Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

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Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

"*The mathematical investigations referred to bring the whole apparatus of a great science to the examination of the properties of a given mechanism, and have accumulated in this direction rich material, of enduring and increasing value. What is left unexamined is however the other, immensely deeper part of the problem, the question:*

How did the mechanism, or the elements of which it is composed, originate?
What laws govern its building up?
Is it indeed formed according to any laws whatever? Or have we simply to accept as data what invention gives us, the analysis of what is thus obtained being the only scientific problem left – as in the case of natural history?"

Reuleaux, **F.**, Theoretische Kinematik, Braunschweig: Vieweg, 1875
Reuleaux, **F.**, The Kinematics of Machinery, London: Macmillan, 1876 and New York: Dover, 1963 (translated by A.B.W. Kennedy)

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Overview

Introduction

Structural parameters of parallel mechanisms

Constraint singularities in parallel mechanisms

Branching singularities in kinematotropic parallel mechanisms

Conclusions

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Introduction

Main objective of this talk:

- **To give an insight on the main criteria for structural synthesis of parallel robotic mechanisms by using the new formulae recently proposed by the author (Gogu 2008) for:**
  - **mobility**
  - **connectivity**
  - **overconstraint**
  - **redundancy**

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Introduction

- mobility
- connectivity
- overconstraint
- redundancy

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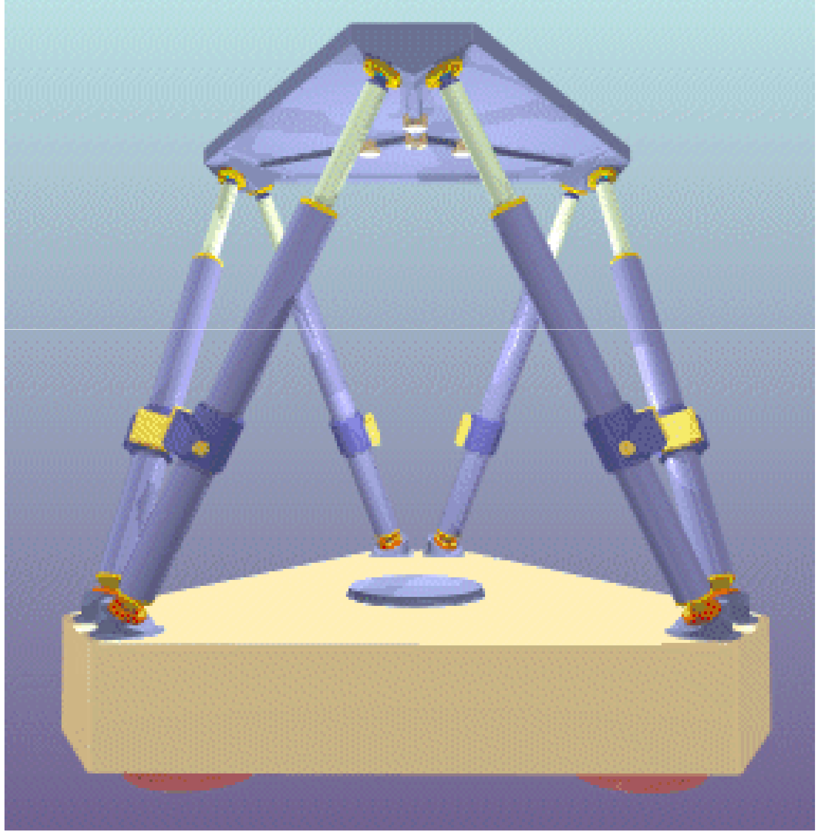

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

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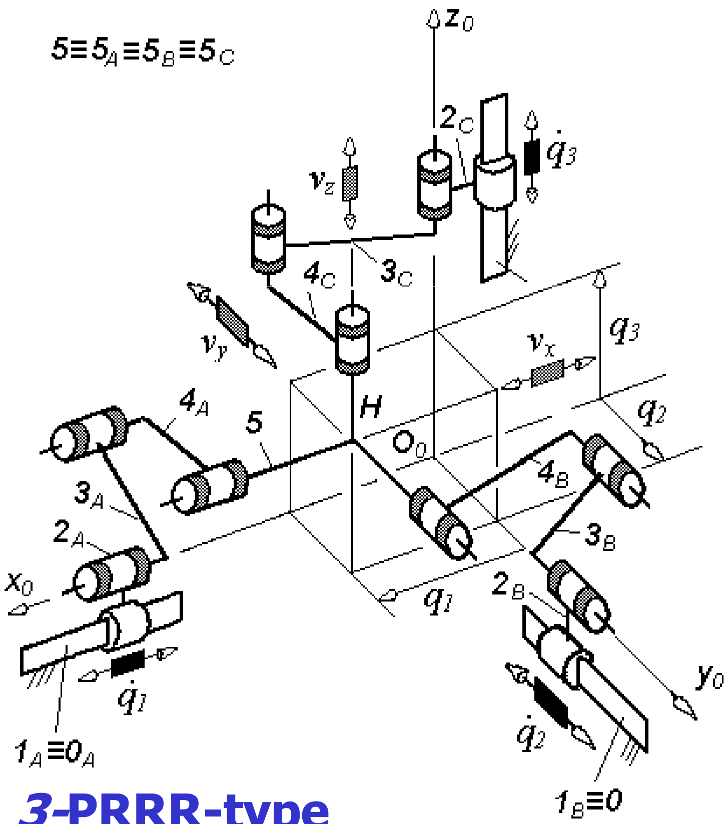
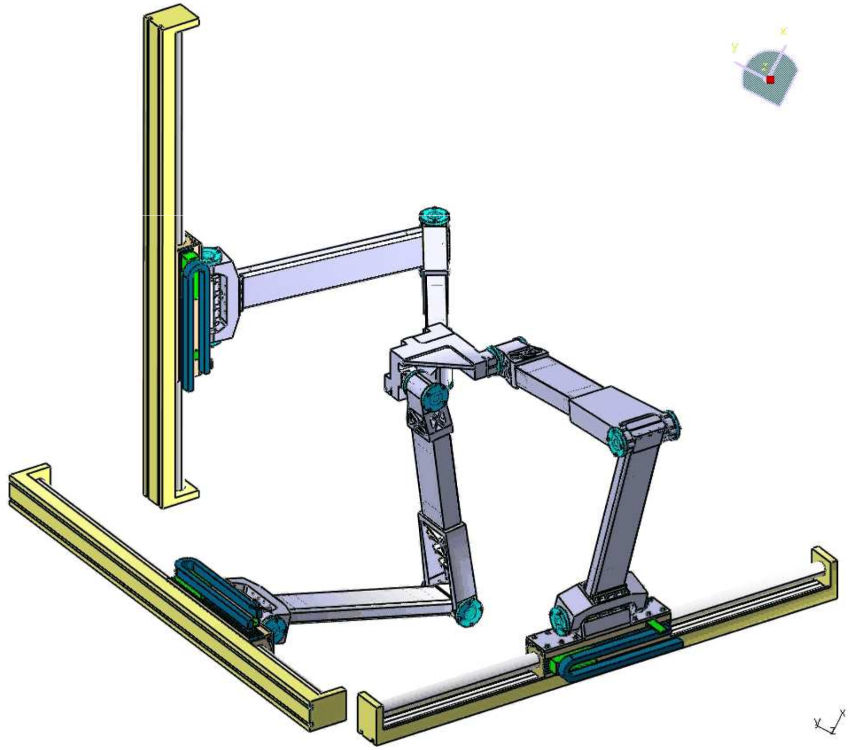


***3*-PRRR-type**



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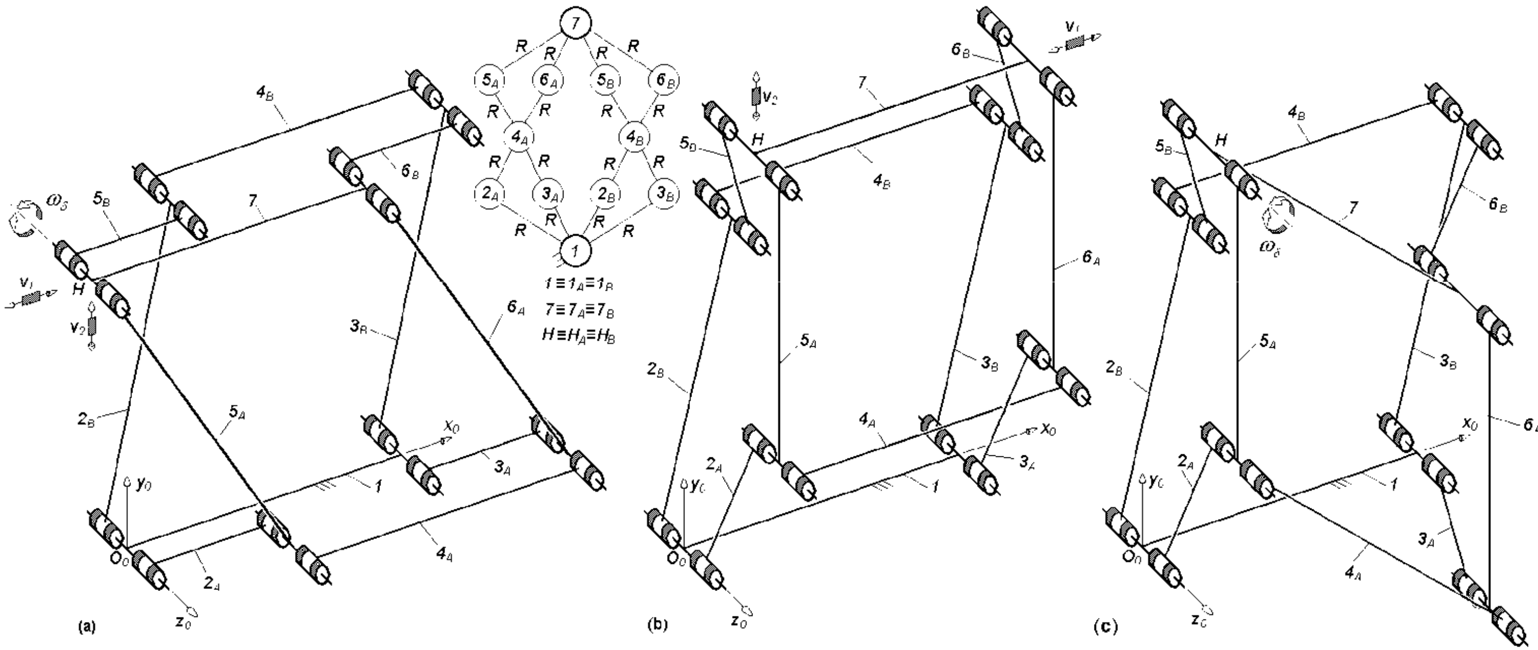

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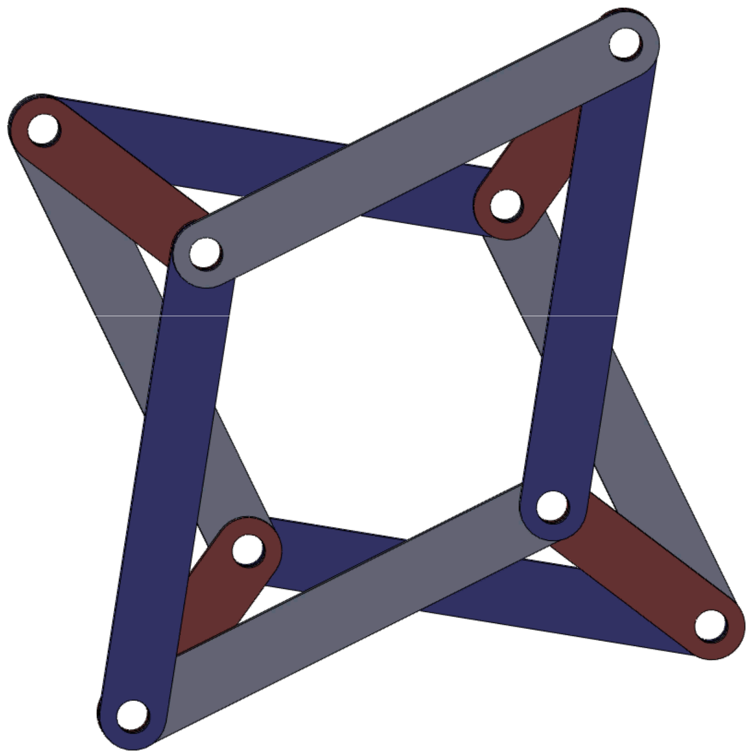

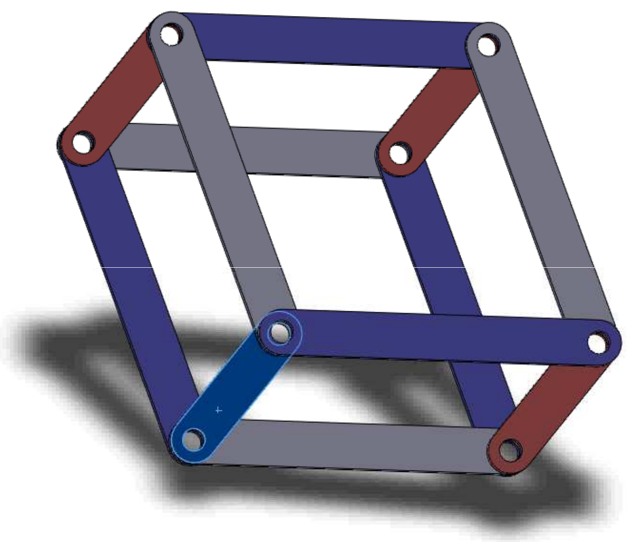

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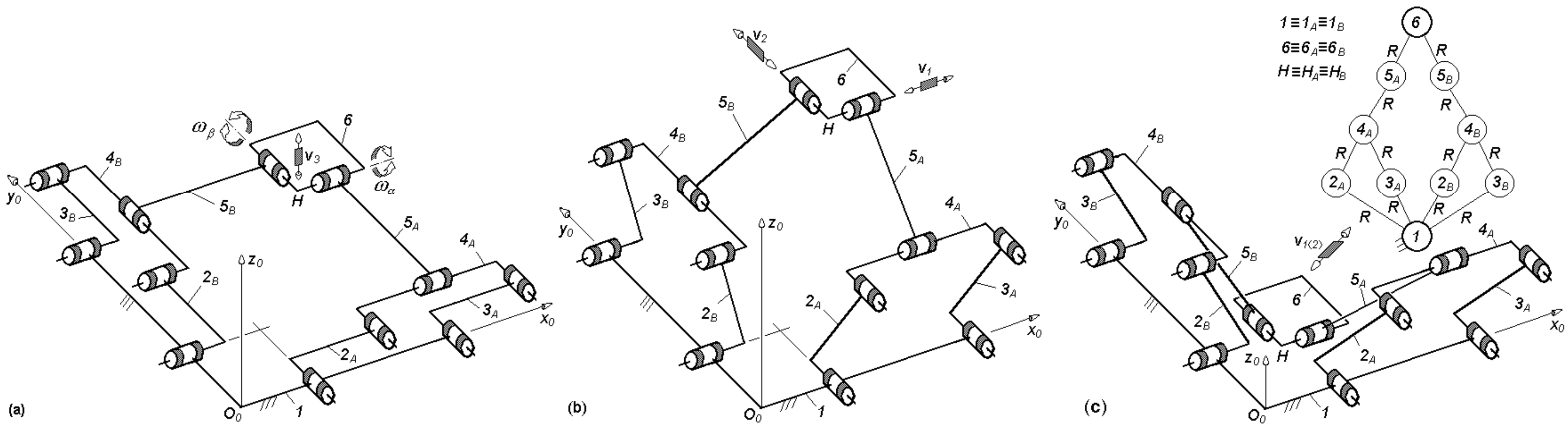

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

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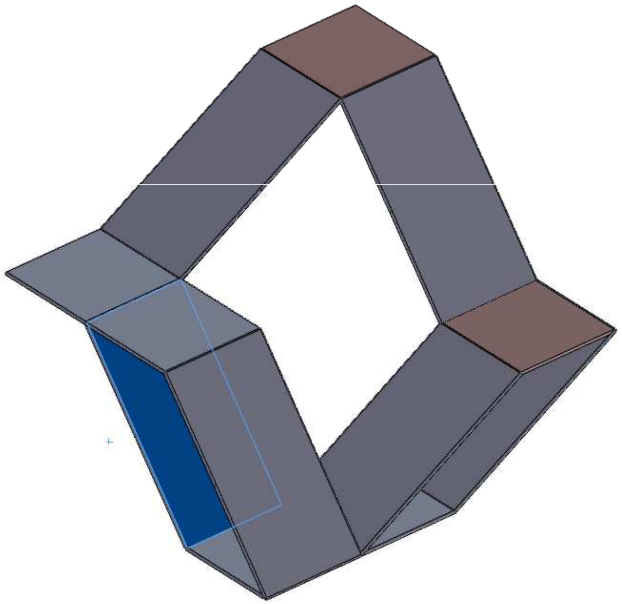
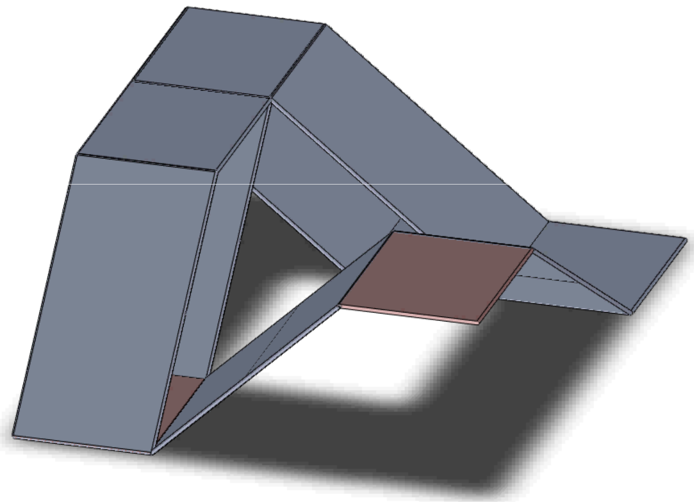
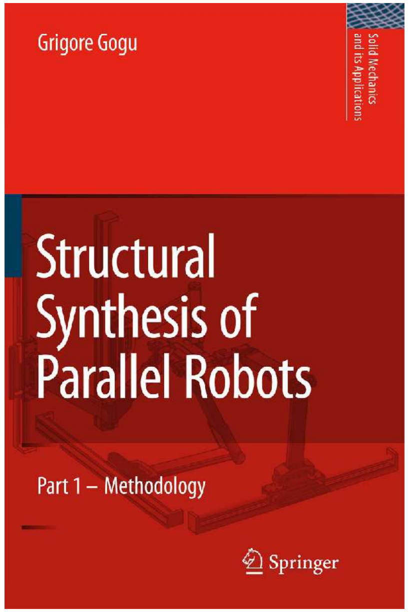




Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Structural parameters of parallel mechanisms

- **The main structural parameters of parallel mechanisms are associated with:**
  - **mobility**
  - **connectivity**
  - **overconstraint**
  - **redundancy**



- **New formulae for the calculation of these parameters in the general case of parallel mechanisms are proposed in (Gogu, 2008).**

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Structural parameters of parallel mechanisms

Mobility (degree of freedom): the number of independent coordinates required to define the configuration of a kinematic chain or mechanism (IFToMM Terminology)

Connectivity between two links of a mechanism: the number of independent finite and/or infinitesimal displacements allowed by the mechanism between the two links.

Number of overconstraints of a mechanism: the difference between the maximum number of joint kinematic parameters that could lose their independence in the closed loops, and the number of joint kinematic parameters that actually lose their independence in the closed loops.

Redundancy of a parallel mechanism : the difference between the mobility of the parallel mechanism and the connectivity of the moving platform.

