A Controller for Avoiding Dynamic Model Degeneracy of Parallel Robots during Singularity Crossing

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General context : Parallel Robots (1)

Advantages

- Speed
- Accuracy
- Stiffness

Drawbacks

- Complex/Expensive architectures
- Limited workspace :
  - Auto-collisions
  - - "Parallel" Singularities

General context : Parallel Robots (2)

Dealing with singularities

- **Design** Carricato and Parenti-Castelli [2002]; Liu et al. [2006]; Gogu [2004]
- **Redundancy** Nahon and Angeles [1989]; Kurtz and Hayward [1992]; Müller [2005]
- **Specific trajectories** Zein et al. [2008]; Campos et al. [2010]
- **Singularity crossing** Ider [2005]; Briot et al. [2015]

Previous work

Dynamic criterion to cross singularities

- **Briot and Arakelian [2008]** Type 2 (parallel) singularities
- **Briot et al. [2015]** LPJTS (leg) singularities

Design of trajectories avoiding IDM degeneracy

Control law

- **Pagis et al. [2015]** Multi-Model CTC limited to Type 2 singularities with restricted criterion
- **Six et al. [2016]** CTC controller in Cartesian space for Type 2 singularity for general criterion

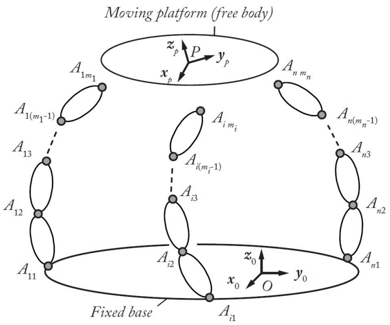
Inverse dynamic model (1)

Dynamic model

$$\boldsymbol{\tau}_a = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_a}\right)^T - \left(\frac{\partial L}{\partial \mathbf{q}_a}\right)^T$$

$$\boldsymbol{\tau}_d = \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}_d}\right)^T - \left(\frac{\partial L}{\partial \mathbf{q}_d}\right)^T$$

$$\mathbf{w}_r = \mathbf{D}^{-T}\left(\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\mathbf{x}}}\right)^T - \left(\frac{\partial L}{\partial \mathbf{x}}\right)^T\right)$$



Inverse dynamic model (2)

Dynamic model

$$\begin{cases} \boldsymbol{\tau} = \boldsymbol{\tau}_a - \boxed{\mathbf{B}^T} \boldsymbol{\lambda}_1 - \boxed{\mathbf{J}_{ta}^T} \boldsymbol{\lambda}_2 \\ \mathbf{w}_r = \boxed{\mathbf{A}_r^T} \boldsymbol{\lambda}_1 - \boxed{\mathbf{J}_t^T} \boldsymbol{\lambda}_2 \\ \boldsymbol{\tau}_d = \boxed{\mathbf{J}_{td}^T} \boldsymbol{\lambda}_2 \end{cases}$$

Kinematic loop-closure equations

- $\mathbf{A}_r \mathbf{t}_r + \mathbf{B}\dot{\mathbf{q}}_a = \mathbf{0}$
- $\mathbf{J}_t \mathbf{t}_r - \mathbf{J}_{ta}\dot{\mathbf{q}}_a - \mathbf{J}_{td}\dot{\mathbf{q}}_d = \mathbf{0}$

Inverse dynamic model

$$\boldsymbol{\tau} = \underbrace{\boldsymbol{\tau}_a - \mathbf{J}_{ta}^T \mathbf{J}_{td}^{-T} \boldsymbol{\tau}_d}_{\mathbf{w}_b} - \mathbf{B}^T \mathbf{A}_r^{-T} \underbrace{\left(\mathbf{J}_t^T \mathbf{J}_{td}^{-T} \boldsymbol{\tau}_d + \mathbf{w}_r\right)}_{\mathbf{w}_d}$$

Type 2 singularities

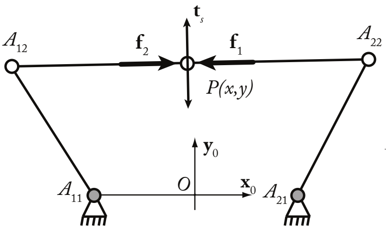


FIGURE – Five-bar mechanism in a Type 2 singularity

Necessary dynamic criterion Briot and Arakelian [2008]

$$\mathbf{t}_s^T \mathbf{w}_d = 0$$

with $\mathbf{t}_s \in \ker \mathbf{A}_r$

Solving the inverse dynamic model (1)

$$\begin{cases} \boldsymbol{\tau} = \mathbf{w}_b - \mathbf{B}^T\boldsymbol{\lambda} \\ \mathbf{w}_d = \mathbf{A}_r^T\boldsymbol{\lambda} \end{cases}$$

