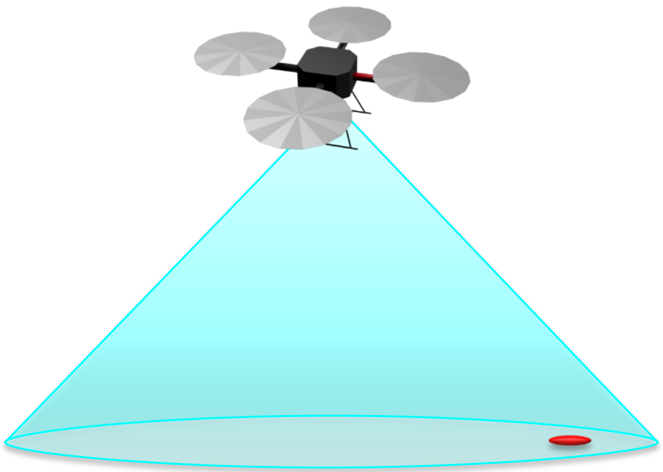
$$\{\lambda_{ij}\} \cup \mathrm{eigs}(\boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{A}}(\boldsymbol{q})\boldsymbol{\mathcal{B}}_{\mathcal{G}}^{\mathcal{A}}(\boldsymbol{q})^T)$$

  - estimating a single metric scale (not multiple distances) would guarantee consistence with the bearing measurements
  - trajectory planning could improve performance (non myopic)

Vision-Based Model Predictive Control of Quadrotor UAVs

Bryan Penin, Riccardo Spica, Paolo Robuffo Giordano and Francois Chaumette

Context



- visual control in minimum time considering
  - quadrotor dynamics (under-actuation)
  - limited actuation (force/torque) and field of view (possibly relaxed)
- partially known state (e.g. target dimension)
  - structure from motion to estimate the state
  - observability is an additional objective/constraint
  - online reactive re-planning (MPC)

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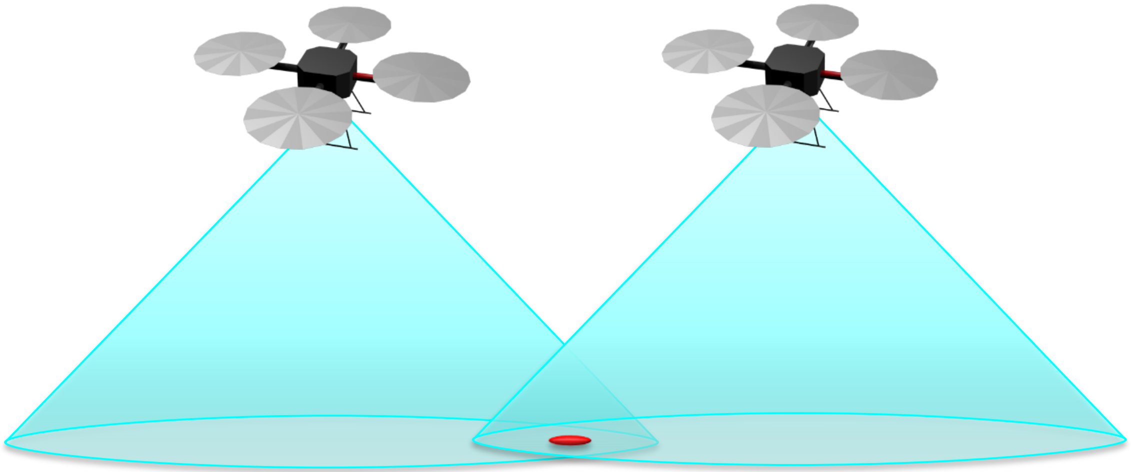

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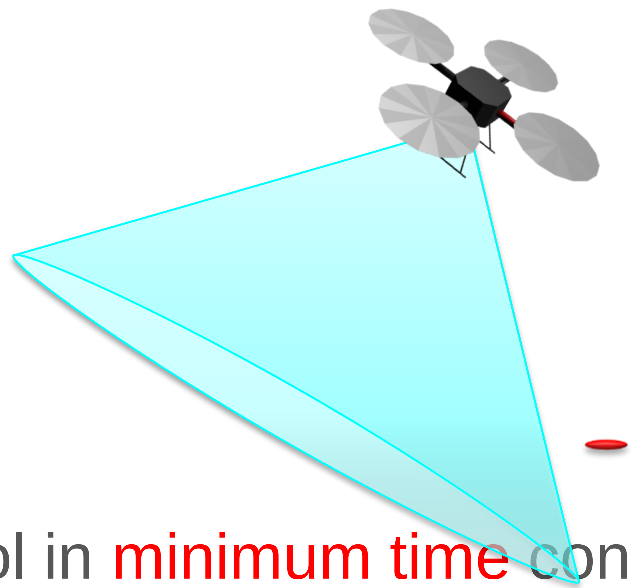