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Structural parameters of parallel mechanisms

Mobility (degree of freedom): the number of independent coordinates required to define the configuration of a kinematic chain or mechanism (IFToMM Terminology)

Formula proposed by Moroskine in 1954 is a general and valid formula for mobility calculation of any mechanism

$$M = \sum_{i=1}^{p} f_i - r$$

p - total number of joints,
f_i - mobility of the i th joint
r - number of joint parameters that lose their independence in the closed loops of the mechanism

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Structural parameters of parallel mechanisms

Mobility calculation: 35 formulae/approaches developed in the last 150 years and critically reviewed in (Gogu, MMT-2005, 2008)

$$M = \sum_{i=1}^{p} f_i - r$$

Extended Chebychev-Grübler-Kutzbach formula:

$$r = \sum_{k=1}^{q} b_k$$

$q=p-m+1$ - total number of independent closed loops in the sense of graph theory

b_k – *motion parameter of k^{th} loop (the rank of the constraint equations of k^{th} loop*)

Structural parameters of parallel mechanisms

Validity limitation of extended Chebychev-Grübler-Kutzbach (CGK) formula was set up in (Gogu, EJM-A/Solids-2005, 2008):

$$r = \sum_{k=1}^{q} b_k$$

if and only if the rank of the linear set of kinematic constraint equations of $(k+1)^{th}$ loop is equal to the dimension of the range of the restriction of $\boldsymbol{F}_{k+1}$ to the kernel of $\boldsymbol{F}_{1\text{-}2\text{-}\ldots\text{-}k}$

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Structural parameters of parallel mechanisms

A parallel mechanism $F \leftarrow G_1\text{-}\ldots\text{-}G_j\text{-}\ldots\text{-}G_k$ in which the characteristic link (end-effector) $n \equiv n_{Gj}$ is connected to the reference link $1 \equiv 1_{Gj}$ by k simple or complex kinematic chains G_j $(1_{Gj}\text{-}2_{Gj}\text{-}\ldots\text{-}n_{Gj})$ is characterized by:

R_{Gj} - the vector space of relative velocities between the distal links n_{Gj} and 1_{Gj} in the kinematic chain G_j disconnected from mechanism F,

R_F - the vector space of relative velocities between the distal links $n \equiv n_{Gj}$ and $1 \equiv 1_{Gj}$ in the mechanism $F \leftarrow G_1\text{-}\ldots\text{-}G_j\text{-}\ldots\text{-}G_k$,

$S_{Gj}=dim(R_{Gj})$ – the connectivity between the distal links n_{Gj} and 1_{Gj} in the kinematic chain G_j disconnected from the mechanism F,

$S_F=dim(R_F)$ - the connectivity between the distal links $n \equiv n_{Gj}$ and $1 \equiv 1_{Gj}$ in the mechanism $F \leftarrow G_1\text{-}\ldots\text{-}G_j\text{-}\ldots\text{-}G_k$.

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Structural parameters of parallel mechanisms

New formulae for the structural parameters of PMs (Gogu, 2008)

Mobility

$$M = \sum_{i=1}^{p} f_i - r$$

Overconstraint

$$N=6q-r$$

Redundancy

$$T=M-S_F$$

$$r = \sum_{i=1}^{k} S_{Gi} - S_F + r_l$$

$$S_F = dim(R_F) = dim(R_{G1} \cap R_{G2} \cap ... \cap R_{Gk})$$

$$r_l = \sum_{i=1}^{k} r_l^{Gi}$$

$$p = \sum_{i=1}^{k} p_{Gi}$$

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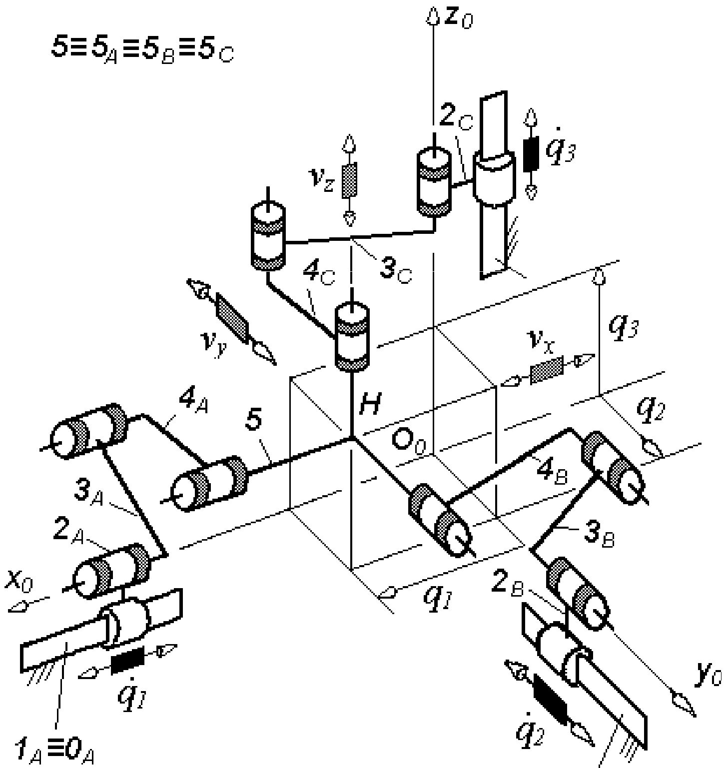
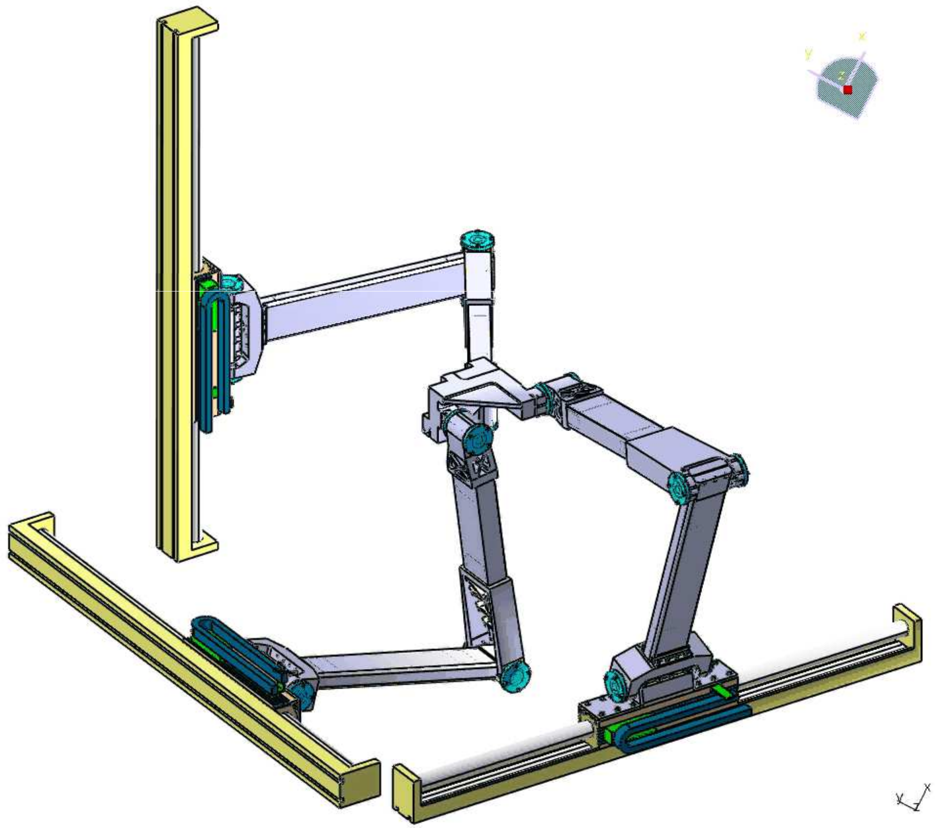
Parallel mechanisms with simple legs
Example – Isoglide3-T3 v1.1



3-PRRR-type



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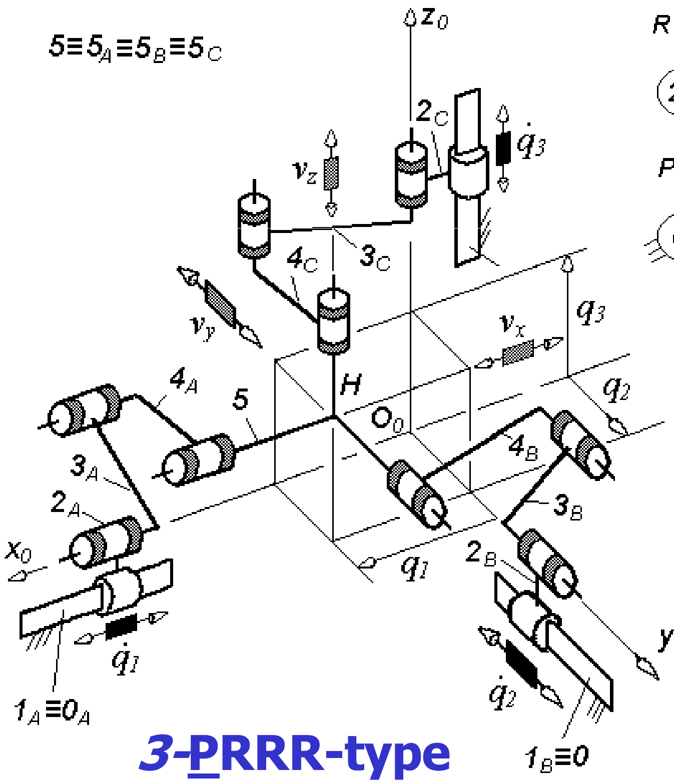
Isoglide3-T3 v1.1

$$p = \sum_{i=1}^{k} p_{Gi}$$



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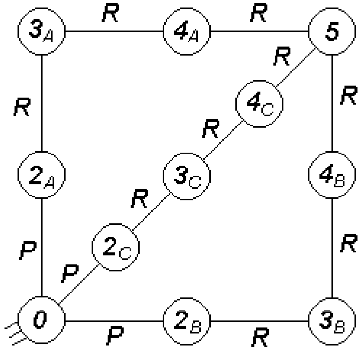

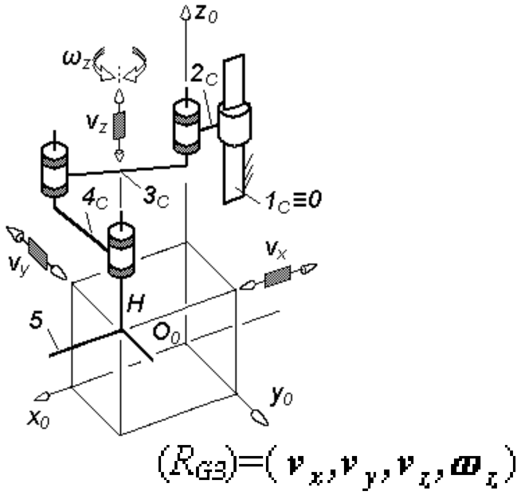

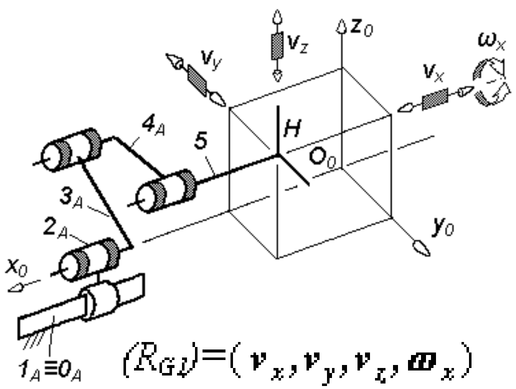

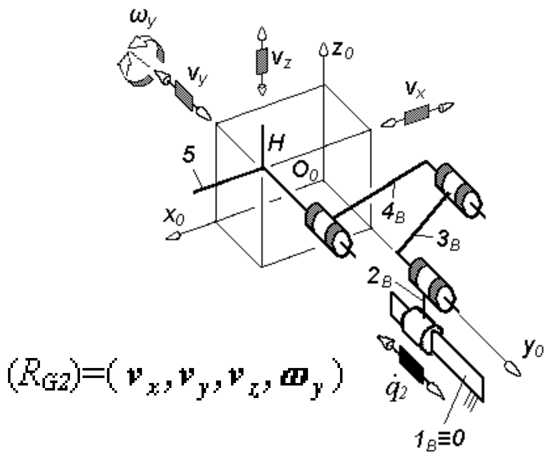

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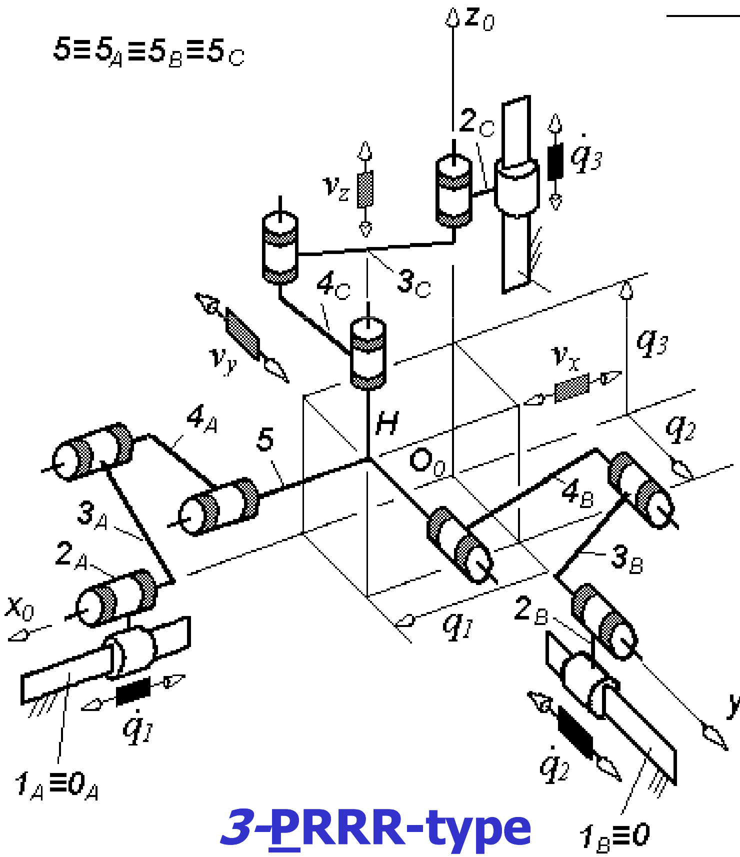
Isoglide3-T3 v1.1



3-PRRR-type

CGK: $M = \sum_{i=1}^{p} f_i - \sum_{k=1}^{q} b_k = 12 - (5+5) = 2$?!

$(R_{G1}) = (v_x, v_y, v_z, \omega_x)$

$(R_{G2}) = (v_x, v_y, v_z, \omega_y)$

$(R_{G3}) = (v_x, v_y, v_z, \omega_z)$

$S_{Gi} = 4,\ i = 1,2,3$

$S_F = \dim(R_{G1} \cap R_{G2} \cap R_{G3}) = 3$

$r = \sum_{i=1}^{k} S_{Gi} - S_F = 3*4-3 = 9$

$M = \sum_{i=1}^{m} f_i - r = 12-9 = 3$

$N = 6q - r = 6*2-9 = 3$

$T = M - S_F = 0$

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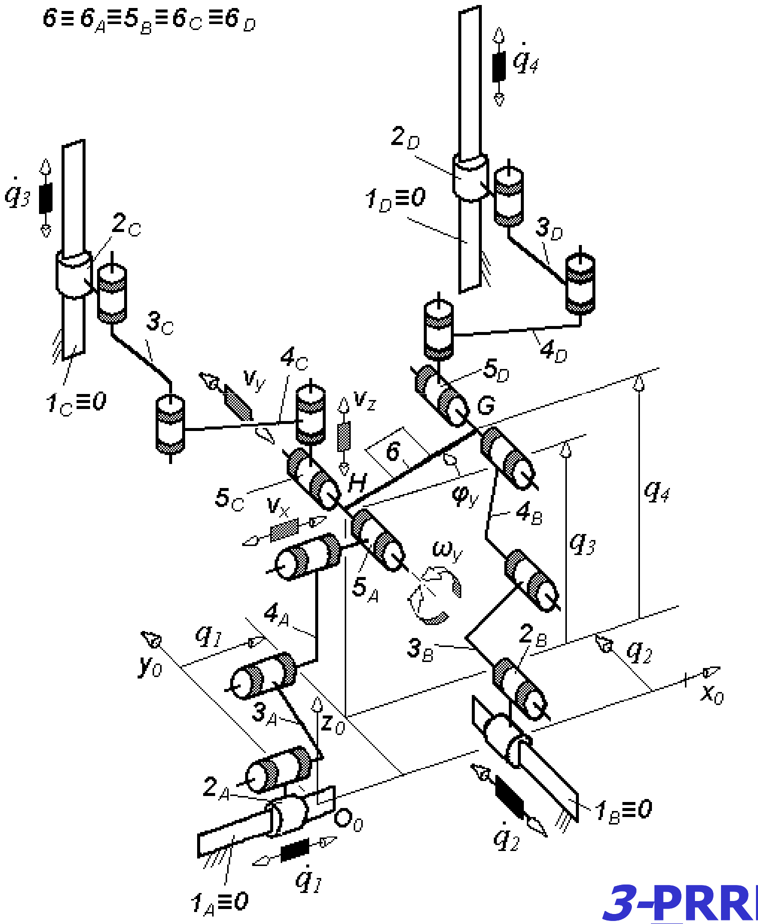
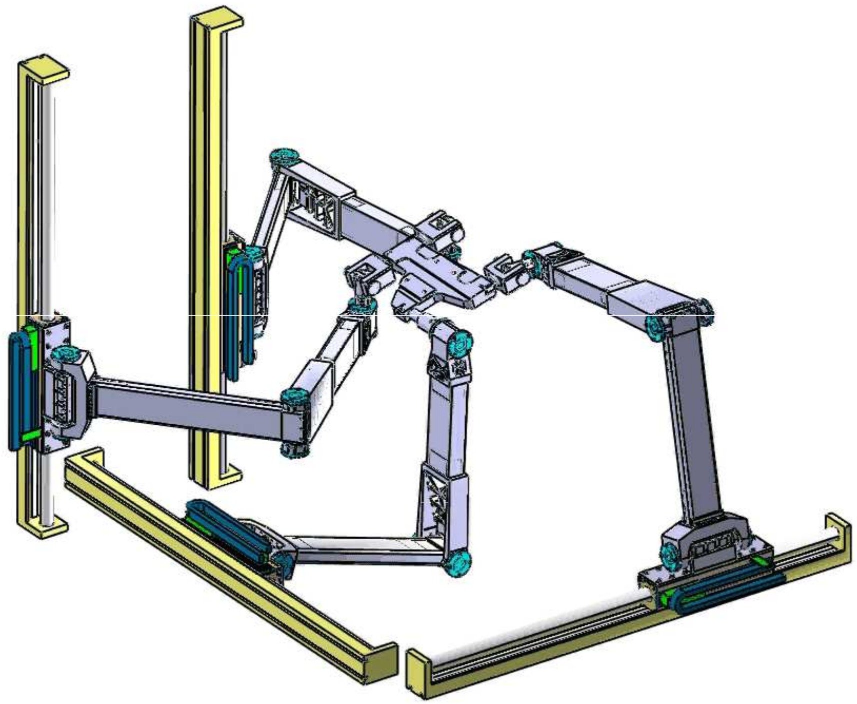
Isoglide4-T3R1 v1.1