A solution in Pagis et al. [2015]

$\boldsymbol{\tau} = \mathbf{w}_b$ with $\mathbf{w}_d = 0$

General criterion

$\mathbf{t}_s^T\mathbf{w}_d = 0 \Leftrightarrow$ Exact solutions to the IDM

General solution to the IDM

$$\underbrace{-\mathbf{A}_r^T\mathbf{B}^{-T}}_{\mathbf{J}_{inv}^T}\boldsymbol{\tau} = -\mathbf{A}_r^T\mathbf{B}^{-T}\mathbf{w}_b + \mathbf{A}_r^T\boldsymbol{\lambda}$$

$$\boxed{\boldsymbol{\tau} = \mathbf{J}_{inv}^{T\,+}(\mathbf{J}_{inv}^T\mathbf{w}_b + \mathbf{w}_d) + \boldsymbol{\nu}, \boldsymbol{\nu} \in \ker \mathbf{J}_{inv}^T}$$

Solving the inverse dynamic model (2)

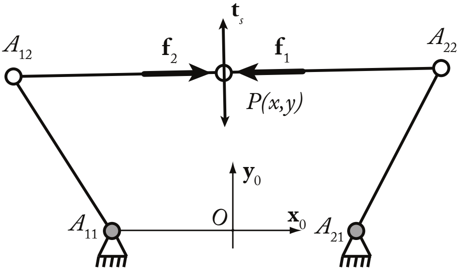


FIGURE – Five-bar mechanism in a Type 2 singularity

Local overconstraint

$\boldsymbol{\nu} = \mathbf{0}$ minimizes the input torques

Solving the inverse dynamic model (3)

IDM as function of platform acceleration

$$\boldsymbol{\tau} = \mathbf{J}_{inv}^{T\,+}(\mathbf{J}_{inv}^{T}\mathbf{w}_b + \mathbf{w}_d)$$

$$\mathbf{w}_b = \mathbf{M}_a\ddot{\mathbf{q}}_a + \mathbf{c}_a \qquad \text{and} \qquad \mathbf{w}_d = \mathbf{M}_x\ddot{\mathbf{x}} + \mathbf{c}_x$$

$$\Downarrow$$

$$\boldsymbol{\tau} = \mathbf{f}(\ddot{\mathbf{q}}_a, \ddot{\mathbf{x}})$$

Expression valid IN and OUT of the singularity (criterion respected)

How to express the IDM as function of $\ddot{\mathbf{q}}_a$ only ?

$$\mathbf{A}_r\ddot{\mathbf{x}} + \mathbf{B}\ddot{\mathbf{q}}_a + \mathbf{b} = \mathbf{0}$$

Extended loop-closure equations (1)

Two equations

- Second order kinematic loop-closure

$$\mathbf{A}_r \ddot{\mathbf{x}} = -\mathbf{B}\ddot{\mathbf{q}}_a - \mathbf{b}$$

  Rank $(n-1)$ in singularity.

- Dynamic criterion ⇔ Free dynamics in the singularity

$$\mathbf{t}_s^T \mathbf{w}_d = \mathbf{t}_s^T (\mathbf{M}_x \ddot{\mathbf{x}} + \mathbf{c}_x) = 0$$

Combined in

$$\mathbf{A}_e \ddot{\mathbf{x}} + \mathbf{B}_e \ddot{\mathbf{q}}_a + \mathbf{b}_e = \mathbf{0}$$

Extended loop-closure equations (2)

From

$$\mathbf{A}_e\ddot{\mathbf{x}} + \mathbf{B}_e\ddot{\mathbf{q}}_a + \mathbf{b}_e = \mathbf{0}$$

Solution to the second order DKM ($\mathbf{A}_e$ always full column rank)

$$\ddot{\mathbf{x}} = -\mathbf{A}_e^{+}(\mathbf{B}_e\ddot{\mathbf{q}}_a + \mathbf{b}_e)$$

Exact solution if and only if the dynamic criterion is respected.

Extended loop-closure equations (3)

DKM when criterion not respected

$$\ddot{\mathbf{x}} = -\mathbf{A}_r^{-1}(\mathbf{B}\ddot{\mathbf{q}}_a + \mathbf{b})$$

Not solvable at singularity

$$\overset{\sigma}{\longleftrightarrow}$$

DKM when criterion respected

$$\ddot{\mathbf{x}} = -\mathbf{A}_e^{+}(\mathbf{B}_e\ddot{\mathbf{q}}_a + \mathbf{b}_e)$$

Always a solution

σ associated to the matrix $\mathbf{A}_r$ condition number.