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Optimization problem

- objective
- constraints
  - current robot state (from vision + IMU)
  - targets in the camera limited field of view
  - desired robot state
  - quadrotor full nonlinear dynamics
  - limited control inputs (propeller speed)

$$\min_{\boldsymbol{u}(s),T} \quad T$$

$$s.t.$$

$$\boldsymbol{\chi}(t) = \boldsymbol{\chi}_t$$

$$\boldsymbol{\beta}_i(s) \in \Omega, \forall s \in [t, t+T]$$

$$\boldsymbol{\chi}(t+T) = \boldsymbol{\chi}^*$$

$$\dot{\boldsymbol{\chi}} = \boldsymbol{h}(\boldsymbol{\chi}, \boldsymbol{u})$$

$$\boldsymbol{u}(s) \in \mathcal{U}, \forall s \in [t, t+T]$$

Optimization problem

- objective
- constraints
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$$\min_{\boxed{\boldsymbol{u}(s)}, T} \quad T$$

infinite dimensional search space

$$s.t.$$

$$\chi(t) = \chi_t$$

$$\boldsymbol{\beta}_i(s) \in \Omega, \forall s \in [t, t+T]$$

$$\chi(t+T) = \chi^*$$

$$\boxed{\dot{\chi} = \boldsymbol{h}(\chi, \boldsymbol{u})}$$

nonlinear differential equality constraints

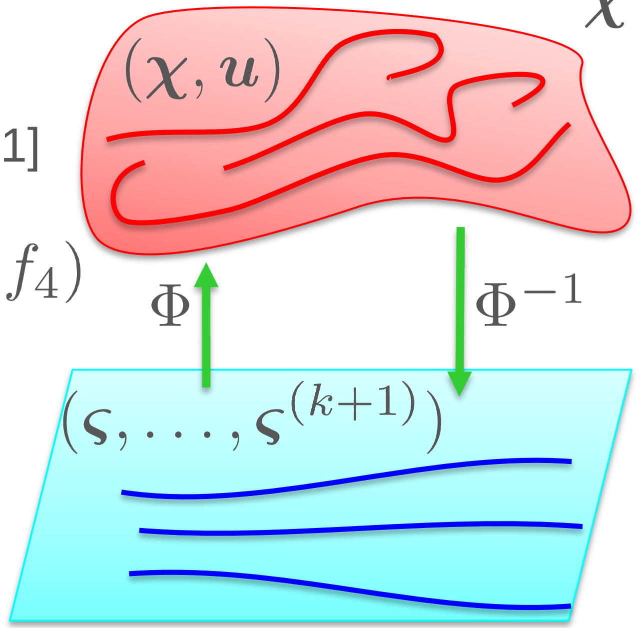
$$\boldsymbol{u}(s) \in \mathcal{U}, \forall s \in [t, t+T]$$

- no analytic solution $\longrightarrow$ numerical optimization

Numerical formulation

$$\dot{\chi} = \boldsymbol{h}(\chi, \boldsymbol{u})$$

- differential flatness
  - for quadrotors [Mistler et al. 2001]



$$\chi = (\boldsymbol{p}, \boldsymbol{v}, \boldsymbol{R}, \boldsymbol{\omega}),\ \boldsymbol{u} = (f_1, f_2, f_3, f_4)$$

$$\varsigma = (\boldsymbol{p}, \psi),\ \boldsymbol{w} = (\boldsymbol{p}^{(4)}, \ddot{\psi})$$

$$\varsigma^{(k+1)} = \boldsymbol{w}$$

- trajectory parametrization

$$\varsigma(s) = \sum_{j=1}^{n} \boxed{B_{j,d}\left(\frac{s-t}{T}\right)} \boxed{\boldsymbol{p}_j},\ \forall s \in [t, t+T]$$