3-PRRRR+1-PRRR-type

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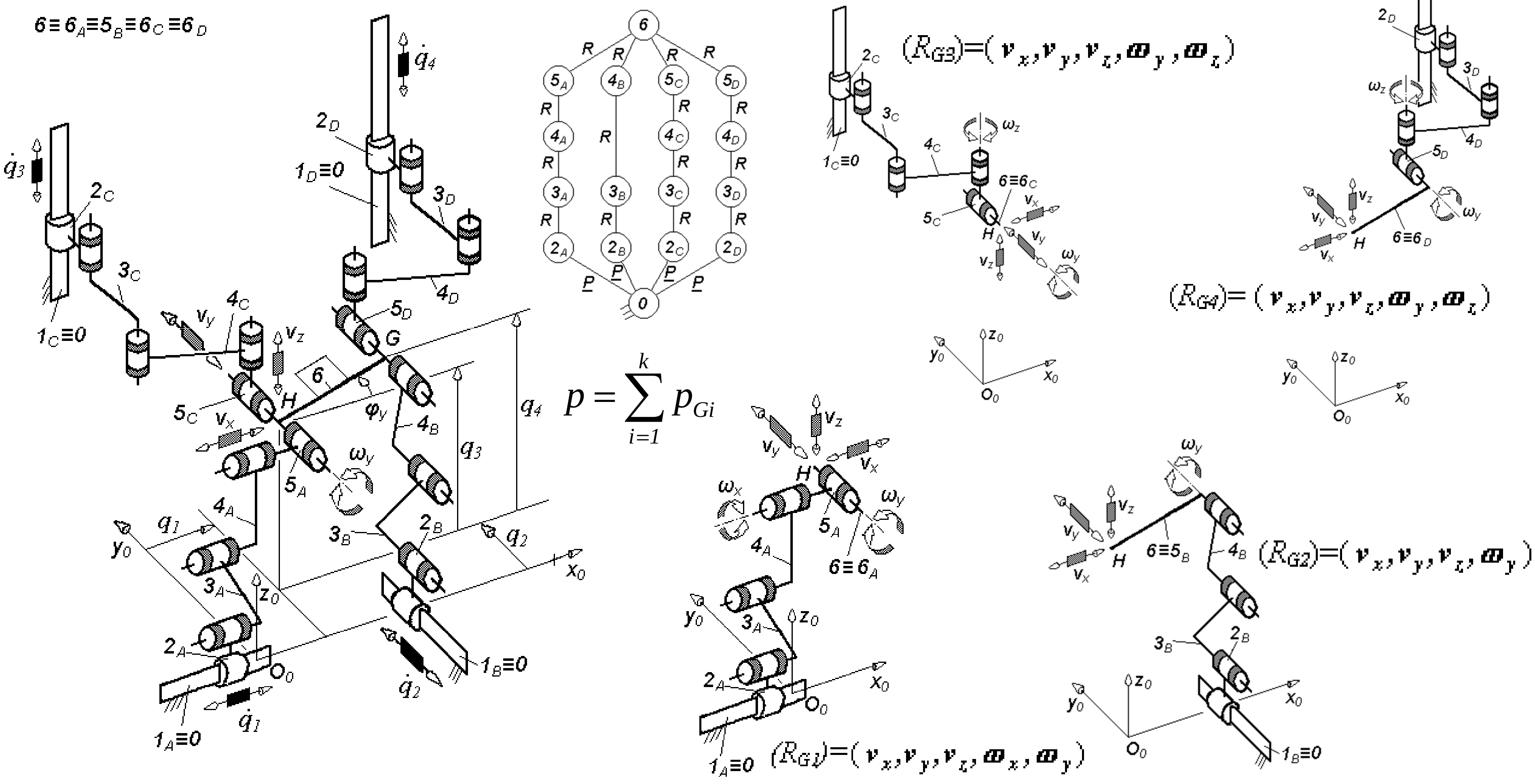
3-PRRRR+1-PRRR-type **Isoglide4-T3R1 v1.1**



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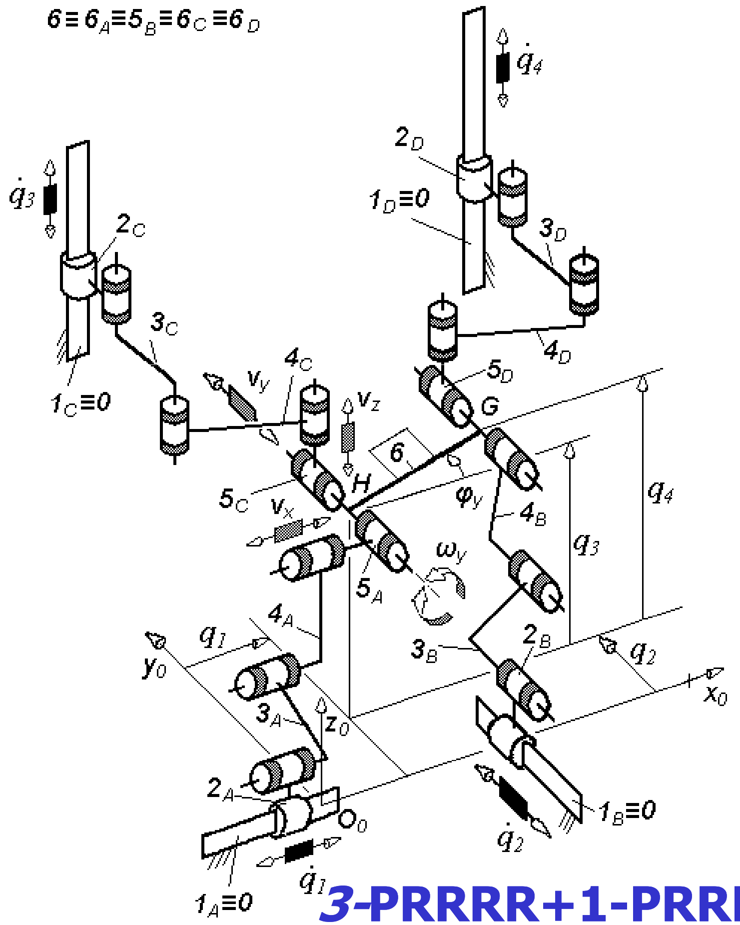
Example – Isoglide4-T3R1 v1.1



3-PRRRR+1-PRRR-type

CGK: $M = \sum_{i=1}^{p} f_i - \sum_{k=1}^{q} b_k = 19 - ? = ?$?!

$(R_{G1}) = (v_x, v_y, v_z, \omega_x, \omega_y)$

$(R_{G2}) = (v_x, v_y, v_z, \omega_y)$

$(R_{G3}) = (v_x, v_y, v_z, \omega_y, \omega_z)$

$(R_{G4}) = (v_x, v_y, v_z, \omega_y, \omega_z)$

$S_{Gi} = 5,\ i = 1, 3, 4$ and $S_{G2} = 4$

$S_F = \dim(R_{G1} \cap R_{G2} \cap R_{G3} \cap R_{G4}) = 4$

$r = \sum_{i=1}^{k} S_{Gi} - S_F = (3*5+4)-4 = 15$

$M = \sum_{i=1}^{p} f_i - r = 19-15 = 4$

$N = 6q - r = 6*3-15 = 3$

$T = M - S_F = 0$

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3-PRRRR+1-PRRRR*-type

Example – Isoglide4-T3R1 v1.1

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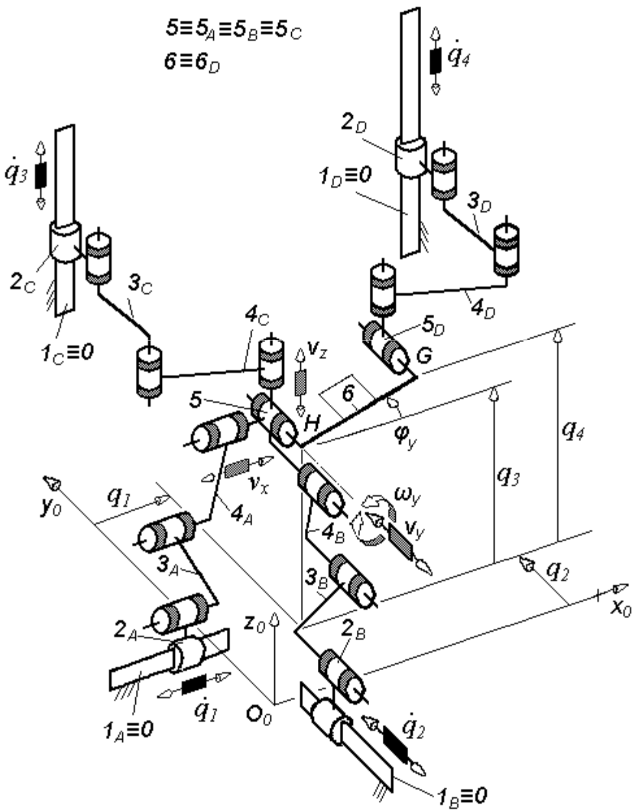
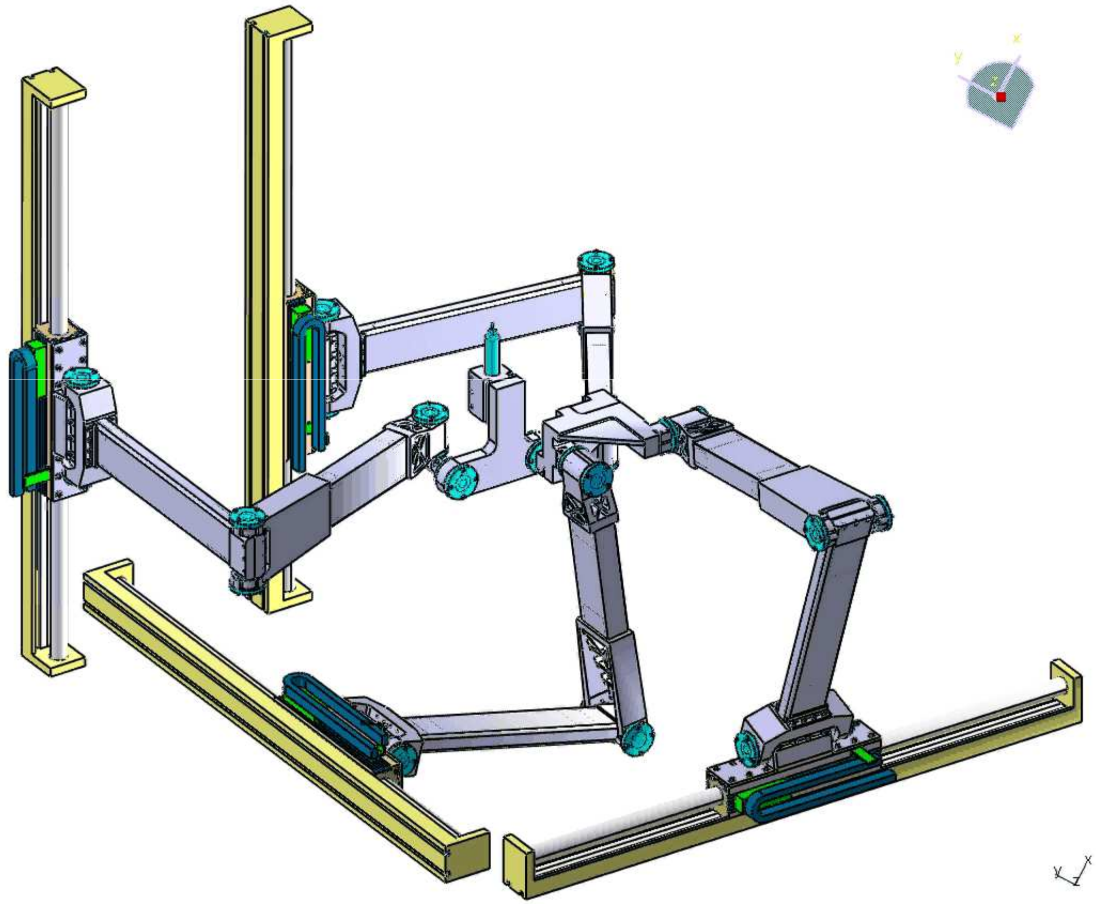
Example – Isoglide4-T3R1 v3.1





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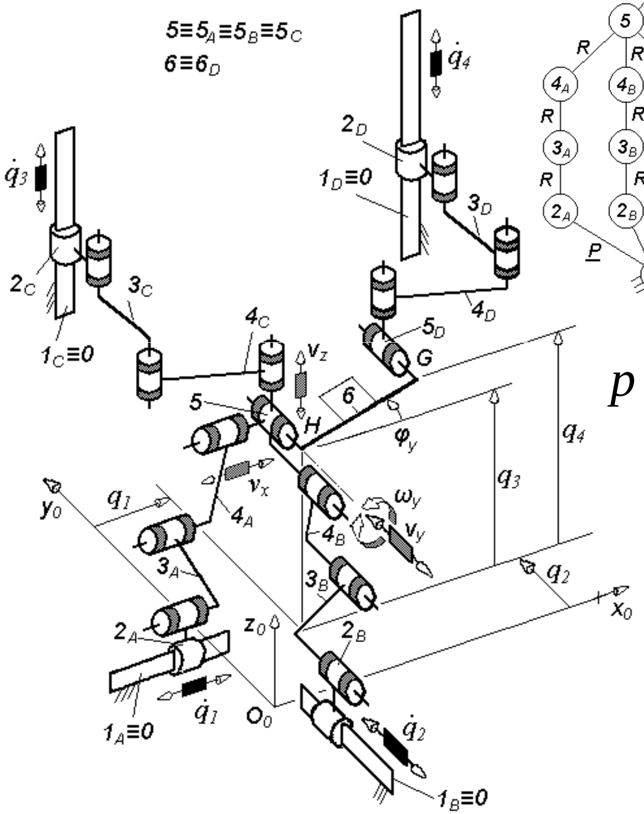
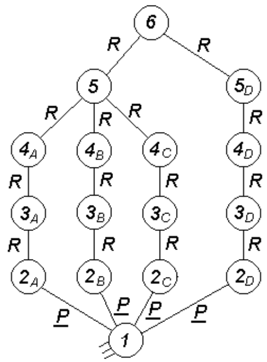
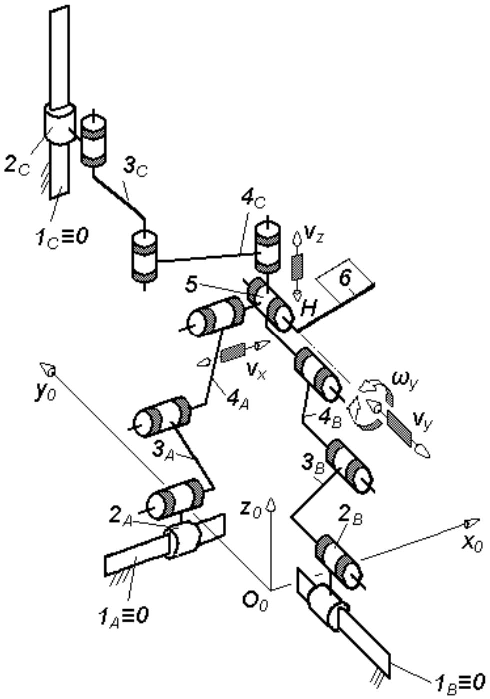
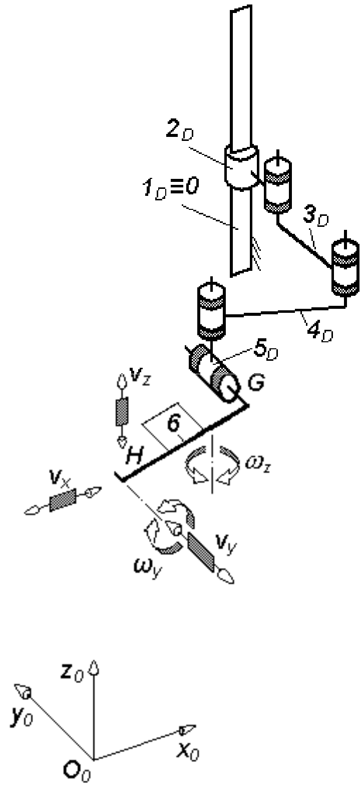
Example – Isoglide4-T3R1 v3.1





$$p = \sum_{i=1}^{k} p_{Gi}$$





$(R_{G1}) = (v_x, v_y, v_z, \omega_y)$ $(R_{G2}) = (v_x, v_y, v_z, \omega_y, \omega_z)$

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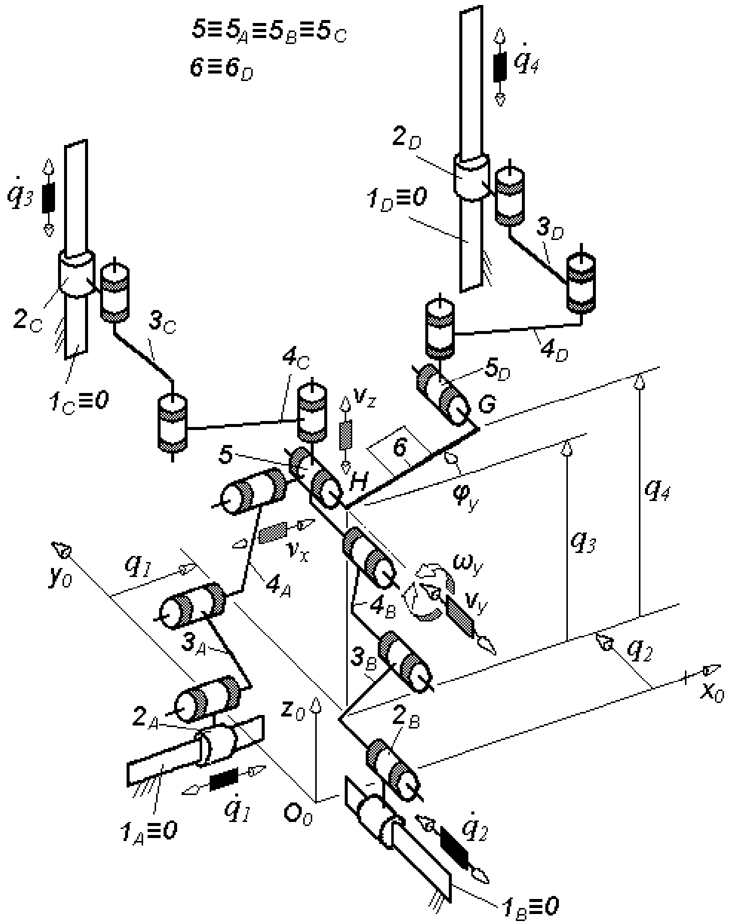
Isoglide4-T3R1 v3.1



CGK: $M = \sum_{i=1}^{p} f_i - \sum_{k=1}^{q} b_k = 18 - ? = ?$?!