Final Inverse Dynamic Model

$$\boldsymbol{\tau} = \mathbf{f}(\ddot{\mathbf{q}}_a, \ddot{\mathbf{x}}) \Rightarrow \boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}}_a + \mathbf{h}$$

Criterion enforcement (1)

Auxiliary input for dynamic control

$$\mathbf{u} = \ddot{\mathbf{q}}_t + \mathbf{K}_d\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e}$$

$$\Downarrow$$

$$\mathbf{v} = -\mathbf{A}_e^+(\mathbf{B}_e\mathbf{u} + \mathbf{b}_e) = \ddot{\mathbf{x}}_t + \mathbf{g}(\mathbf{e})$$

Dynamic criterion respect ?

In the singularity neighbourhood

Projection of the auxiliary control input $\mathbf{v}$ into the achievable solution space to avoid IDM degeneracy

$$\mathbf{t}_s^T\mathbf{w}_d(\mathbf{v}) = 0$$

Criterion enforcement (2)

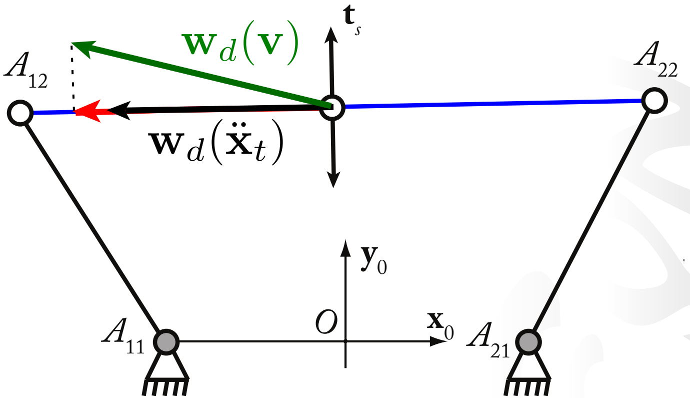
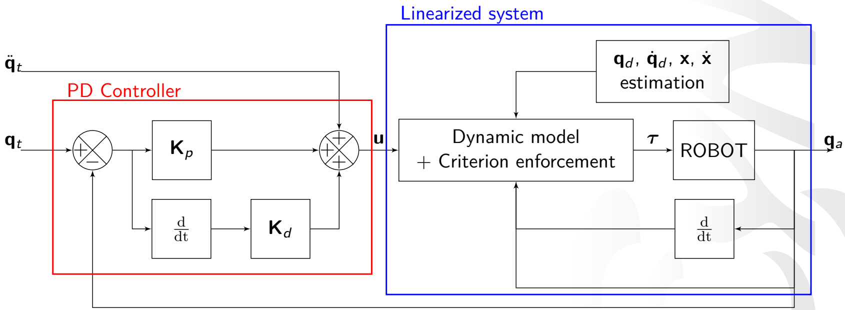


FIGURE – Projecting the auxiliary control into achievable solution space

Controller scheme



Single point crossing trajectory

Speed x0.25

Multiple points crossing trajectory

Conclusion

Obtained result

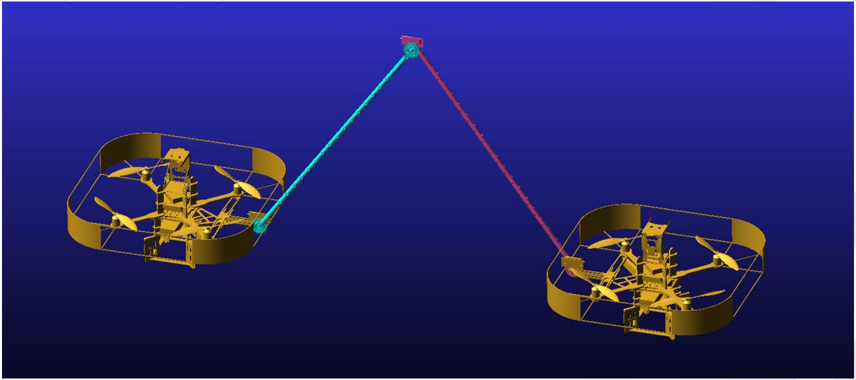
- A controller in **joint space** able to track trajectories crossing type 2 singularities under the respect of the **general** dynamic criterion.
- Theoretical results transposable to **LPJTS singularities**.

Under review

- Controller design to cross the singularity without trajectory planning (Balderas-Hill et al.).

Future work (1)

Design of a flying parallel robot



Future work (2)

Motivations

- Repartition of the effort over several drones
- Rigid links allow to apply efforts on the effector
- No additional actuator load
- Reconfiguration of the tool for take-off/landing.

Explored issues

- Identification
- Dynamic control of the system
  - "Backstepping" control
  - No natural decoupling in the dynamic model
- Apply reconfiguration techniques

Thank you for your attention