B-spline basis functions sufficiently smooth to guarantee continuity of χ

limited number of control points to be numerically optimized

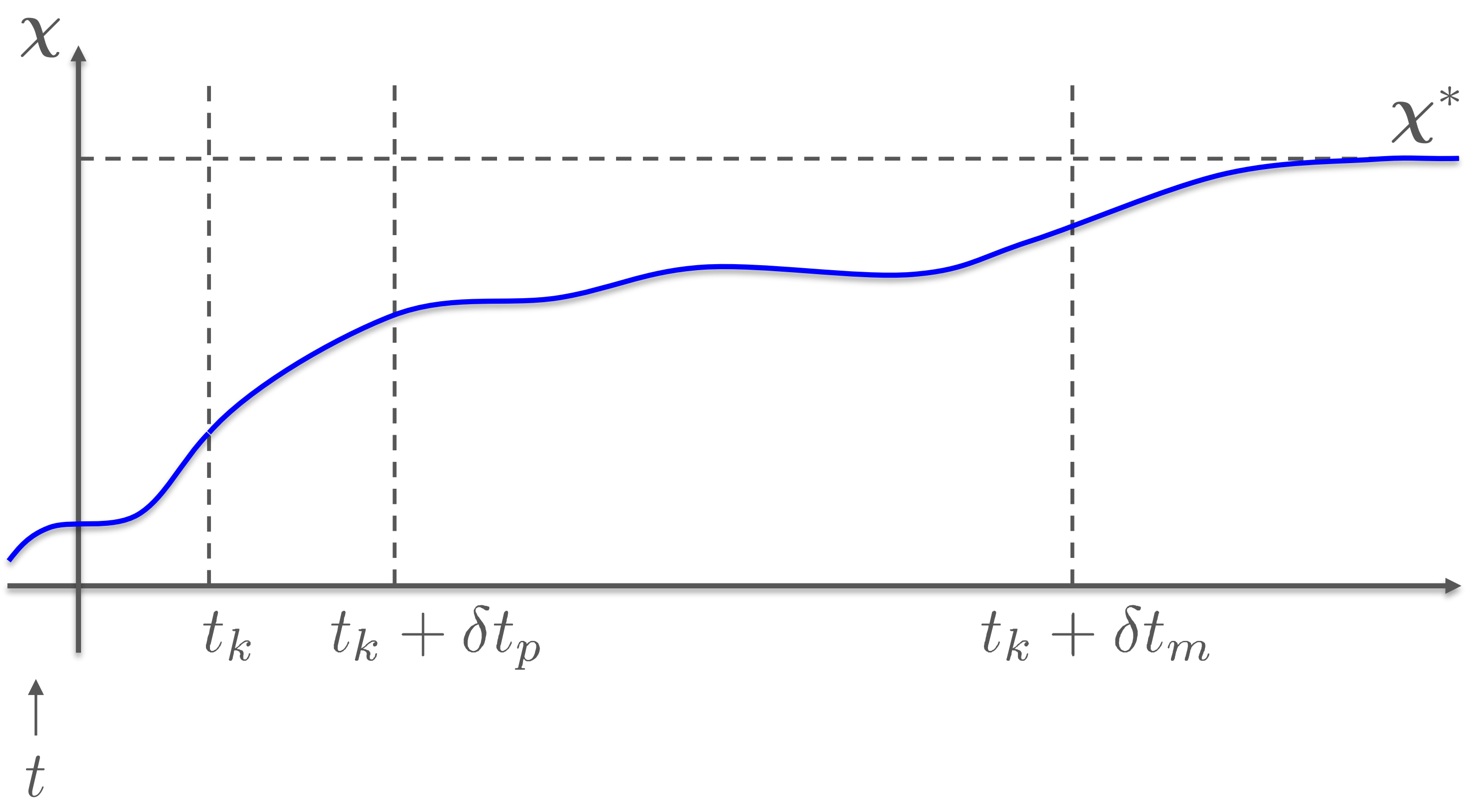
Overall architecture

- optimization problem ($\boldsymbol{\sigma}_{\boldsymbol{\chi}} = (\boldsymbol{p}, \dot{\boldsymbol{p}}, \ddot{\boldsymbol{p}}, \dddot{\boldsymbol{p}}, \psi, \dot{\psi})$)