$(R_{G1}) = (v_x, v_y, v_z, \omega_y)$

$(R_{G2}) = (v_x, v_y, v_z, \omega_y, \omega_z)$

$S_{G1} = 4$, $S_{G2} = 5$

$S_F = dim(R_{G1} \cap R_{G1}) = 4$

$$r = \sum_{i=1}^{k} S_{Gi} - S_F + r_l = 4+5-4+9 = 14$$

$$M = \sum_{i=1}^{m} f_i - r = 18-14 = 4$$

$$N = 6q - r = 6*3-14 = 4$$

$T = M - S_F = 0$

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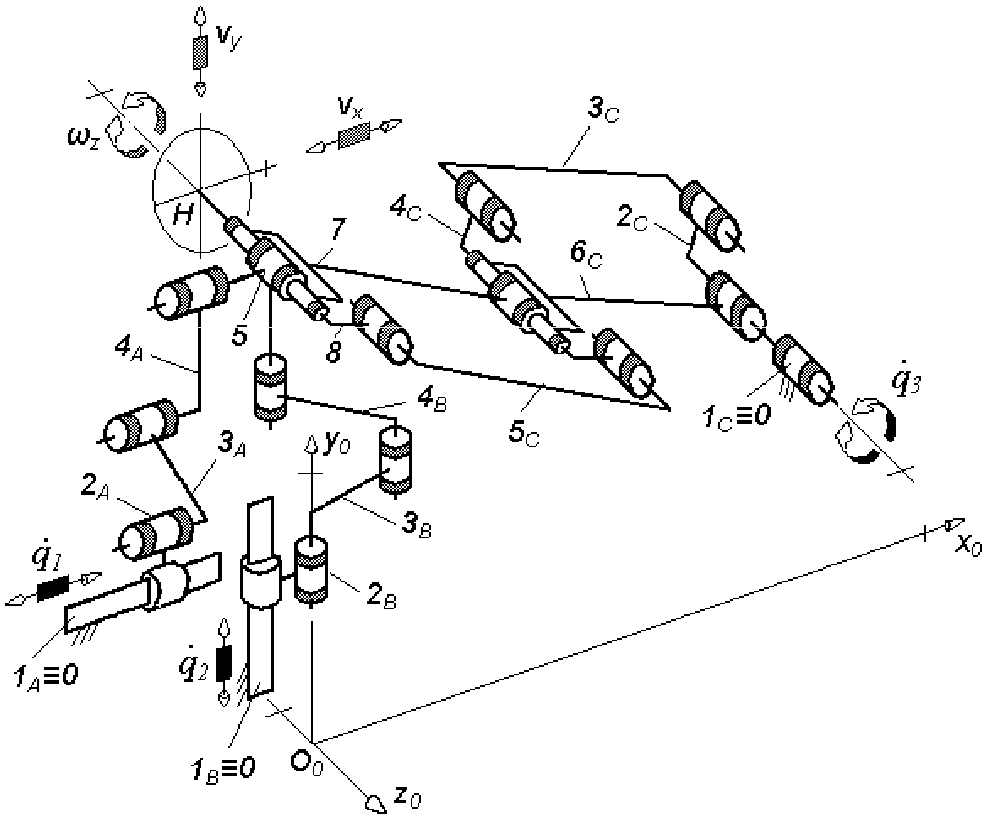
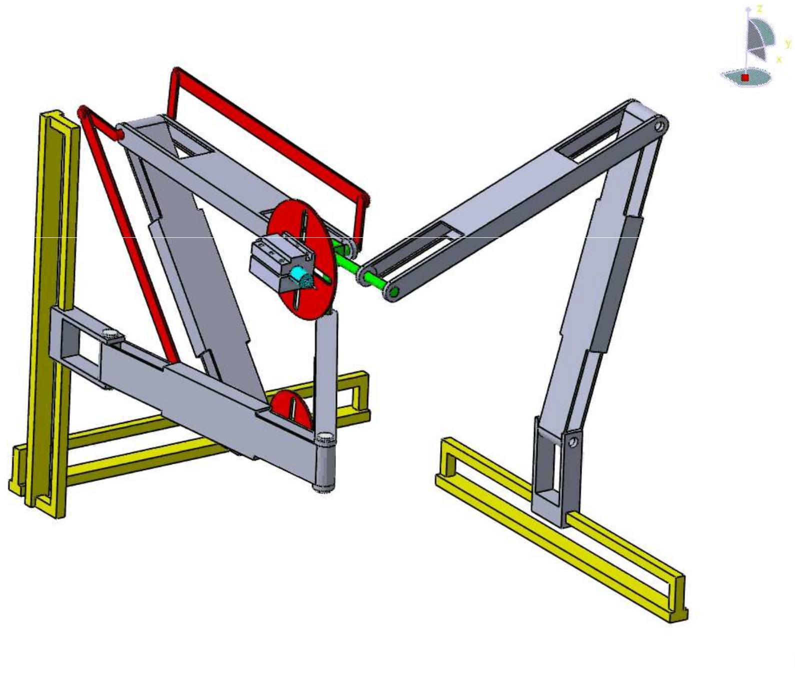
Isoglide3-T2R1

$$p = \sum_{i=1}^{k} p_{Gi}$$





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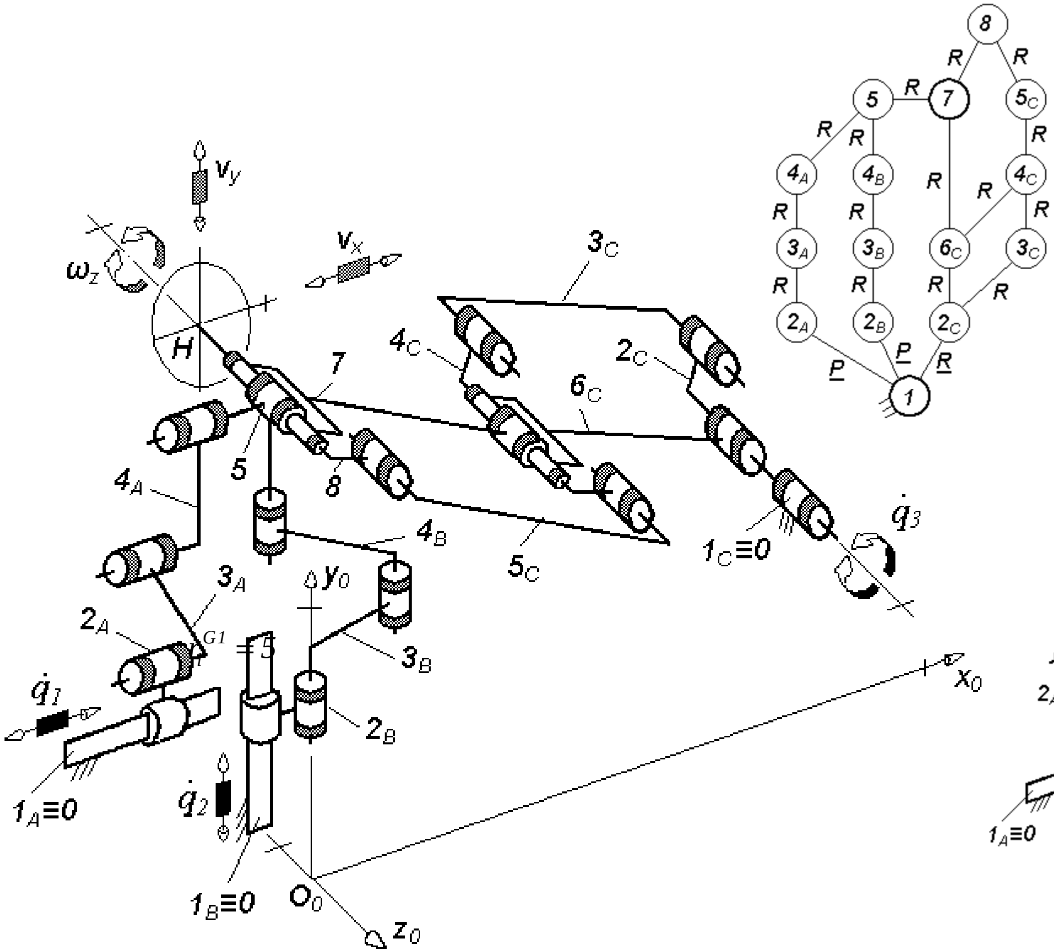
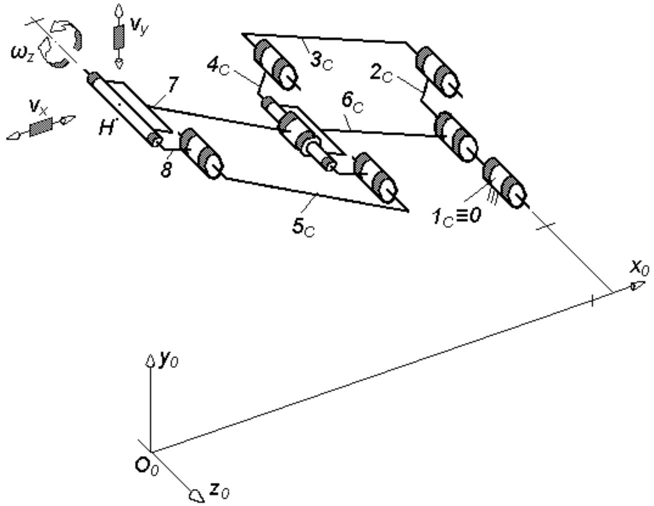
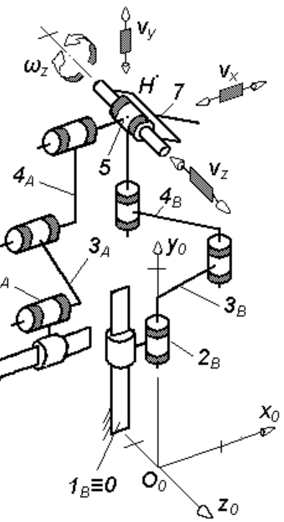
Isoglide3-T2R1







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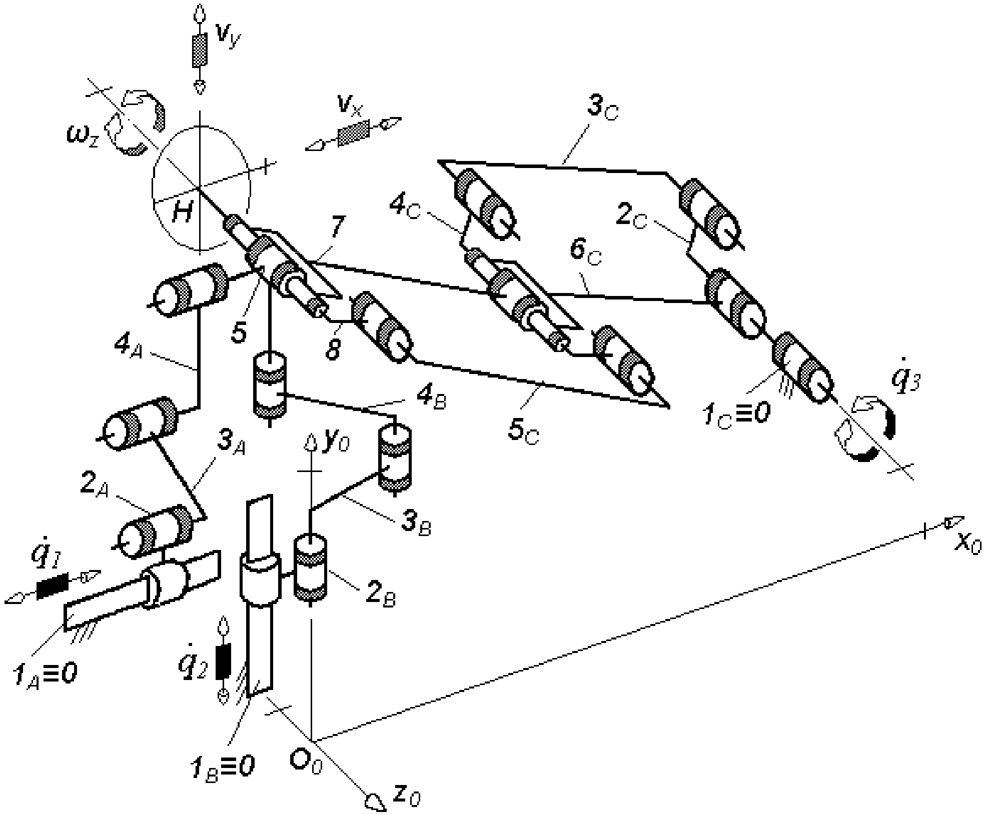
Isoglide4-T3R1 v3.1

CGK: $$M = \sum_{i=1}^{p} f_i - \sum_{k=1}^{q} b_k = 18 - ? = ?$$?!



$(R_{G1}) = (v_x, v_y, v_z, \omega_y)$

$(R_{G2}) = (v_x, v_y, \omega_z)$

$S_{G1} = 4$, $S_{G2} = 3$

$S_F = dim(R_{G1} \cap R_{G1}) = 3$

$r_l^{G1} = 5,\ r_l^{G2} = 6$

$$r = \sum_{i=1}^{k} S_{Gi} - S_F + r_l = 4+3-3+11 = 15$$

$$M = \sum_{i=1}^{m} f_i - r = 18-15 = 3$$

$N = 6q - r = 6*4-15 = 9$

$T = M - S_F = 0$

Constraint singularities

- The term of constraint singularity have been recently coined by Zlatanov et al. (2002a) to characterize the configuration of lower mobility parallel manipulators in which both the connectivity of the moving platform and the mobility of the parallel mechanism increase their instantaneous values.

- This type of singularity was initially identified as a configuration space singularity of the 3-UPU robot at SNU - Seoul National University (Han et al. 2002).

- At its home position this translational parallel robot exhibits finite motions even with all active prismatic joints locked.

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Constraint singularities

Tools used in the analysis of the constraint singularities:

- **the rank and the condition number of the Jacobian matrix of the loop closure equations (Han et al. 2002),**
- **the screw theory (Zlatanov et al., 2002a, 2002b),**
- **the augmented Jacobian matrix (Joshi and Tsai, 2002),**
- **the linear complex approximation (Wolf and Shoham, 2002; Wolf et all. 2002),**
- **Morse function theory and differential forms associated with the constraint functions (Liu at al. 2003).**
- **Theory of linear transformations (Gogu 2008)**

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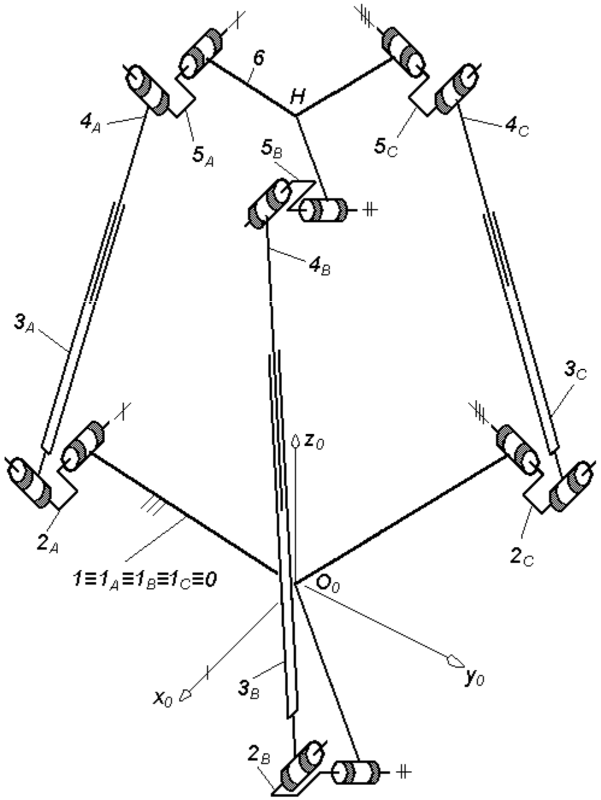
Constraint singularities



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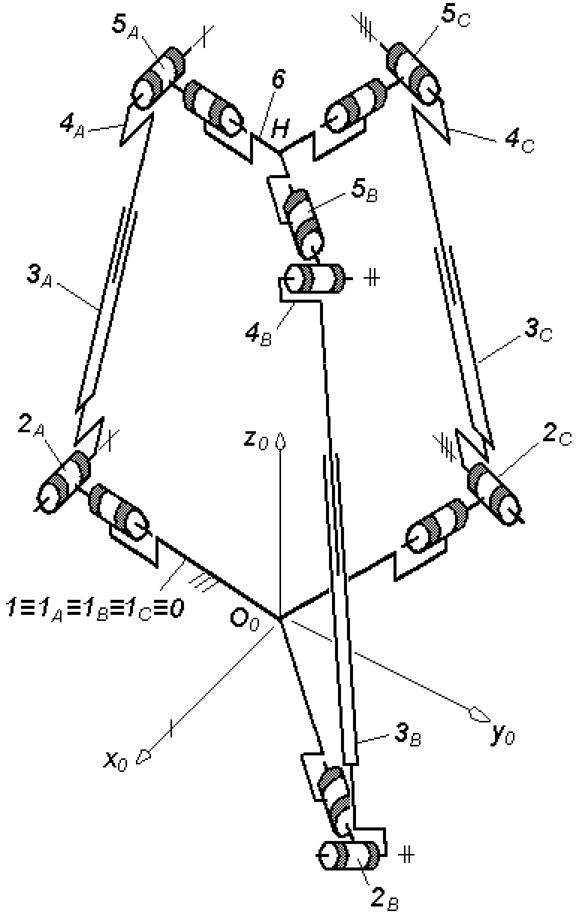

3-UPU parallel mechanisms

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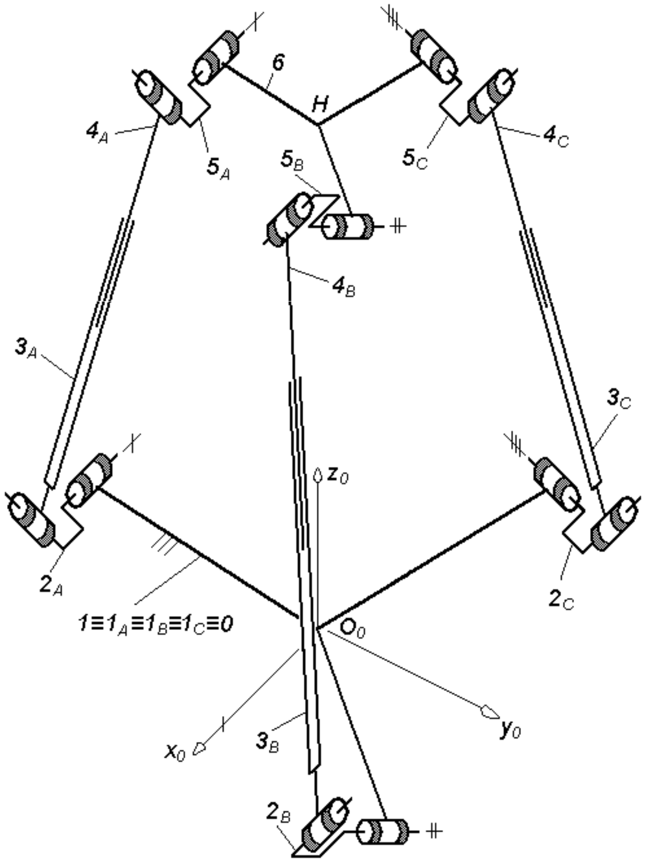
No constraint singularities



$S_{Gi}=5$

$(R_{G1})=(v_x, v_y, v_z, \omega_x, \omega_y)$

$(R_{G2})=(v_x, v_y, v_z, \omega_x, \omega_z)$

$(R_{G3})=(v_x, v_y, v_z, \omega_y, \omega_z)$

$$S_F = dim(R_F) = dim(R_{G1} \cap R_{G2} \cap R_{G3})$$

$S_F=3$ $\qquad r_l = 0$

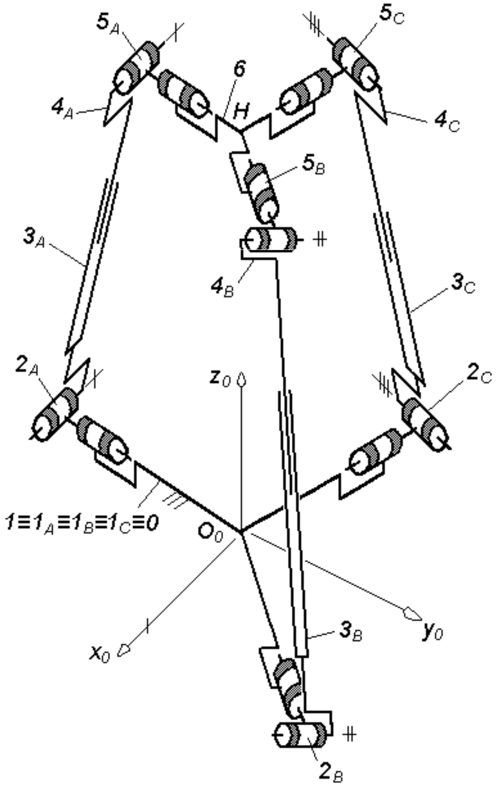
$$r = \sum_{i=1}^{k} S_{Gi} - S_F + r_l = 12$$

$$M = \sum_{i=1}^{p} f_i - r = 3$$

$T=M-S_F=0$ $\qquad N=6q-r=0$

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Constraint singularities



$$({}^iR_{G1})=({}^iR_{G2})=({}^iR_{G3})=(v_x,\ v_y,\ v_z,\ \omega_x,\ \omega_y)$$