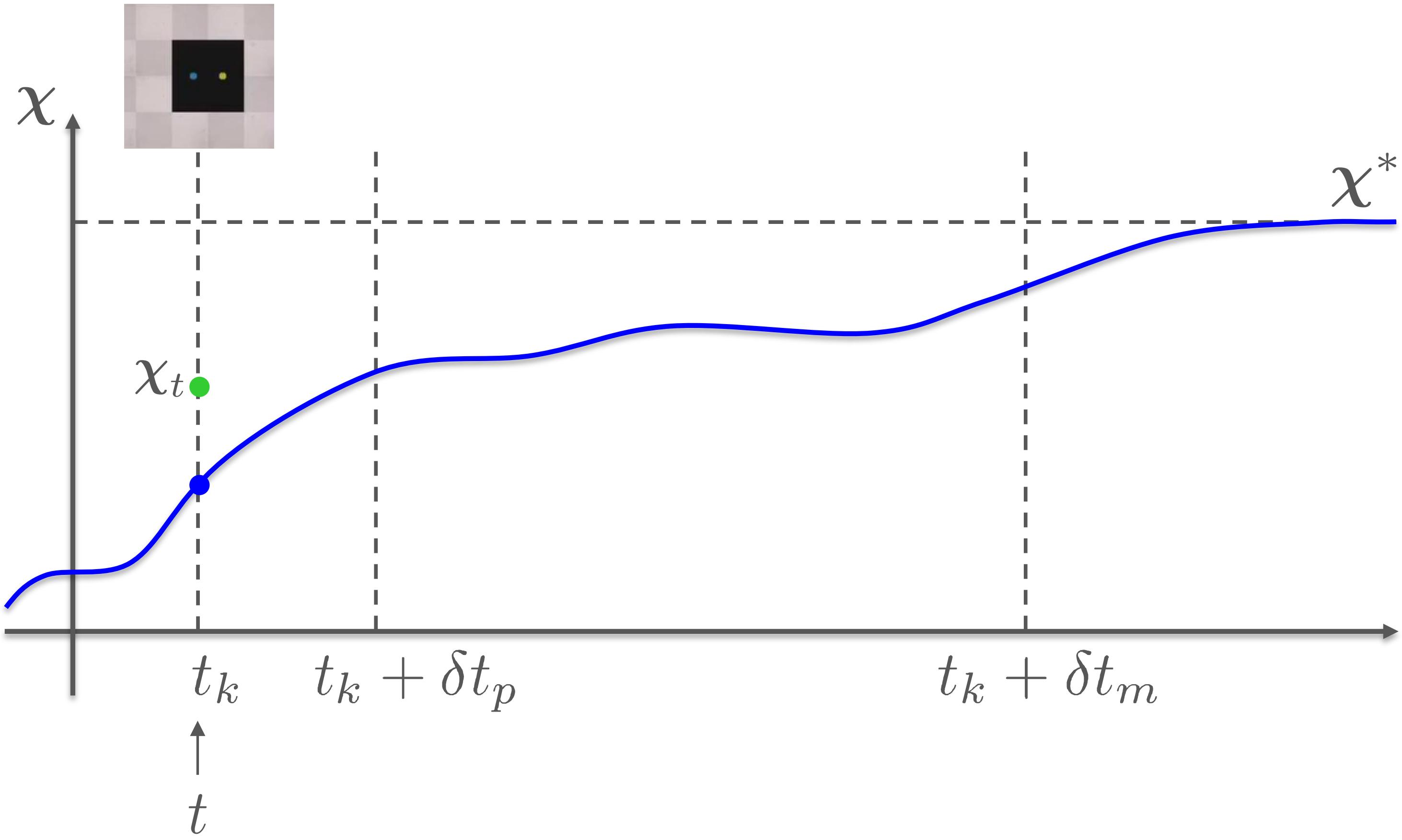
$$\begin{aligned}
\min_{\boldsymbol{p}_1,\dots\boldsymbol{p}_n,T} \quad & T \\
s.t. \quad & \boldsymbol{\sigma}_\chi(t) = \boldsymbol{\sigma}_t = \phi_\chi^{-1}(\boldsymbol{\chi}_t), \\
& \boldsymbol{\sigma}_\chi(t+T) = \boldsymbol{\sigma}^* = \phi_\chi^{-1}(\boldsymbol{\chi}^*), \\
& \boldsymbol{\beta}_i(s) \in \Omega, \forall s \in [t, t+T], i = 1, \dots, N, \\
& \boldsymbol{u}(s) \in \mathcal{U}, \forall s \in [t, t+T],
\end{aligned}$$

- numerical resolution takes $\delta t_p = 40\,\mathrm{ms}$
- visual measurements ($\rightarrow \chi_t$) available every $\delta t_m = 80\,\mathrm{ms}$
- IMU odometry and a nonlinear trajectory tracker [Lee et al. 2010] to track the generated optimal trajectories
- online re-planning to cope with uncertainties

Online re-planning



Online re-planning



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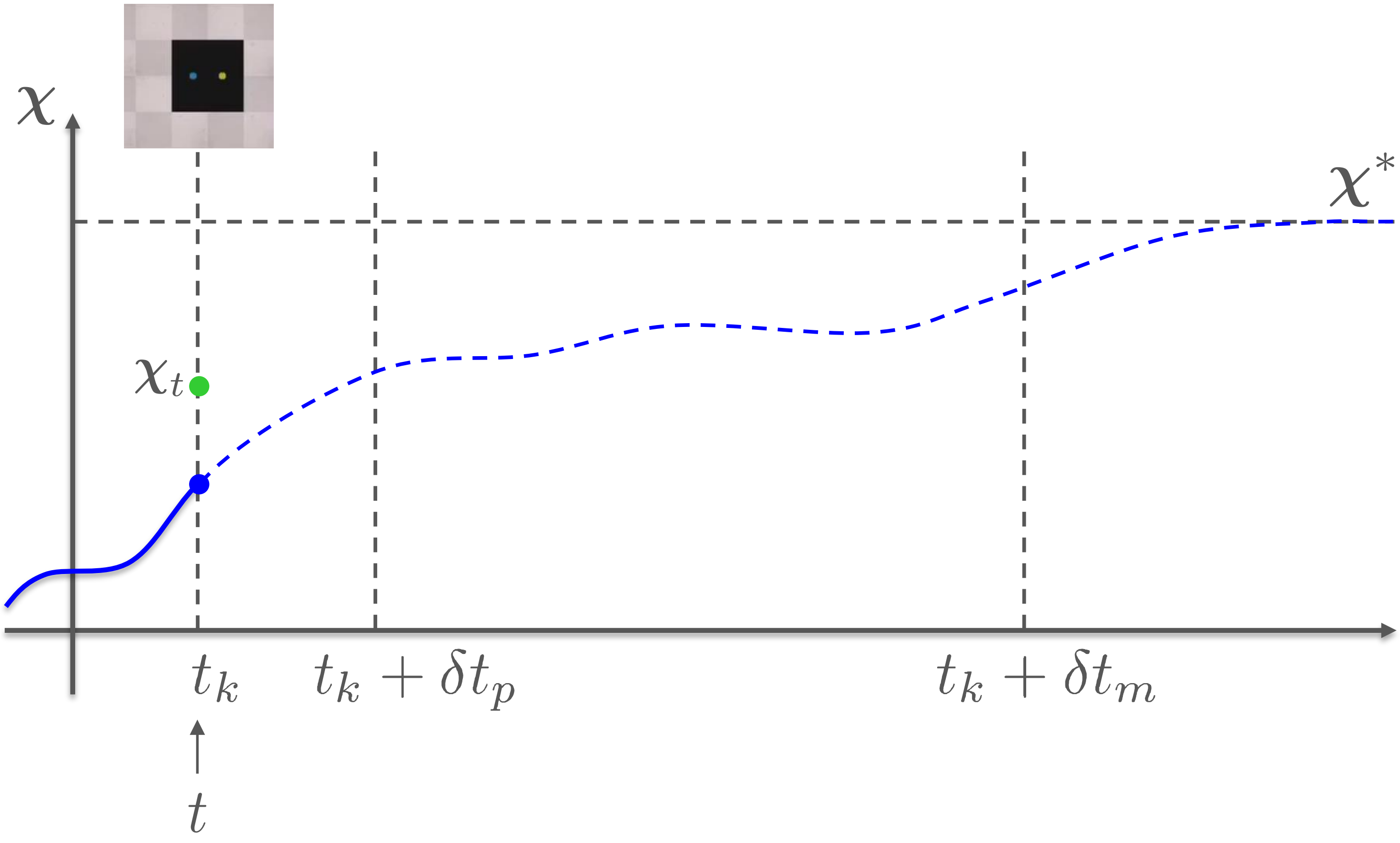
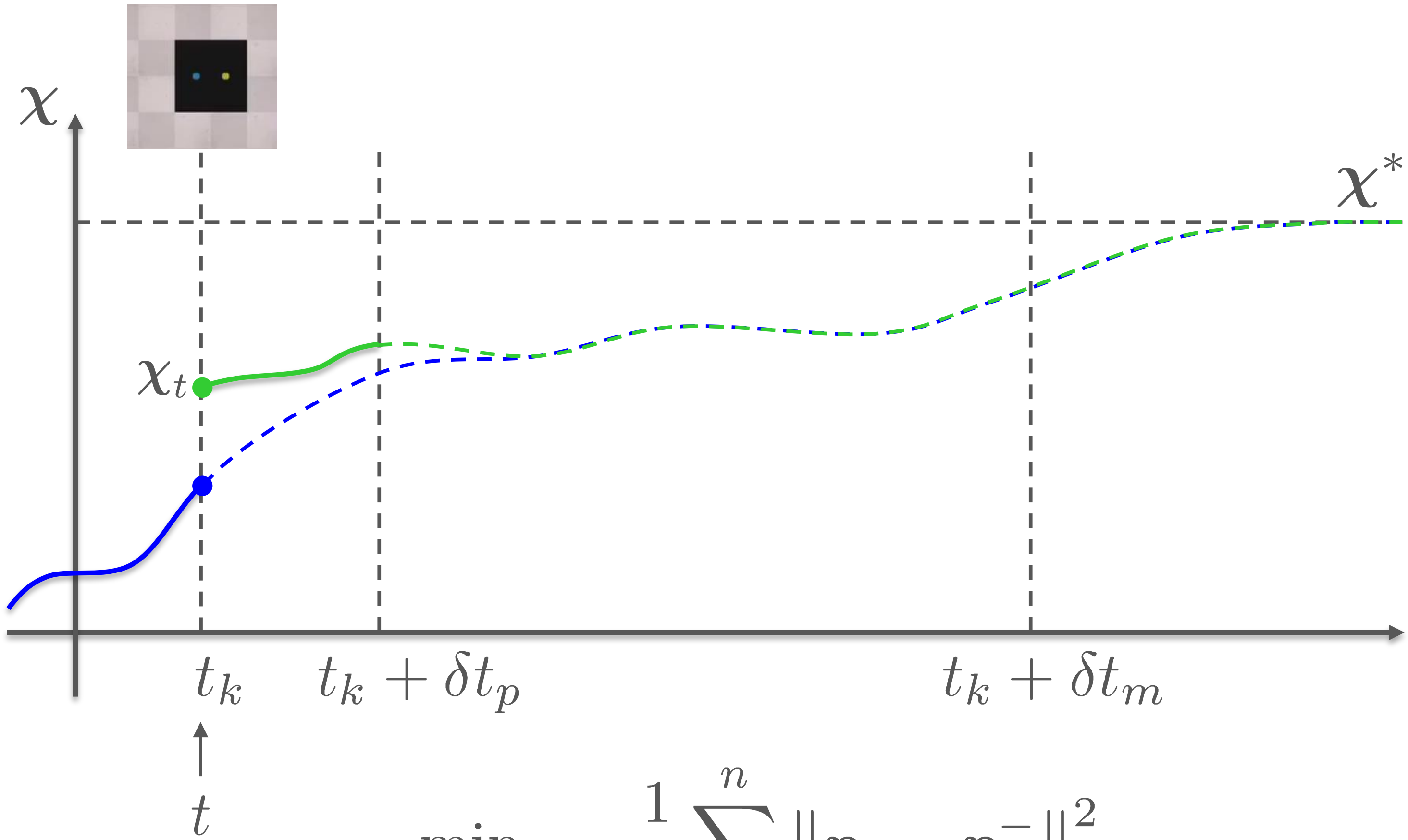