$${}^iS_{Gi}=5$$

$${}^iS_F = dim({}^iR_F) = dim({}^iR_{G1} \cap {}^iR_{G2} \cap {}^iR_{G3})$$

$${}^iS_F=5 \quad {}^ir_l = 0$$

$${}^ir = \sum_{i=1}^{k} {}^iS_{Gi} - {}^iS_F + {}^ir_l = 10$$

$${}^iM = \sum_{i=1}^{p} f_i - {}^ir = 5$$

$${}^iT={}^iM-{}^iS_F=0$$

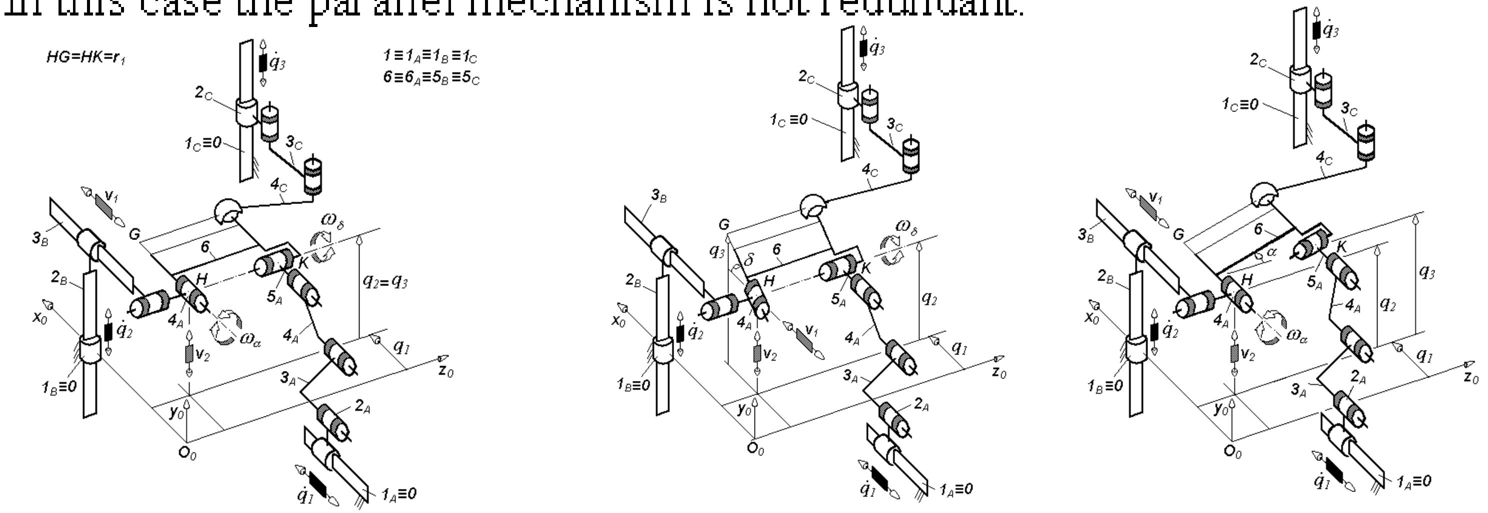
$${}^iN=6q-{}^ir=2$$

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

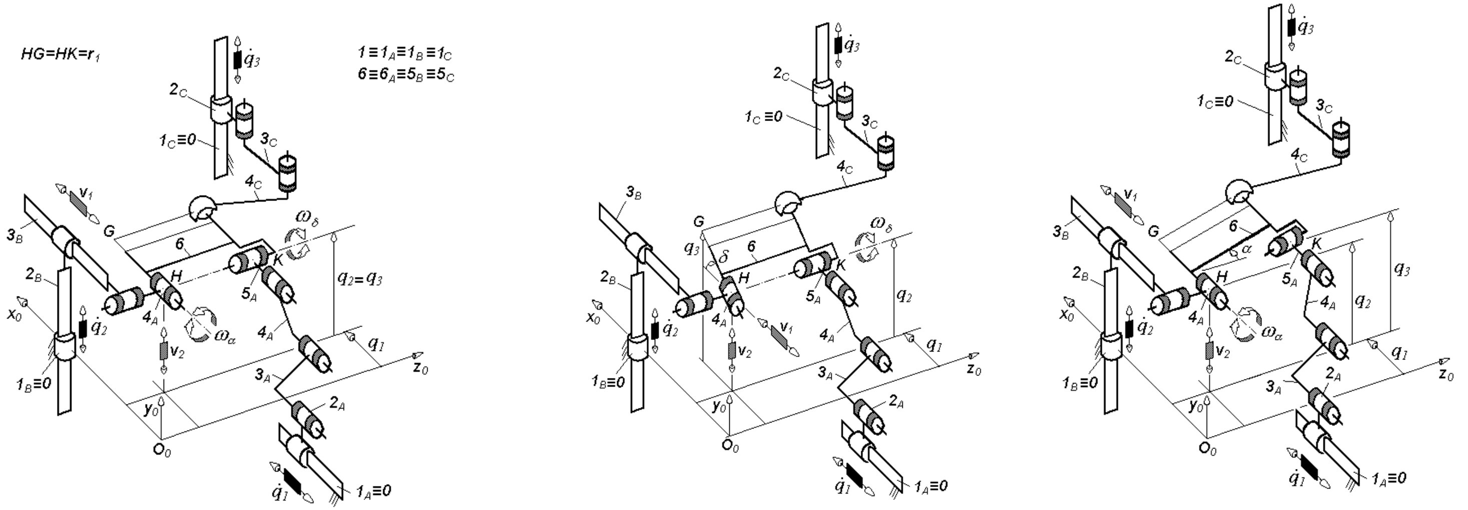
Definition 1 – Bifurcation of type BCS1: A bifurcation of type BCS1 occurs when a parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$, get out from a constraint singularity (CS) in different branches characterized by the same degree of mobility and the same connectivity of the moving platform but with different bases of the vector space of relative velocities between the moving and the fixed platforms.

In this case the parallel mechanism is not redundant.



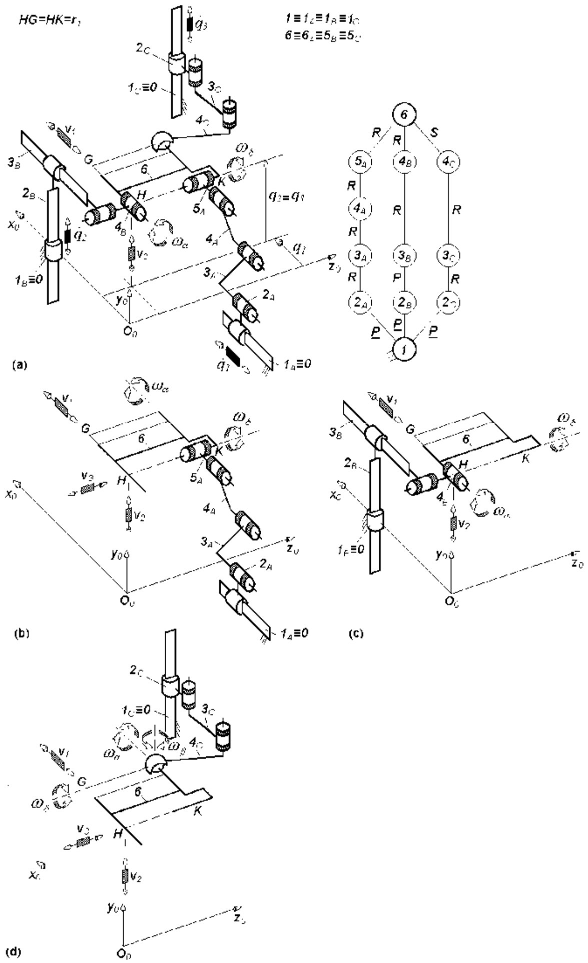
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

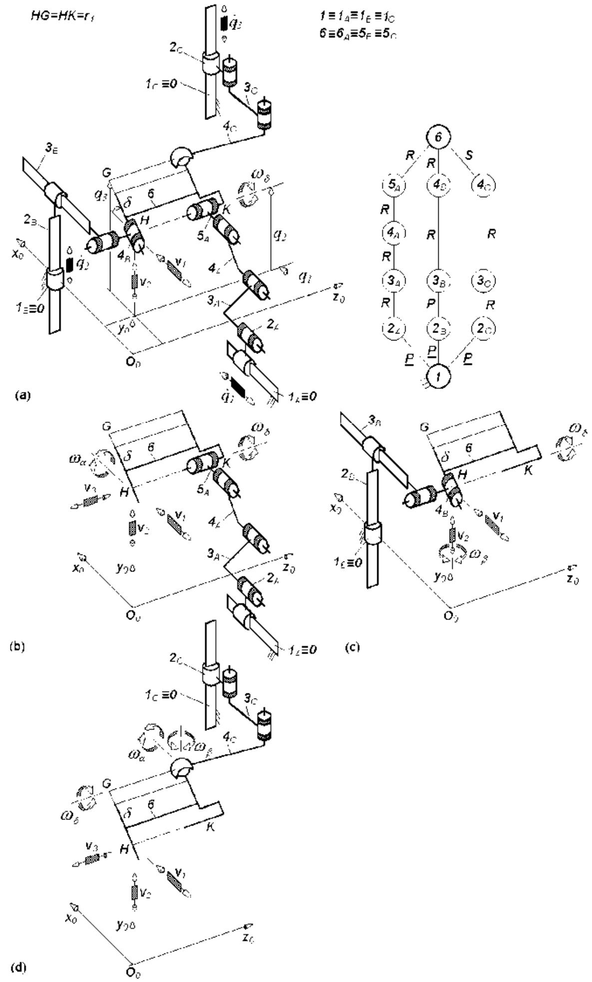
Bifurcation in constraint singularities

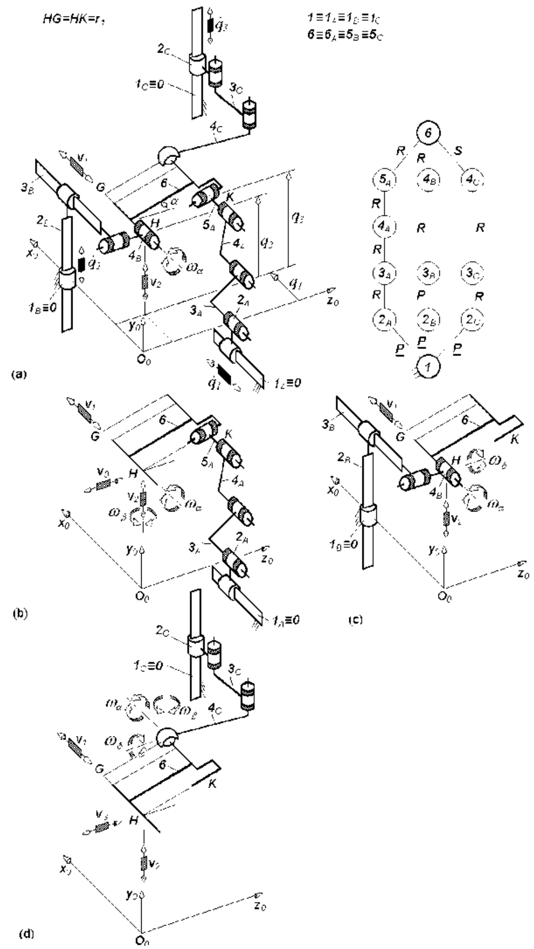


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities





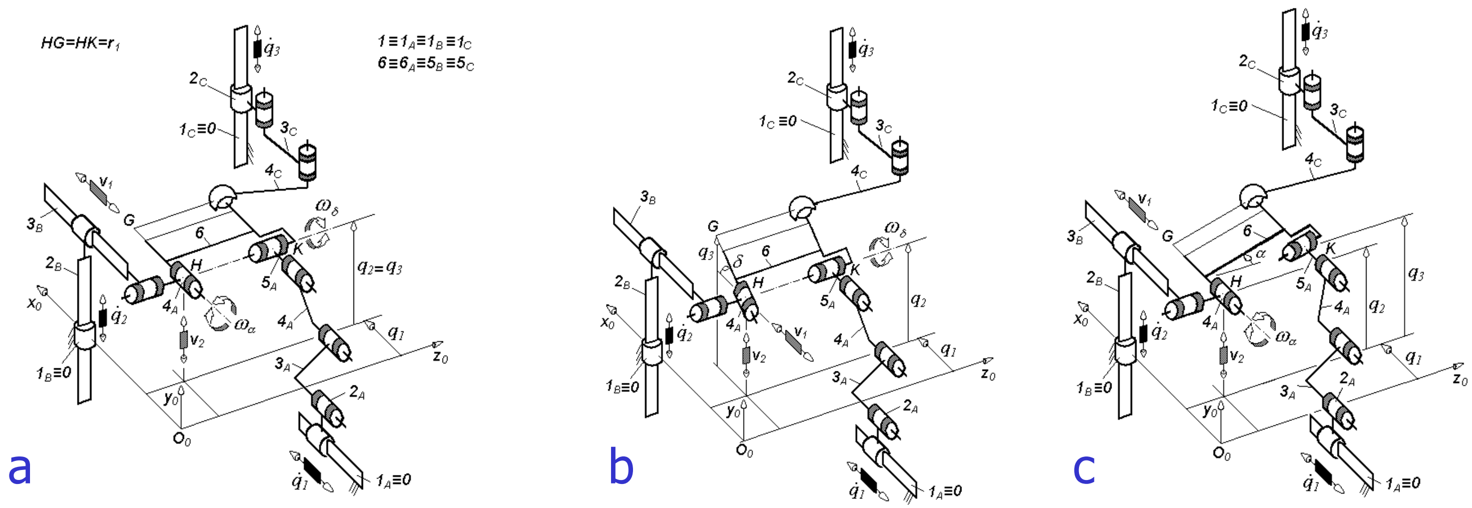


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

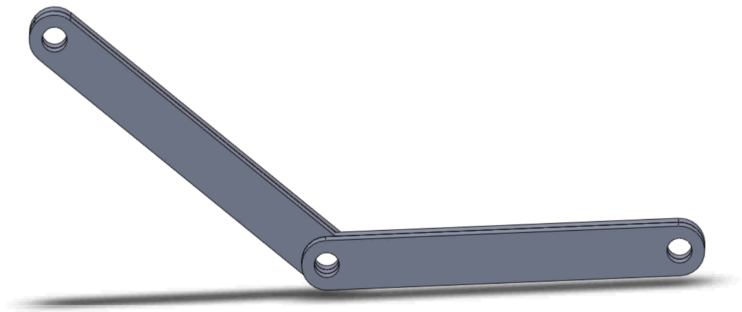
Structural parameters		
Instantaneous parameters in branching singularity in Fig. 4a	Full-cycle parameters in the branch in Fig. 4b	Full-cycle parameters in the branch in Fig. 4c
${}^iS_{G1}=5,\ {}^iS_{G2}=4,\ {}^iS_{G3}=6$ $({}^iR_{G1})=(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\delta)$ $({}^iR_{G2})=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\delta)$ $({}^iR_{G3})=(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\beta,\boldsymbol{\omega}_\delta)$ $({}^iR_F)=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\delta)$ ${}^iS_F=4,\ {}^ir_l=0,\ {}^ir=11$ ${}^iM=4,\ {}^iN=1,\ {}^iT=0$	$S_{G1}=5,\ S_{G2}=4,\ S_{G3}=6$ $(R_{G1})=(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\beta,\boldsymbol{\omega}_\delta)$ $(R_{G3})=$ $(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\beta,\boldsymbol{\omega}_\delta)$ $(R_F)=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $S_F=3,\ r_l=0,\ r=12$ $M=3,\ N=0,\ T=0$	$S_{G1}=5,\ S_{G2}=4,\ S_{G3}=6$ $(R_{G1})=(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\ \boldsymbol{\omega}_\beta)$ $(R_{G2})=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\delta)$ $(R_{G3})=$ $(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\boldsymbol{\omega}_\alpha,\boldsymbol{\omega}_\beta,\boldsymbol{\omega}_\delta)$ $(R_F)=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\alpha)$ $S_F=3,\ r_l=0,\ r=12$ $M=3,\ N=0\ T=0$

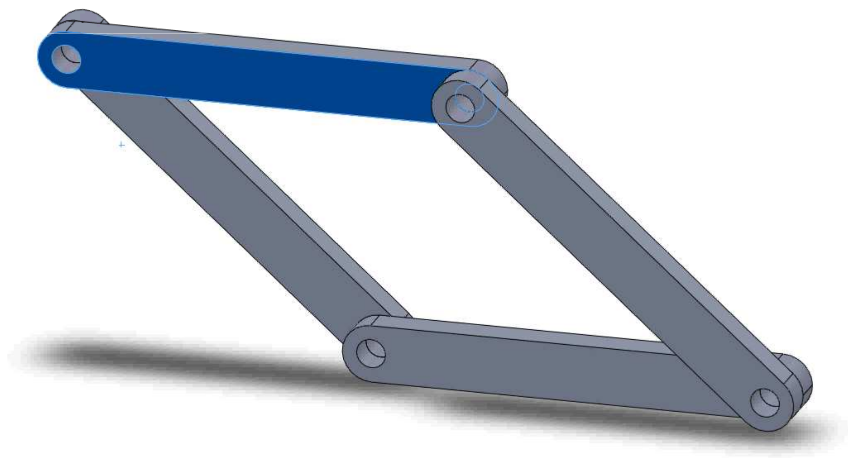
Fig. 4



Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

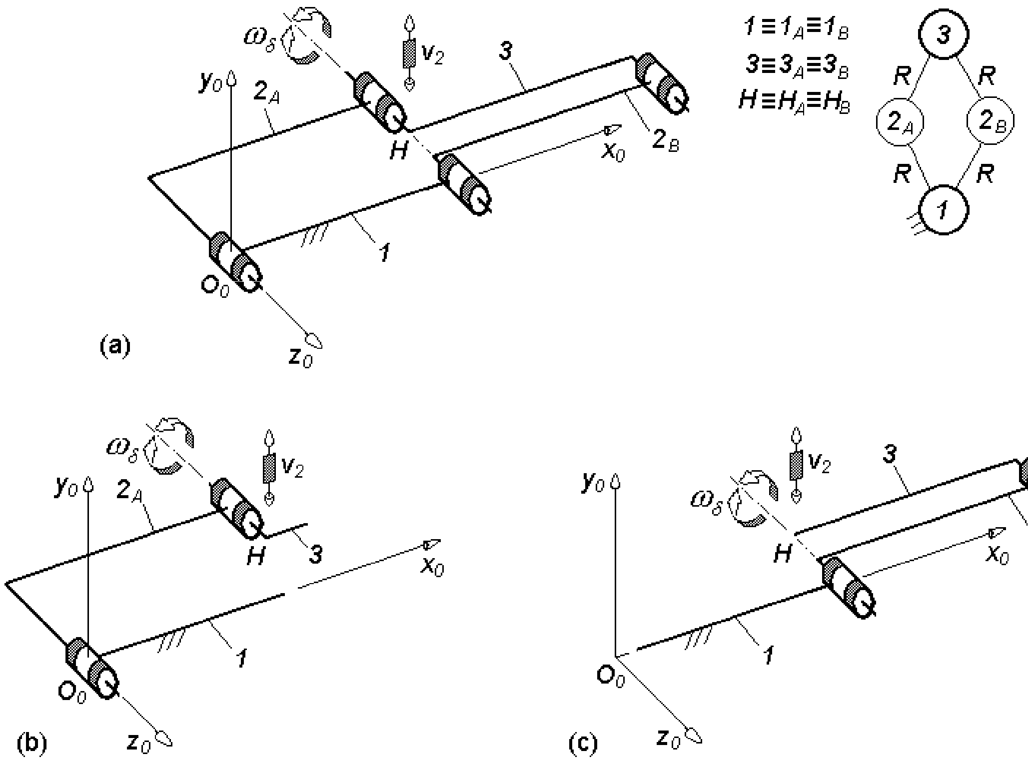
Bifurcation in constraint singularities





Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

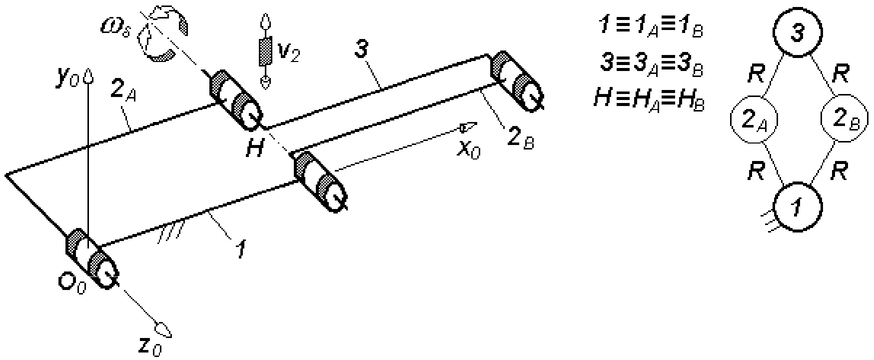


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

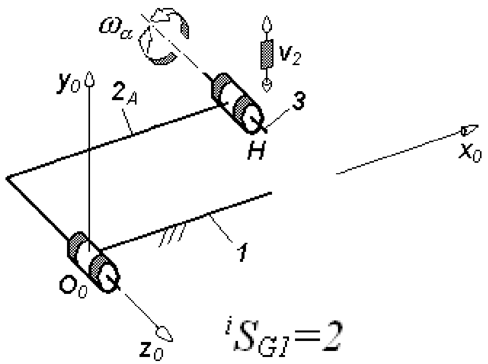
$^{i}S_F=2$

$(^{i}R_F)=(\boldsymbol{v}_2,\ \boldsymbol{\omega}_\delta)$



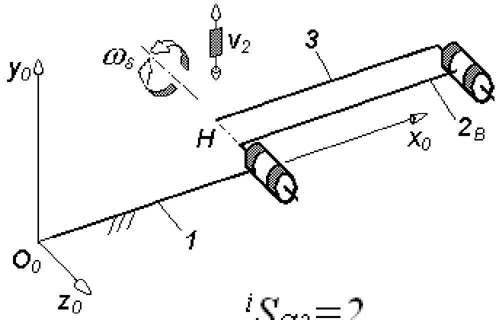
$^{i}r_l=0,\ ^{i}r=2$

$^{i}M=2,\ ^{i}N=4,\ \ ^{i}T=0$



$^{i}S_{G1}=2$

$(^{i}R_{G1})=(\boldsymbol{v}_2,\ \boldsymbol{\omega}_\delta)$



$^{i}S_{G2}=2$

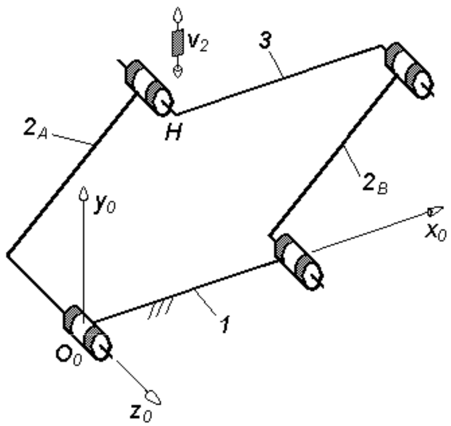
$(^{i}R_{G2})=(\boldsymbol{v}_2,\ \boldsymbol{\omega}_\delta)$