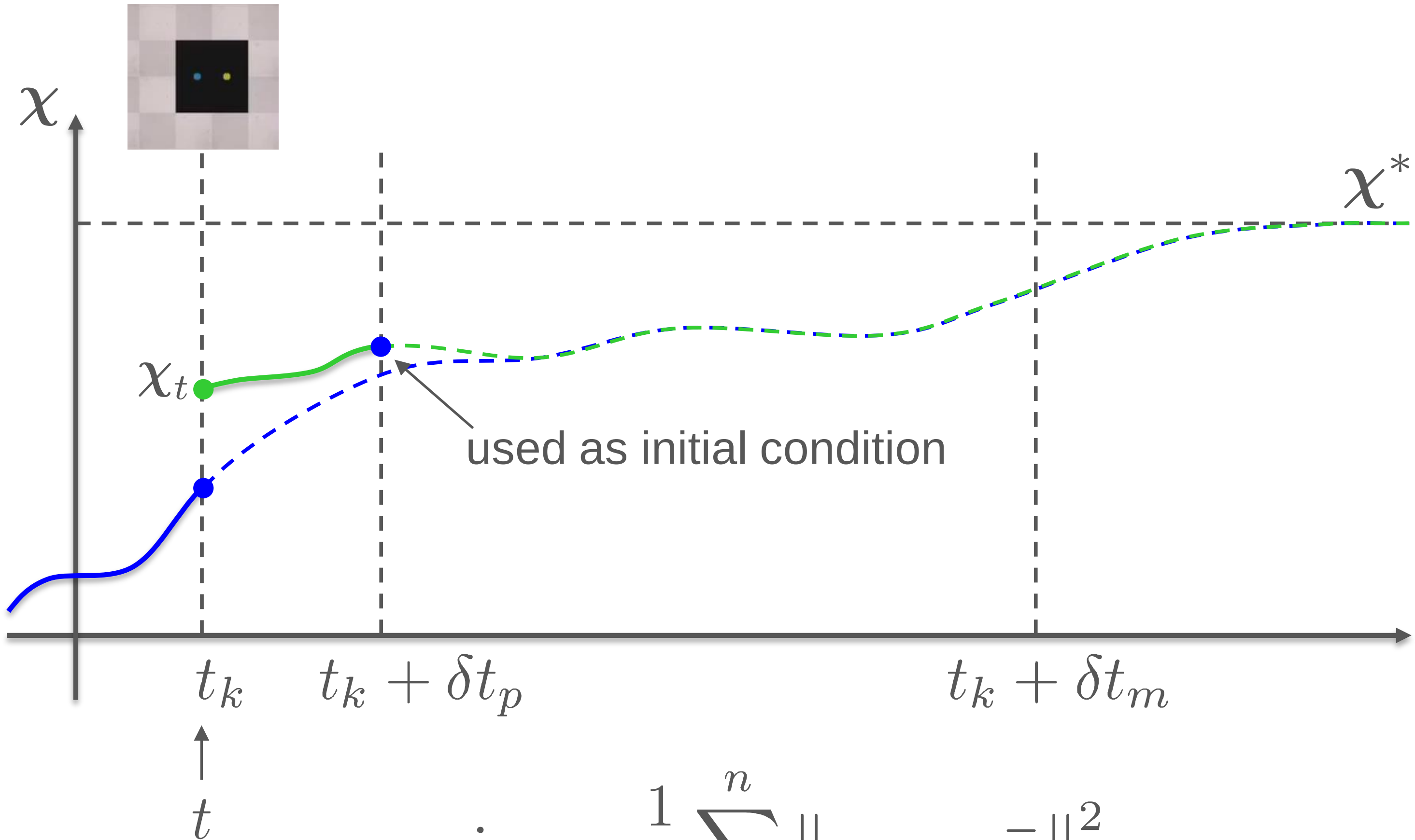
Online re-planning



$$\min_{\boldsymbol{p}_1^-, \ldots, \boldsymbol{p}_n^-} \quad \frac{1}{2} \sum_{j=1}^{n} \left\| \boldsymbol{p}_j - \boldsymbol{p}_j^- \right\|^2$$

$$s.t. \quad \chi(t) = \chi_t$$

Online re-planning



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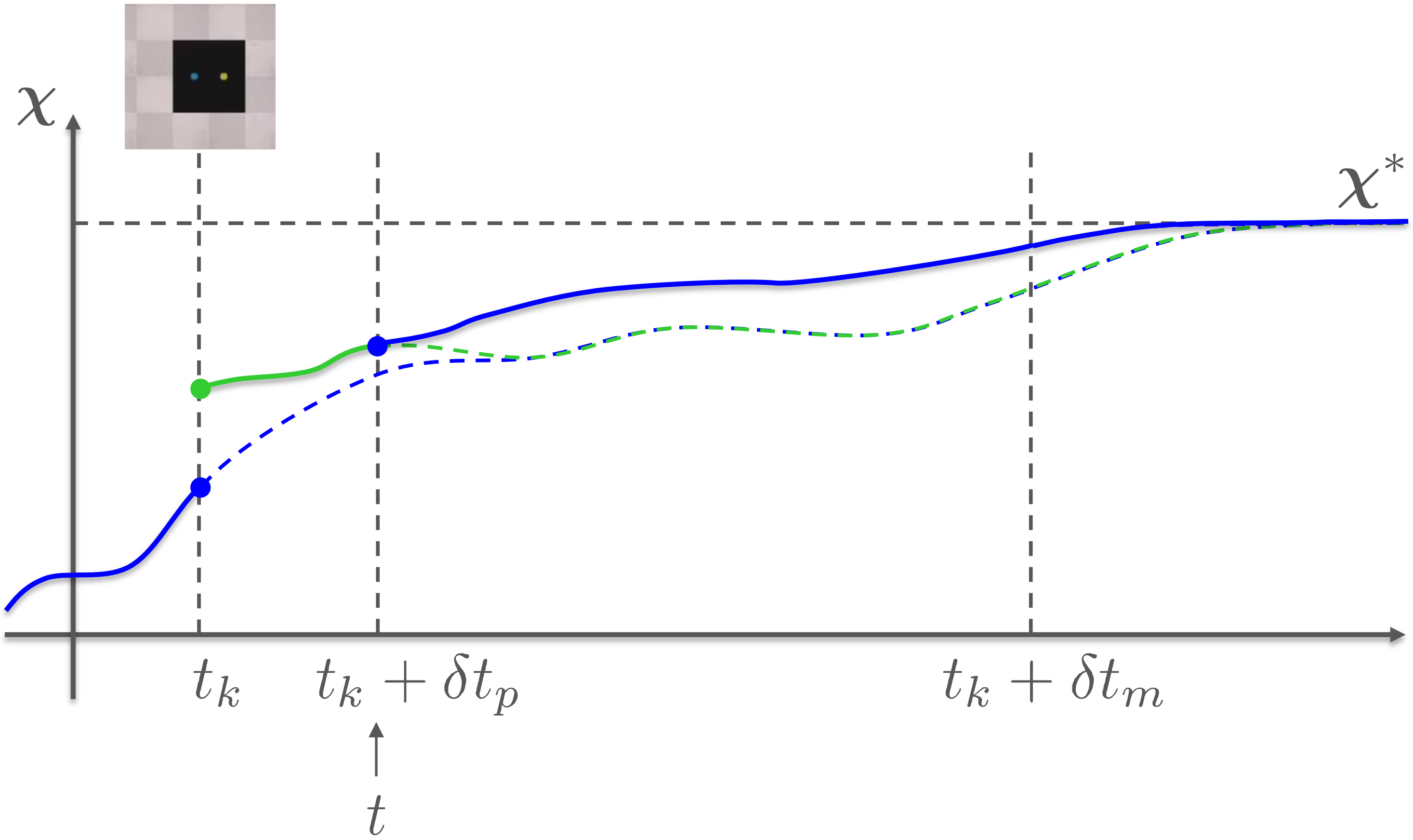
Online re-planning