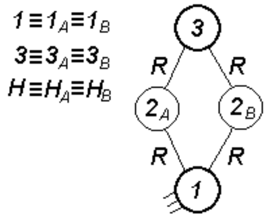
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

$S_F=1$

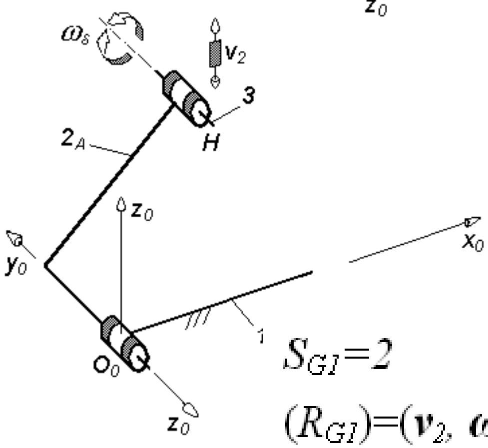
$(R_F)=(\mathbf{v}_2)$





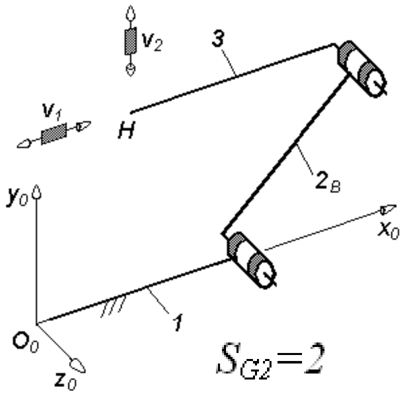
$r_l=0,\ r=3$

$M=1,\ N=3,\ T=0$



$S_{G1}=2$

$(R_{G1})=(\mathbf{v}_2,\ \omega_\delta)$



$S_{G2}=2$

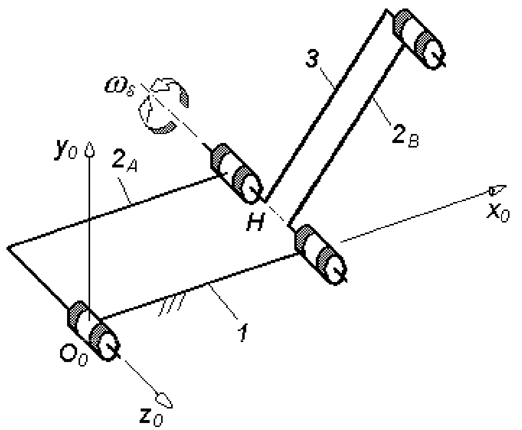
$(R_{G2})=(\mathbf{v}_1,\ \mathbf{v}_2)$

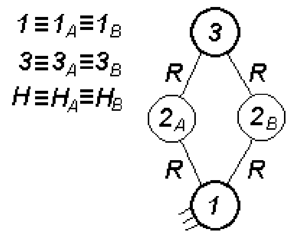
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

$S_F=1$

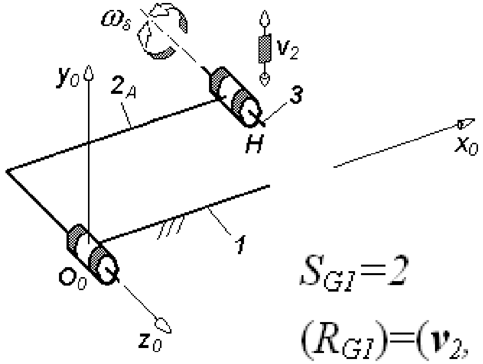
$(R_F)=(\omega_\delta)$





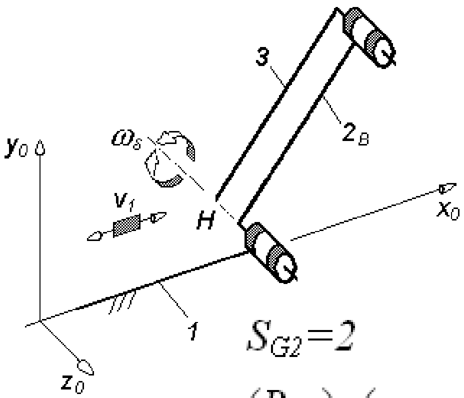
$r_l=0,\ r=3$

$M=1,\ N=3,\ T=0$



$S_{G1}=2$

$(R_{G1})=(\boldsymbol{v}_2,\ \boldsymbol{\omega}_\delta)$



$S_{G2}=2$

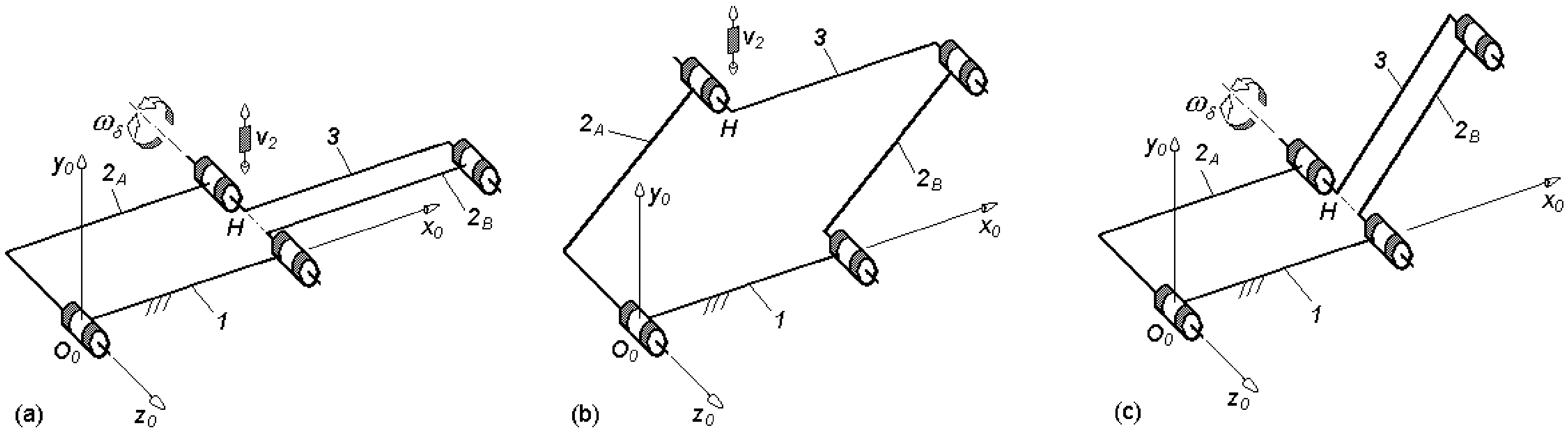
$(R_{G2})=(\boldsymbol{v}_1,\ \boldsymbol{\omega}_\delta)$

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Bifurcation in constraint singularities

Fig. 5

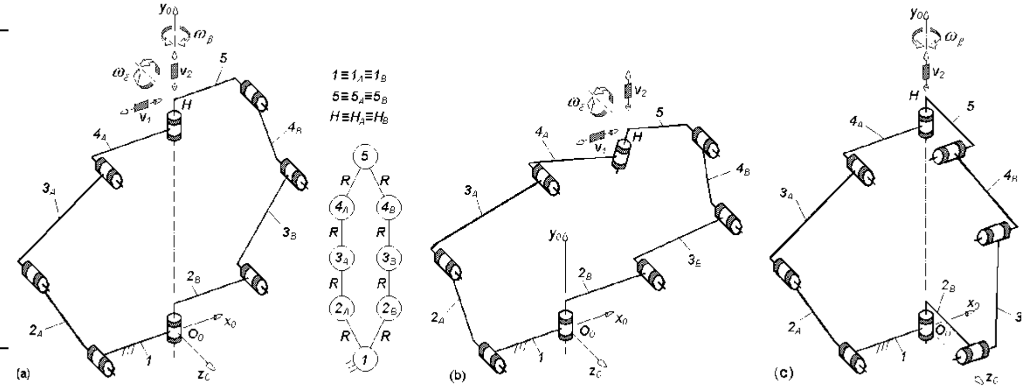
Structural parameters		
Instantaneous parameters in branching singularity in Fig. 5a	Full-cycle parameters in the branch in Fig. 5b	Full-cycle parameters in the branch in Fig. 5.c
${}^iS_{G1}={}^iS_{G2}=2$ $({}^iR_{G1})=({}^iR_{G2})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $({}^iR_F)=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ ${}^iS_F=2$, ${}^ir_l=0$, ${}^ir=2$ ${}^iM=2$, ${}^iN=4$, ${}^iT=0$	$S_{G1}=S_{G2}=2$ $(R_{G1})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1,\mathbf{v}_2)$ $(R_F)=(\mathbf{v}_2)$ $S_F=1$, $r_l=0$, $r=3$ $M=1$, $N=3$, $T=0$	$S_{G1}=S_{G2}=2$ $(R_{G1})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1,\boldsymbol{\omega}_\delta)$ $(R_F)=(\boldsymbol{\omega}_\delta)$ $S_F=1$, $r_l=0$, $r=3$ $M=1$, $N=3$, $T=0$



Bifurcation in constraint singularities

Definition 2 – Bifurcation of type BCS2: A bifurcation of type BCS2 occurs when a parallel mechanism $F \leftarrow G_1\text{-}G_2\text{-}\ldots G_k$, get out from a constraint singularity in different branches characterized by distinct values of mobility and connectivity of the moving platform.

In this case, the parallel mechanism is kinematotropic.



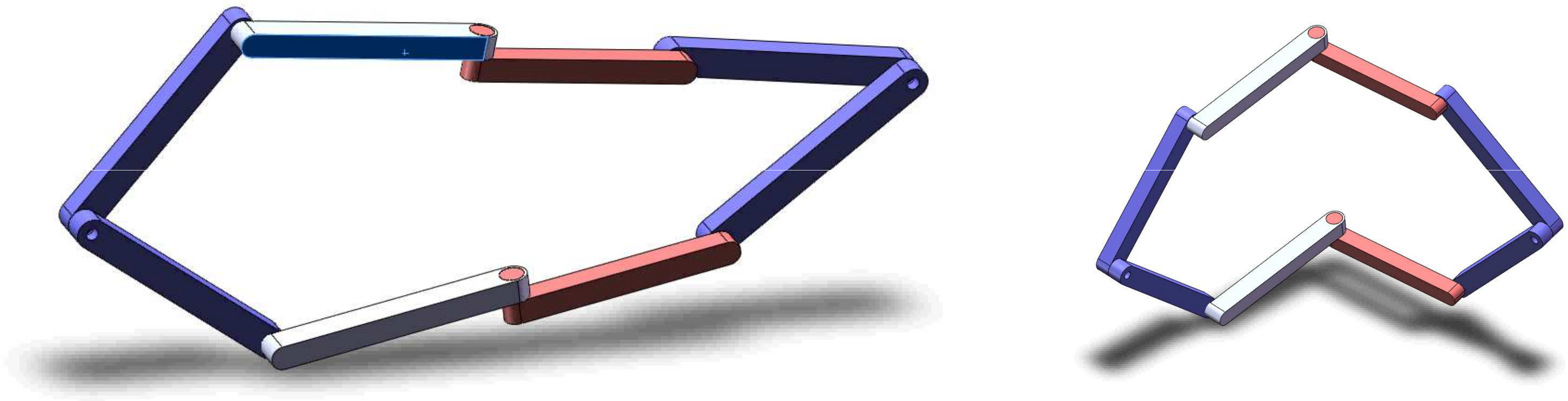
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Branching singularities in kinematotropic parallel mechanisms

- The term kinematotropic was coined by K. Wohlhart (1996) to define the linkages that permanently change their full-cycle mobility when passing by an instantaneous singularity from one branch to another.
- A branch refers to a free-of-singularity configuration of the mechanism in which each structural parameter keeps the same value for the full-cycle of the same branch.
- The singularity transitory phase when passing from one branch to another is called a branching singularity (BS).
- Various single and multi-loop kinematotropic mechanisms have been presented in the literature (Wohlhart 1996, Galletti and Fanghella 2001, Fanghella et al. 2006).

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Branching singularities in kinematotropic parallel mechanisms



Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Fig. 1

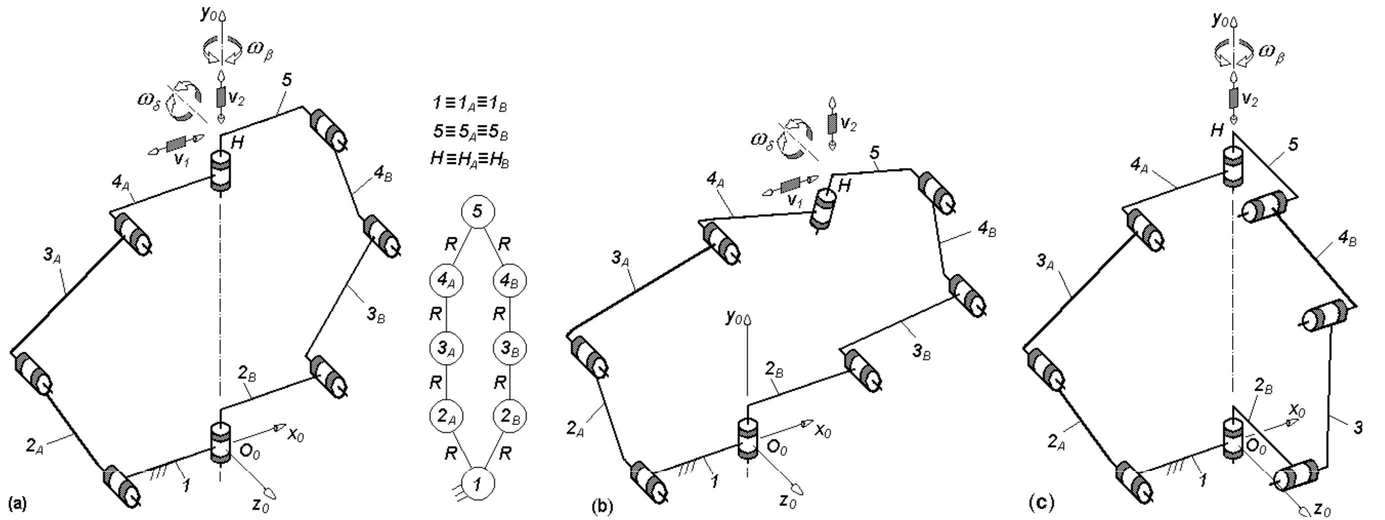
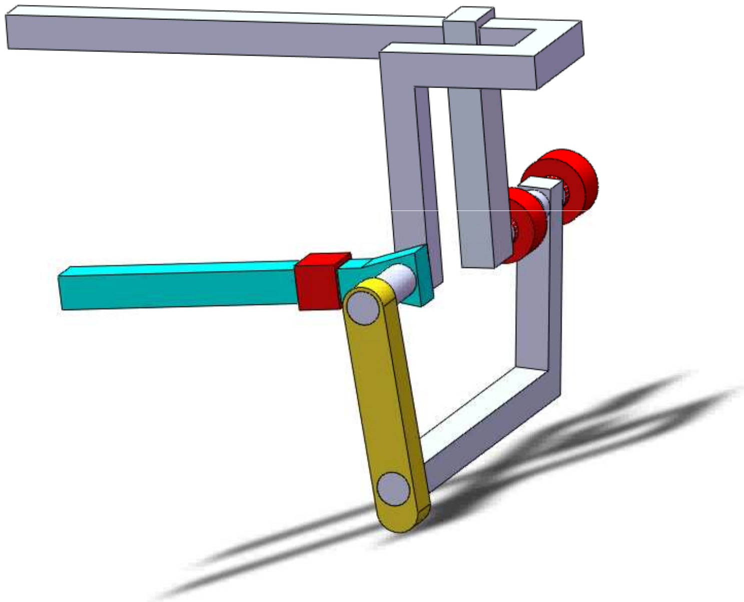


Fig.	Type of BS	Structural parameters		
		Instantaneous parameters in branching singularity in Fig. 1a	Full-cycle parameters in the branch in Fig. 1b	Full-cycle parameters in the branch in Fig. 1c
Fig. 1	BS-A	${}^iS_{G1}={}^iS_{G2}=4$ $({}^iR_{G1})=({}^iR_{G2})=$ $(\mathbf{v}_1, \mathbf{v}_2, \omega_\beta, \omega_\delta)$ $({}^iR_F)=(\mathbf{v}_1, \mathbf{v}_2, \omega_\beta, \omega_\delta)$ ${}^iS_F=4, {}^ir_l=0, {}^ir=4$ ${}^iM=4, {}^iN=2, {}^iT=0$	$S_{G1}=S_{G2}=4$ $(R_{G1})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\alpha, \omega_\delta)$ $(R_{G2})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\beta, \omega_\delta)$ $(R_F)=(\mathbf{v}_1, \mathbf{v}_2, \omega_\delta)$ $S_F=3, r_l=0, r=5$ $M=3, N=1, T=0$	$S_{G1}=S_{G2}=4$ $(R_{G1})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\beta, \omega_\delta)$ $(R_{G2})=(\mathbf{v}_2, \mathbf{v}_3, \omega_\alpha, \omega_\beta)$ $(R_F)=(\mathbf{v}_2, \omega_\beta)$ $S_F=2, r_l=0, r=6$ $M=2, N=0, T=0$

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Branching singularities in kinematotropic parallel mechanisms



Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Fig. 2

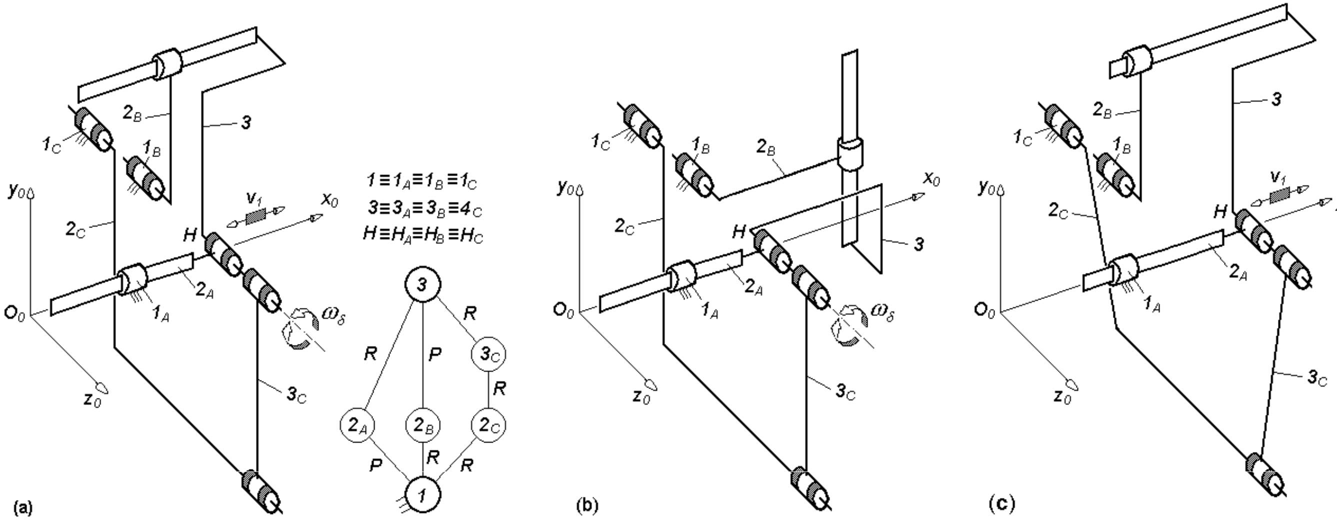
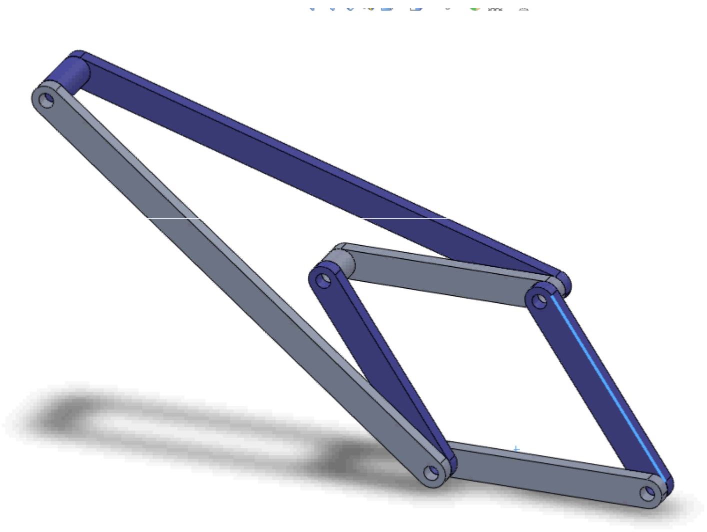
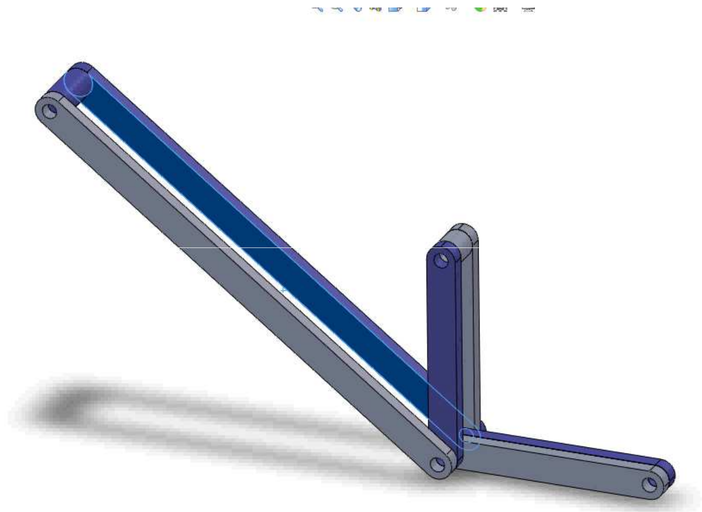


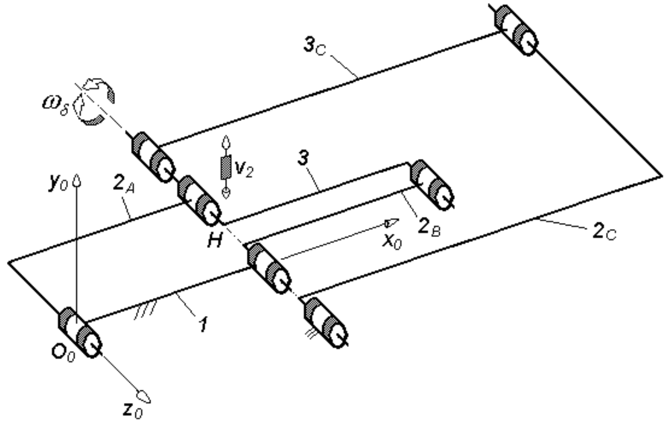
Fig.	Type of BS	Structural parameters		
		Instantaneous parameters in branching singularity in Fig. 2a	Full-cycle parameters in the branch in Fig. 2b	Full-cycle parameters in the branch in Fig. 2c
Fig. 2	BS-B	${}^iS_{G1}={}^iS_{G2}={}^iS_{G3}=2$, $({}^iR_{G1})=({}^iR_{G2})=({}^iR_{G3})=$ $(\mathbf{v}_1, \boldsymbol{\omega}_\delta)$ $({}^iR_F)=(\mathbf{v}_1, \boldsymbol{\omega}_\delta)$ ${}^iS_F=2$, ${}^ir_l=0$, ${}^ir=4$ ${}^iM=3$, ${}^iN=8$, ${}^iT=1$	$S_{G1}=S_{G2}=S_{G3}=2$ $(R_{G1})=(R_{G3})=(\mathbf{v}_1, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_2, \boldsymbol{\omega}_\delta)$ $(R_F)=(\boldsymbol{\omega}_\delta)$ $S_F=1$, $r_l=0$, $r=5$ $M=2$, $N=7$, $T=1$	$S_{G1}=S_{G2}=2$, $S_{G3}=3$ $(R_{G1})=(\mathbf{v}_1, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1, \mathbf{v}_2)$, $(R_{G3})=$ $(\mathbf{v}_1, \mathbf{v}_2, \boldsymbol{\omega}_\delta)$, $(R_F)=(\mathbf{v}_1)$ $S_F=1$, $r_l=0$, $r=6$ $M=1$, $N=6$, $T=0$

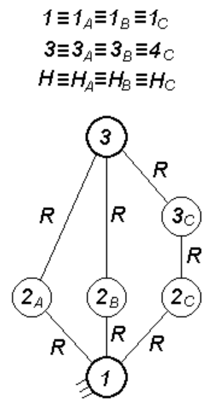
Branching singularities in kinematotropic parallel mechanisms

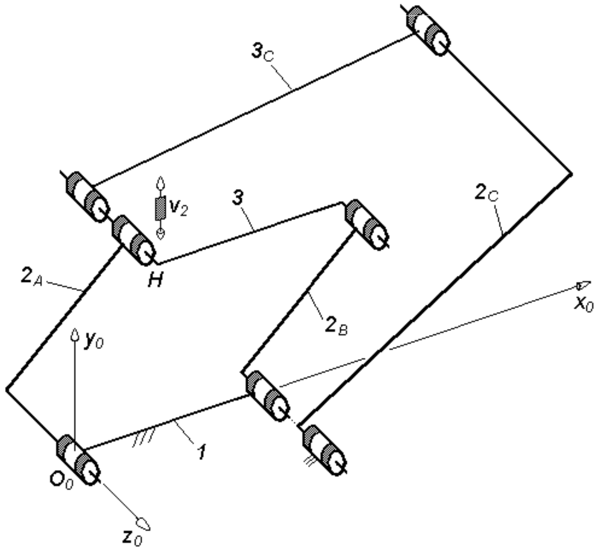


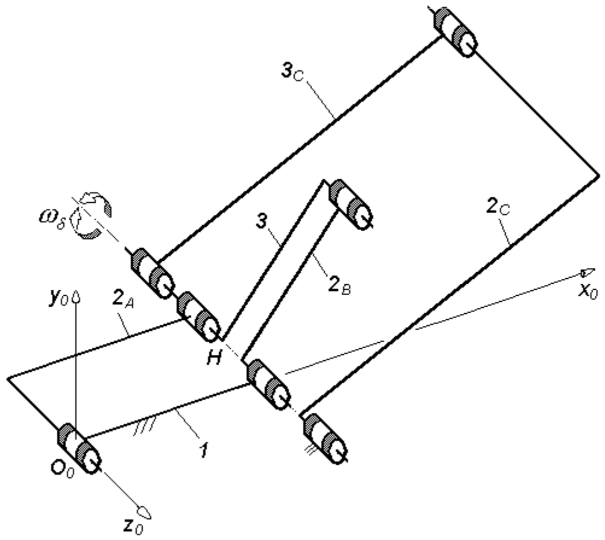


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

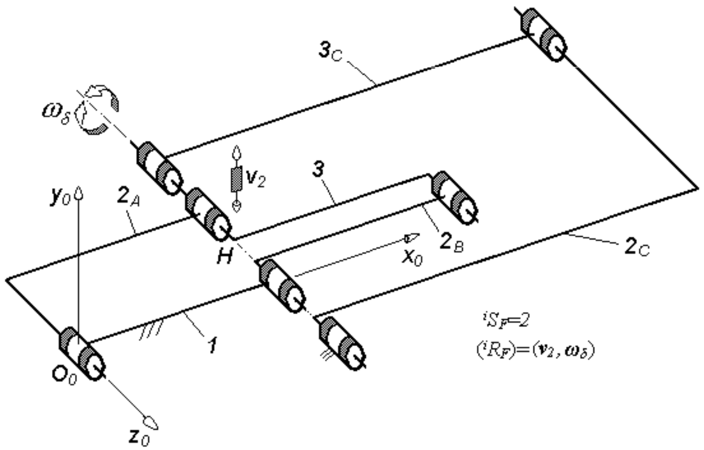


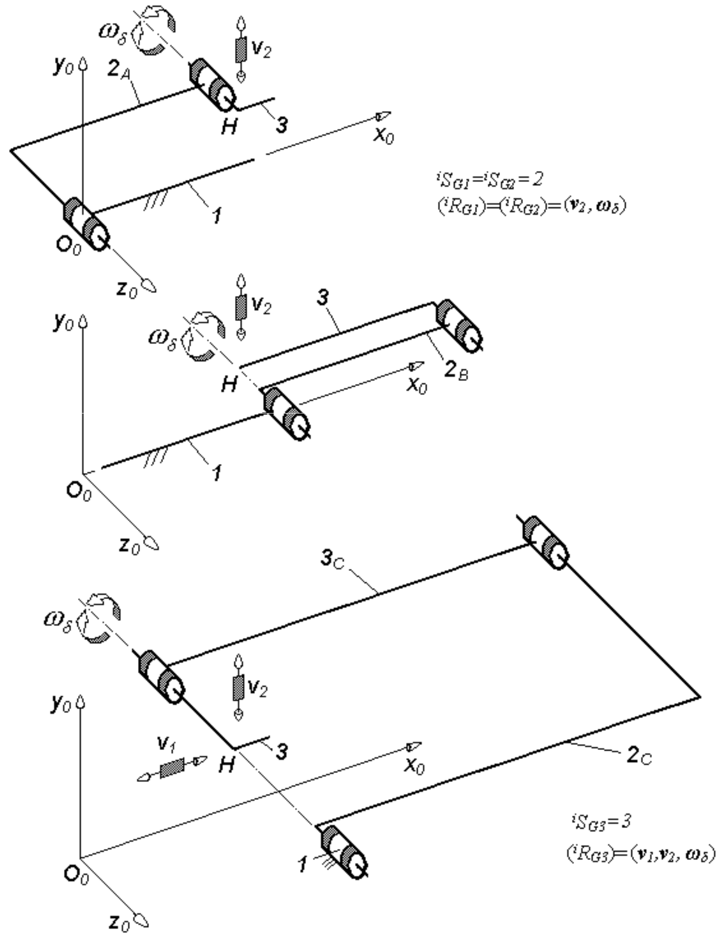




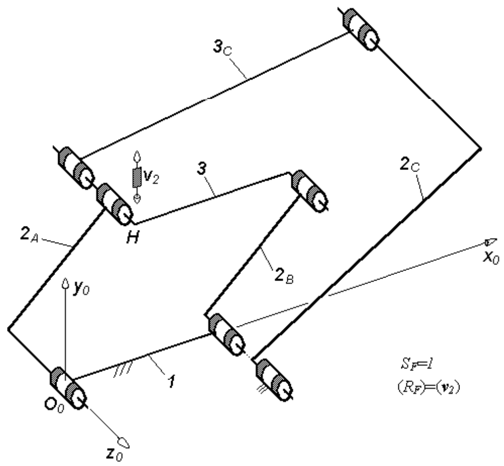


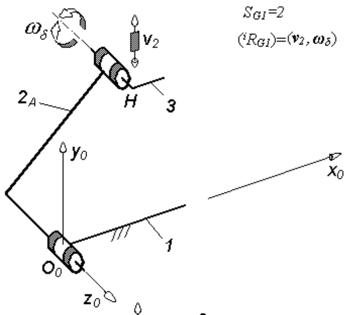
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

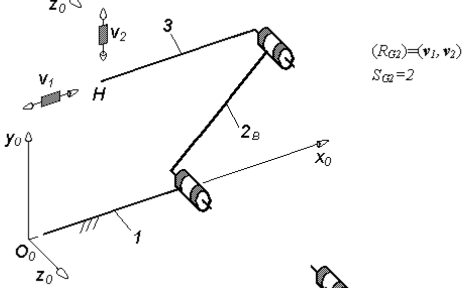


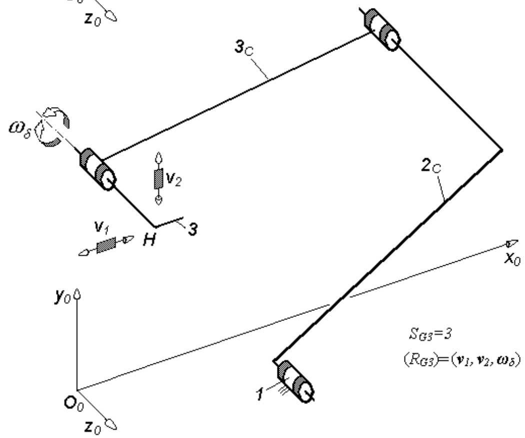


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

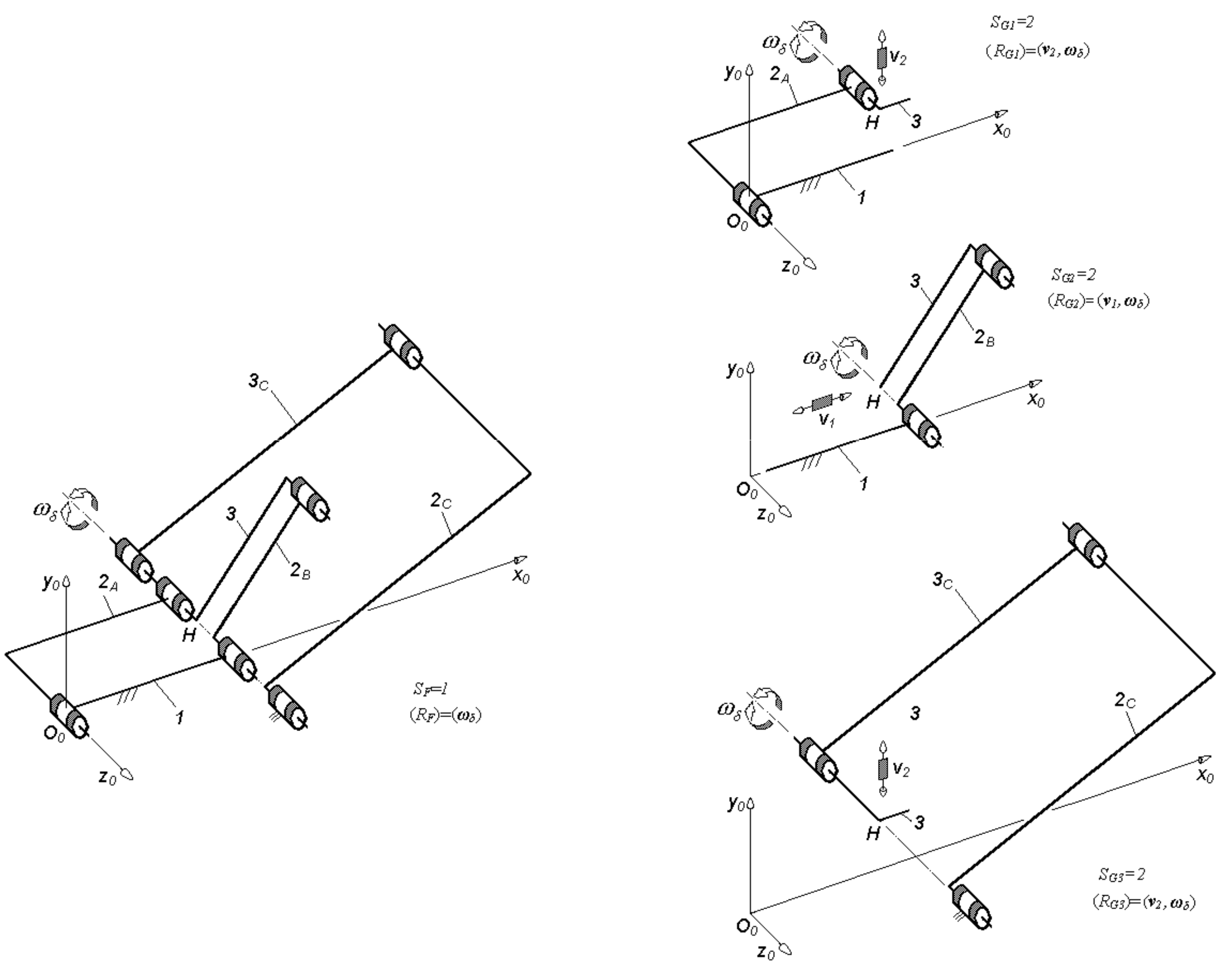








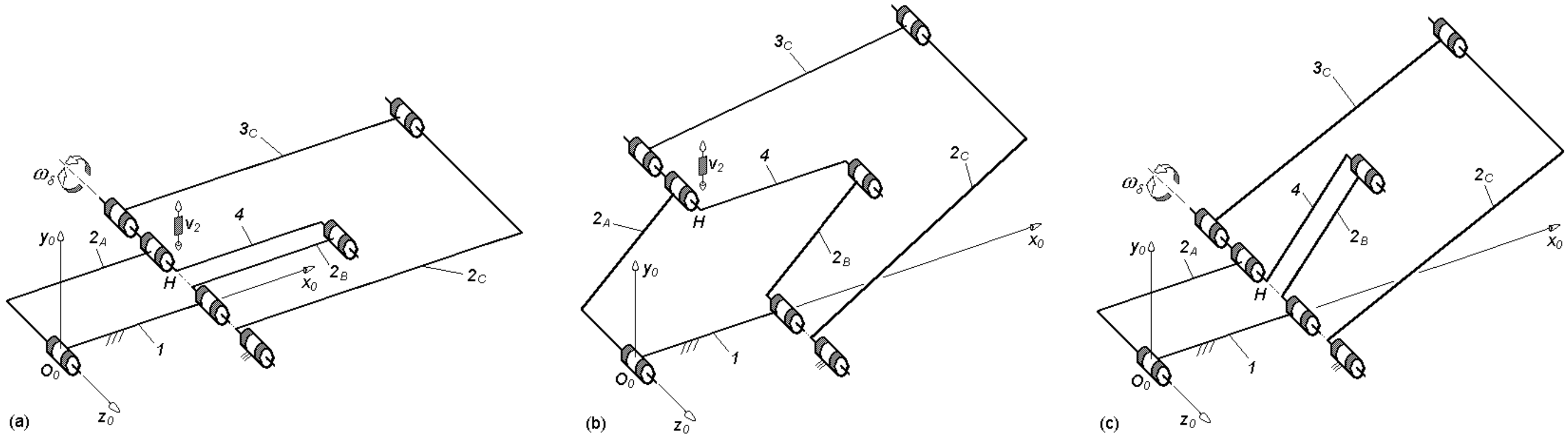
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy



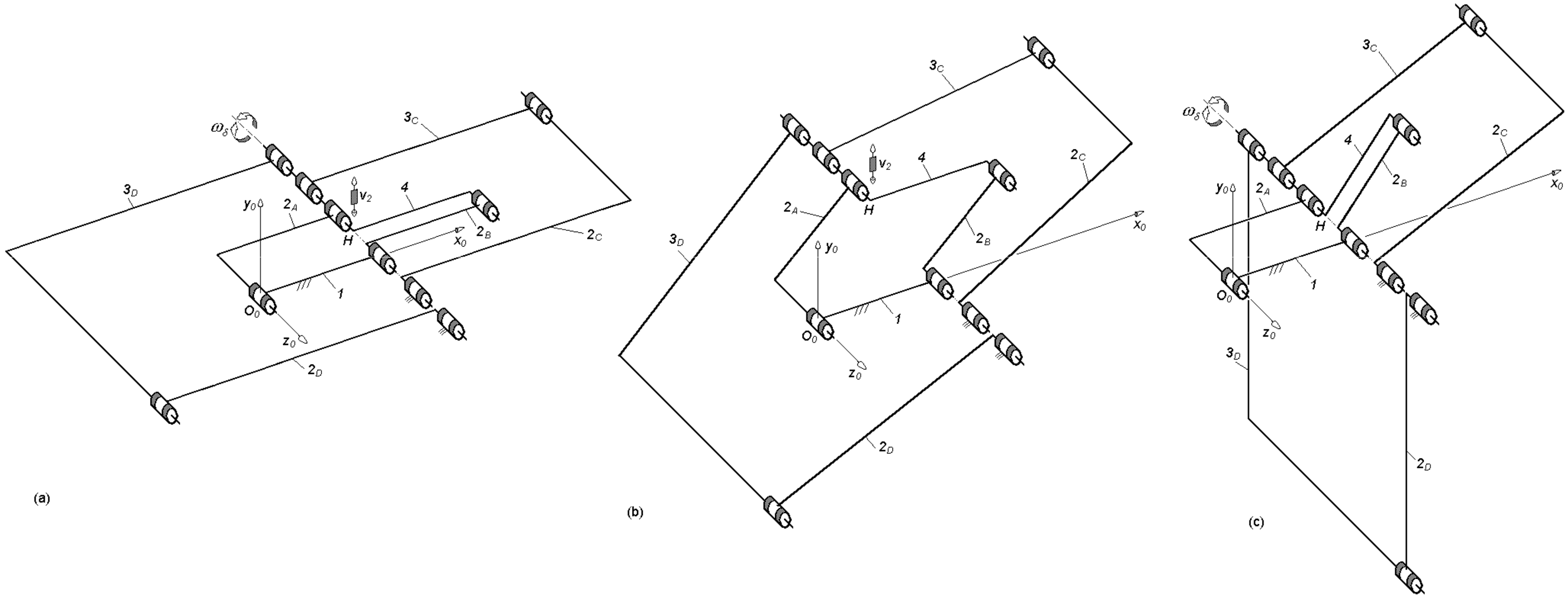
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Structural parameters		
Instantaneous parameters in branching singularity in Fig. 8a	Full-cycle parameters in the branch in Fig. 8b	Full-cycle parameters in the branch in Fig. 8c
${}^{i}S_{G1}={}^{i}S_{G2}={}^{i}S_{G3}=2$, $({}^{i}R_{G1})=({}^{i}R_{G2})=({}^{i}R_{G3})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $({}^{i}R_F)=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ ${}^{i}S_F=2$, ${}^{i}r_l=0$, ${}^{i}r=4$ ${}^{i}M=3$, ${}^{i}N=8$, ${}^{i}T=1$	$S_{G1}=S_{G2}=2$, $S_{G3}=3$ $(R_{G1})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1,\mathbf{v}_2)$, $(R_{G3})=(\mathbf{v}_1,\mathbf{v}_2,\boldsymbol{\omega}_\delta)$, $(R_F)=(\mathbf{v}_2)$ $S_F=1$, $r_l=0$, $r=6$ $M=1$, $N=6$, $T=0$	$S_{G1}=S_{G2}=S_{G3}=2$ $(R_{G1})=(R_{G3})=(\mathbf{v}_2,\boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_1,\boldsymbol{\omega}_\delta)$ $(R_F)=(\boldsymbol{\omega}_\delta)$ $S_F=1$, $r_l=0$, $r=5$ $M=2$, $N=7$, $T=1$

Fig. 8

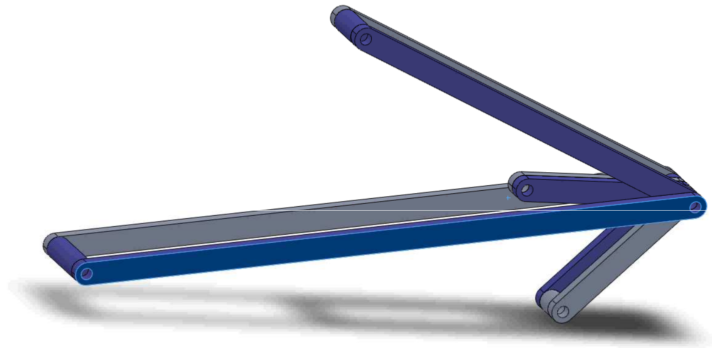


Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy



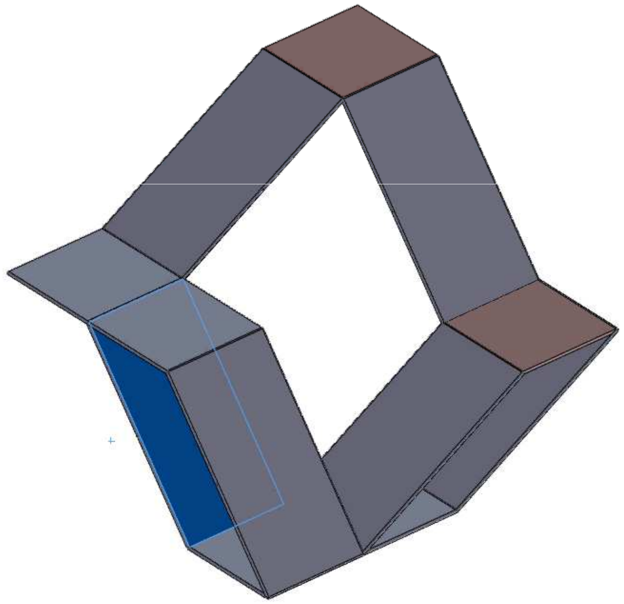
Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

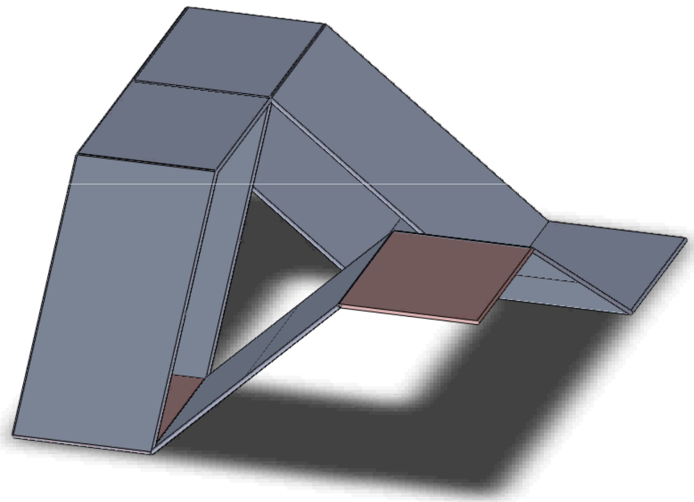
Branching singularities in kinematotropic parallel mechanisms



Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Branching singularities in kinematotropic parallel mechanisms





Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Fig. 3

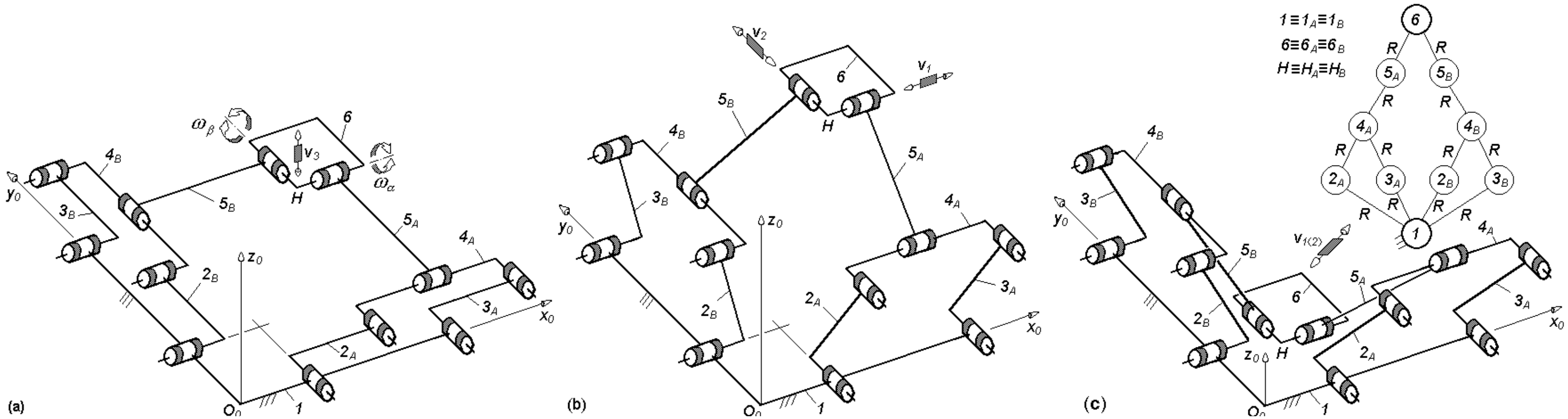
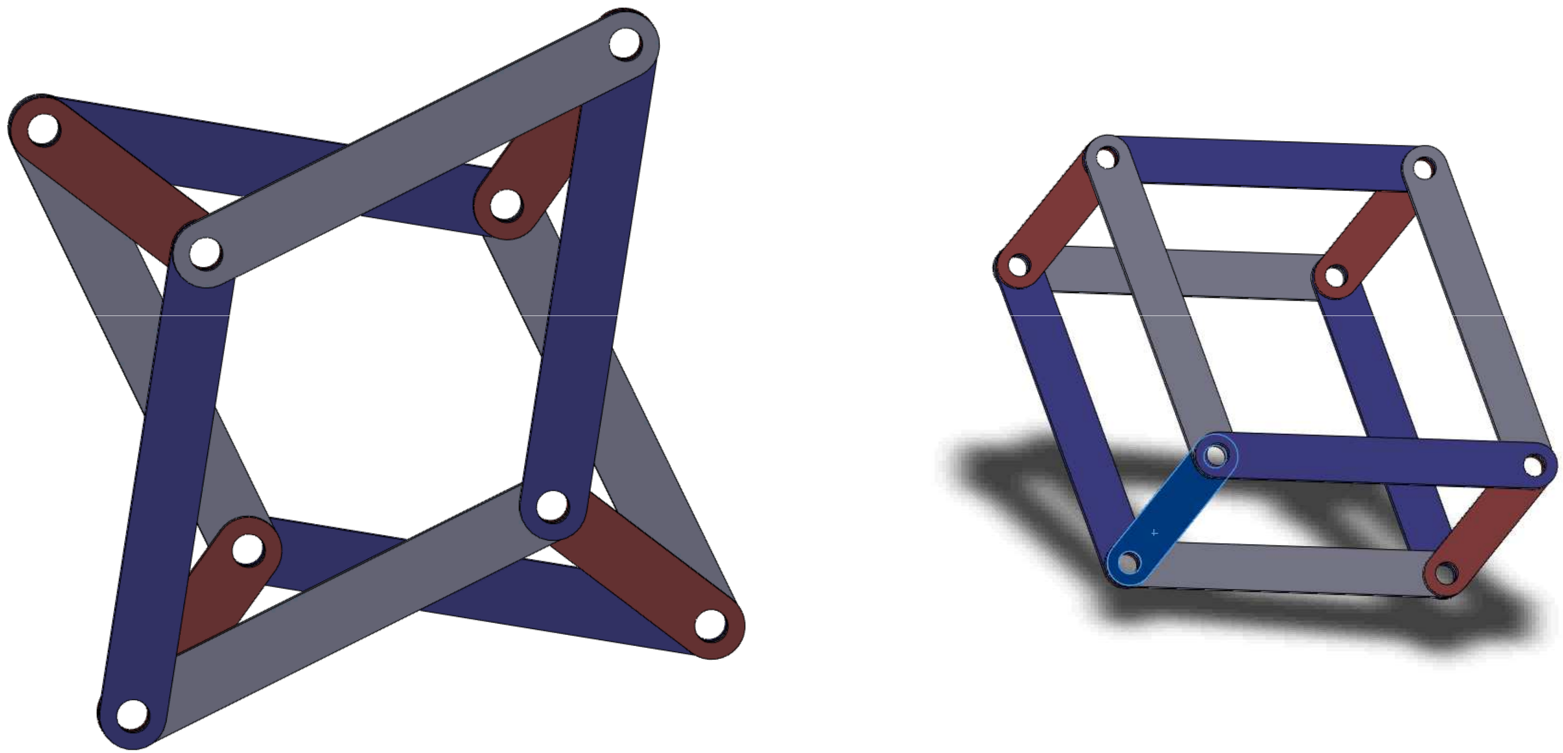


Fig.	Type of BS	Structural parameters		
		Instantaneous parameters in branching singularity in Fig. 3a	Full-cycle parameters in the branch in Fig. 3b	Full-cycle parameters in the branch in Fig. 3c
Fig. 3	BS-C	${}^iS_{G1}={}^iS_{G2}=3$ $({}^iR_{G1})=({}^iR_{G2})=$ $(\mathbf{v}_3, \omega_\alpha, \omega_\beta)$ $({}^iR_F)=(\mathbf{v}_3, \omega_\alpha, \omega_\beta)$ ${}^iS_F=3$, ${}^ir_l^{G1}={}^ir_l^{G2}=2$ ${}^ir_l=4$, ${}^ir=7$ ${}^iM=5$, ${}^iN=11$, ${}^iT=2$	$S_{G1}=S_{G2}=3$ $(R_{G1})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\alpha)$ $(R_{G2})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\beta)$ $(R_F)=(\mathbf{v}_1, \mathbf{v}_2)$ $S_F=2$, $r_l^{G1}=r_l^{G2}=3$ $r_l=6$, $r=10$ $M=2$, $N=8$, $T=0$	$S_{G1}=S_{G2}=3$ $(R_{G1})=(\mathbf{v}_1, \mathbf{v}_2, \omega_\alpha)$ $(R_{G2})=(\mathbf{v}_1, \mathbf{v}_3, \omega_\beta)$ $(R_F)=(\mathbf{v}_1)$ $S_F=1$, $r_l^{G1}=r_l^{G2}=3$, $r_l=6$, $r=11$ $M=1$, $N=7$, $T=0$

Branching singularities in kinematotropic parallel mechanisms



Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

Fig. 4

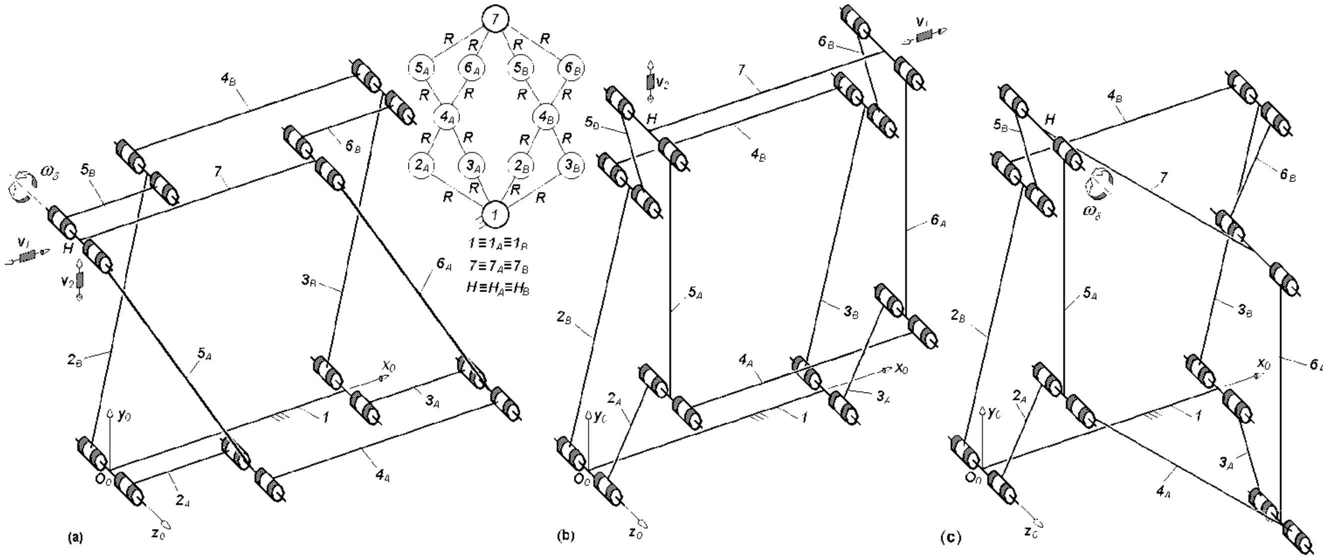


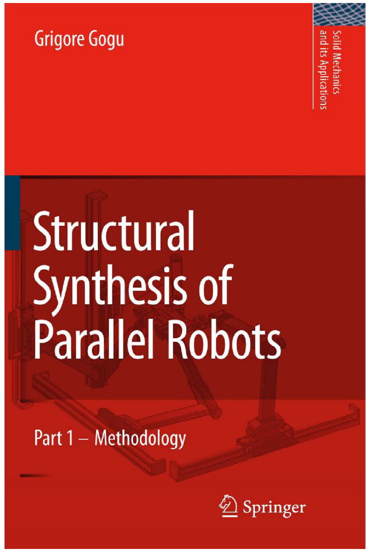
Fig.	Type of BS	Structural parameters		
		Instantaneous parameters in branching singularity in Fig. 4a	Full-cycle parameters in the branch in Fig. 4b	Full-cycle parameters in the branch in Fig. 4c
Fig. 4	BS-D	${}^{i}S_{G1}={}^{i}S_{G2}=3$ $({}^{i}R_{G1})=({}^{i}R_{G2})=$ $(\mathbf{v}_1, \mathbf{v}_2, \boldsymbol{\omega}_\delta)$ $({}^{i}R_F)=(\mathbf{v}_1, \mathbf{v}_2, \boldsymbol{\omega}_\delta)$ ${}^{i}S_F=3,\ {}^{i}r_l^{G1}={}^{i}r_l^{G2}=5$ ${}^{i}r_l=10,\ {}^{i}r=13$ ${}^{i}M=3,\ {}^{i}N=17,\ {}^{i}T=0$	$S_{G1}=S_{G2}=2$ $(R_{G1})=(\mathbf{v}_1, \mathbf{v}_2)$ $(R_{G2})=(\mathbf{v}_1, \mathbf{v}_2)$ $(R_F)=(\mathbf{v}_1,\mathbf{v}_2)$ $S_F=2,\ r_l^{G1}=r_l^{G2}=6$ $r_l=12,\ r=14$ $M=2, N=16, T=0$	$S_{G1}=S_{G2}=2$ $(R_{G1})=(\mathbf{v}_1, \boldsymbol{\omega}_\delta)$ $(R_{G2})=(\mathbf{v}_2, \boldsymbol{\omega}_\delta)$ $(R_F)=(\boldsymbol{\omega}_\delta)$ $S_F=1,\ r_l^{G1}=r_l^{G2}=6,$ $r_l=12,\ r=15$ $M=1, N=15, T=0$

Criteria for structural synthesis of parallel robots: mobility, connectivity, overconstraints and redundancy

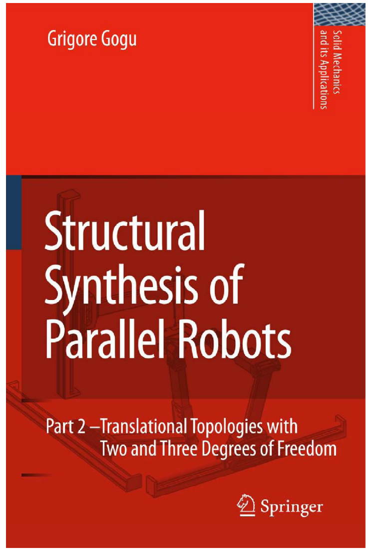
Conclusions

• **The new formulae of mobility, connectivity, overconstraint and redundancy of parallel robots, recently proposed by the author, are usefol for structural synthesis and singularity analysis of parallel mechanisms .**

Gogu, G. Structural Synthesis of Parallel Robots,
Part 1: Methodology,
Springer, 2008, ISBN 978-14020-5102-9, 714 pages



Gogu, G. Structural Synthesis of Parallel Robots,
Part 2: Translational Topologies with Two and Three Degrees of Freedom,
Springer, 2009, ISBN 978-14020-9793-5, 779 pages



• **The bifurcation in constraint singularities can be easily identified by inspection with no need to calculate the Jacobian / augmented Jacobian.**