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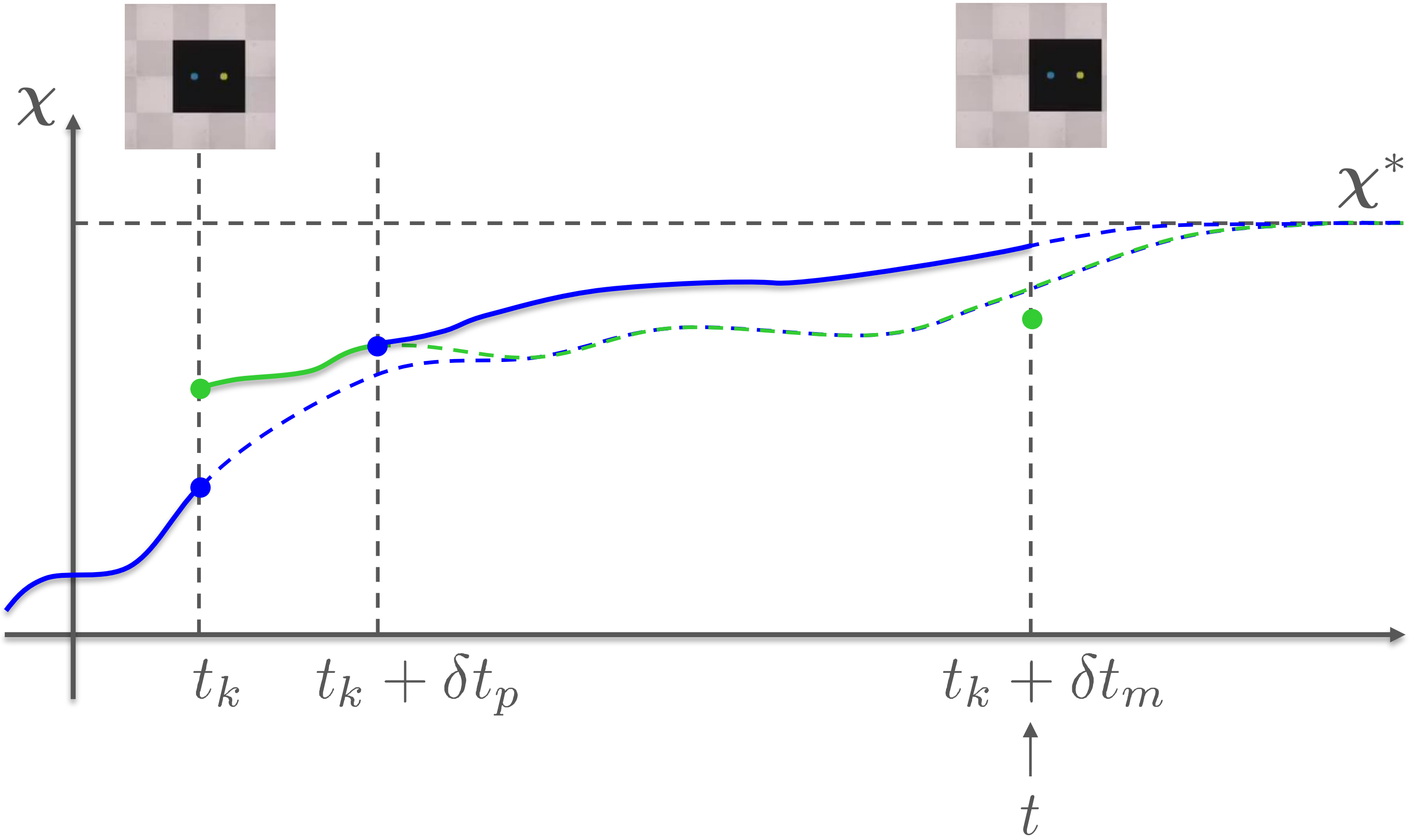

Simulation results

B. Penin, R. Spica, P. Robuffo Giordano, and F. Chaumette, “Vision-Based Minimum-Time Trajectory Generation and Control of a Quadrotor UAV,” In IEEE ICRA 2017, Singapore, May 2017, **under review**.

Conclusions

- we generate minimum time trajectories considering
  - full underactuated dynamics (no near-hovering)
  - system limitations in terms of
    - propeller thrust
    - camera field of view

- future work
  - speed up optimization with C++ implementation (NLOPT)
  - real experiments
  - relax visibility constraints introducing “blind” phases
  - introduce additional constraints (e.g. active estimation)

Thank you for your attention

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